Discrete Mathematics
Examples of Ch. 2

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Example

Explicitly express the following sets.

1. $A = \{x \mid x \in \varnothing\}$.
2. $B = \{x \in \mathbb{N} \mid 0 \leq x^2 \leq 50\}$.
3. $C = \{x^2 \mid x \in \mathbb{N} \wedge 0 \leq x^2 \leq 50\}$.
4. $D = \{x^2 \in \mathbb{N} \mid 0 \leq x^2 \leq 50\}$.

Example

Let $A = \{1, 2, 3\}$, $B = \{3, 4\}$ and $C = \{A, B\}$. Write down the following sets and their cardinality.

1. $A \cap B$.
2. $B - A$.
3. 2^A.
4. 2^{A-B}.
5. C^2.
6. B^A.
7. $A \times B \times C$.

Example

Prove $\overline{A \cap B} = \overline{A} \cup \overline{B}$ by (1) logical equivalences; (2) membership table; and (3) explanation of $\overline{A \cap B} \subseteq \overline{A} \cup \overline{B}$ and $\overline{A} \cup \overline{B} \subseteq \overline{A \cap B}$.

Example

Wath are the distributive laws of sets?

Example

Give an example of the principle of inclusion-exclusion.

Example

Give a definition in logical expression of following terminologies about function $f : A \longrightarrow B$.

1. One-to-one.
2. Onto.
3. Increasing.
4. Strictly increasing.
5. Domain.
6. Codomain.
7. Range.
8. The image of $S \subseteq A$.
9. The pre-image of $T \subseteq B$.

Example

Give and simply the negative statement of the answer of Q1 and Q2 in the previous question set.

Example

Prove that $f(x) = \sin x$ is not a one-to-one function on $[0, \pi]$.

Example

Prove that $f(x) = x^2$ is not a onto function from $[0, 10]$ to $[0, 101]$.

Example

Let $f(x) = x^2 + 2x + 1$ and $g(x) = \sin x$. Asnwer the following questions.

1. $(f \circ g)(0)$.
2. $(f \circ g)([0, \pi/4])$.
3. Is it correct that $f(S \cup T) = f(S) \cup f(T)$ for all S, T that are contained in the domain of f.

Example

Study the ceiling and floor functions.

Example

Let $f(x) = x^2 + 2x + 1$ and $g(x) = \sin x$. Asnwer the following questions.

1. $(f \circ g)(0)$.
2. $(f \circ g)([0, \pi/4])$.
3. Is it correct that $f(S \cup T) = f(S) \cup f(T)$ for all S, T that are contained in the domain of f.

Example

Prove that the cardinality of the set of all positive even numbers and the set of all positive odd numbers are the same.

Example

Prove that $\mathbb{N}$ and $\mathbb{Z}$ have the same cardinality.

Example

Prove that rational numbers are countable.

Example

Give values of the following equations.

1. $\sum_{i=1}^{5} \sum_{j=1}^{10} i^2 + ij + j^2$.
2. $\sum_{x \in A} x^2$ for $A = \{1, 3, 5, 7\}$.
3. $\prod_{i=1}^{10} 2$.