# An Omni-vision Based Localization Method for Automatic Helicopter Landing Assistance on Standard Helipads ${ }^{\dagger}$ 

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#### Abstract

An omni-vision-based localization method for automatic helicopter landing assistance on a helipad with a circled H -shape is proposed. The landing process includes two stages: approaching and alignment. Three types of image features, circle, line, and point, are used to derive skillfully analytic equations for computing the helicopter height, distance, and orientation with respect to the landing site. Experimental results with good location estimation accuracy are also shown. ${ }^{\dagger}$


Keywords - helicopter landing, helipad, localization, omni-image.

## I. Introduction

Vision- based automatic helicopter landing is useful for safe helicopter aviation or unmanned air vehicle (UAV) applications. This approach uses cameras to acquire environment images and applies image analysis techniques to conduct helicopter localization and flight guidance during the landing process. See Fig. 1 for an illustration.


Figure 1.Illustration of helicopter landing on a helipad with a circled H -shape.

Many vision-based methods[1-3] have been proposed for unmanned helicopter flying, in which traditional projective cameras with fixed fields of view (FOV) were used. It is advantageous to use omni-cameras to enlarge the viewing scope and so speed up the automatic landing process. In this direction, Hrabar and Sukhatme [4] designed an omni-vision system which tries to find the centroid of the H-shape on the helipad to generate commands for guiding the helicopter. Demonceaux, et al. [5] proposed a helicopter posture computation method using catadioptric omni-images of the horizon line to estimate the pitch and roll angles of the helicopter. Bazin et al. [6] extended the method of [5] to estimate the

[^0]yaw angle of the helicopter using the information of vanishing points for automatic flight in urban areas.

Most of the above methods do not investigate the full details of the automatic landing process. Also, when the H-shape or T-shape is used as a landmark for localization, the geometric information existing in the landmark was not fully exploited. In this study, we investigate omni-vision-based helicopter localization techniques as assistance to automatic helicopter landing using the circled H -shape on a helipad as the landmark. The features on the landmark, including point, line, and circle, and their mutual relation properties are used systematically for helicopter location estimation. The landing process is divided into two stages - approaching and aligning.

In the remainder of this paper, in Section II we introduce the idea of the proposed helicopter localization method. Then, in Section III we present the techniques for estimations of the helicopter position, orientation, and height in the proposed two-stage helicopter landing process. Some experimental results are presented in Section IV, followed by conclusions in Section V.

## II. Idea of Proposed Method

The proposed two-stage automatic helicopter landing process includes: (1) approaching - maneuvering the helicopter to approach the helipad, and (2) aligning - maneuvering the helicopter to fly between, and then in alignment with, the two outmost boundary lines of the H -shape. In each stage, certain geometric information contained in single omni-images of the circled H -shape is used to estimate the real-world location information (including the position, orientation, or/and height) of the helicopter. More details are described in the following algorithm. For used notations, see Fig. 2.

## Algorithm 1. Two-stage automatic landing process.

## Stage 1. Approaching the helipad.

1. Take an omni-image $I_{1}$ of the helipad $M$.
2. Find the circular shape $S$ in $I_{1}$ by approximating it as an ellipse $S^{\prime}$ by the Hough transform.
3. Compute the orientation $\theta_{1}$ and distance $d_{1}$ of the helicopter with respect to the center $C$ of the H -shape using the information of the radius of $S$ and the parameters of $S^{\prime}$.
4. Maneuver the helicopter to approach $M$ using the information of $\theta_{1}$ and $d_{1}$ by adjusting the helicopter orientation to decrease the value of $\theta_{1}$ while moving the helicopter forward.
5. Repeat Steps 1 through 4 until the helicopter is at a
pre-defined distance to $M$.

## Stage 2. Aligning with the $\boldsymbol{H}$-shape boundaries.

## 6. Take an image $I_{2}$ of $M$.

7. Find the leftmost and rightmost boundary lines $L_{\ell}$ and $L_{r}$ of the H -shape in $I_{2}$ by approximating them as conic sections $L_{\ell}^{\prime}$ and $L_{r}{ }^{\prime}$ using the Hough transform.
8. Compute the helicopter height $h$ over the helipad using the information of the known distance $d_{\mathrm{m}}$ between $L_{\ell}$ and $L_{r}$.
9. Compute the distances $d_{\ell}$ and $d_{r}$ of the helicopter with respect to $L_{\ell}$ and $L_{r}$, respectively, as well as the orientation $\theta_{2}$ of the helicopter with respect to $L_{\ell}$, using the information of $L_{\ell}{ }^{\prime}, L_{r}{ }^{\prime}$, and $h$.
10. Maneuver the helicopter to fly between $L_{\ell}$ and $L_{r}$ and adjust its orientation to be parallel to $L_{\ell}$ using the information of $d_{t}, d_{r}$, and $\theta_{2}$.
11. Repeat Steps 6 through 10 until $\theta_{2}$ approaches zero (i.e., until it is equal to a pre-selected small value).


Figure 2. Detail of a circled H -shape on a standard helipad.

## III. Proposed Localization Techniques

The techniques proposed for the two stages of the helicopter landing process are described in this section.

### 3.1 Proposed Techniques for Approaching Stage

The major steps in the first stage for approaching the helipad include the following tasks.

## (A) Task A-Circular shape detection

A camera coordinate system (CCS) and an image coordinate system (ICS) are set up on the omni-camera system for use in this study, which includes a traditional perspective camera and a hyperboloidal-shaped mirror, as depicted in Fig. 3. The camera coordinates, denoted as $(X, Y, Z)$, are used to specify the position of each space point in the real world, and the image coordinates, denoted as $(u, v)$, are used to specify the position of the corresponding image point. The perspective camera and the mirror are assumed to be aligned so that the omni-camera system becomes single-viewpointed and that the optical axis of the perspective camera coincides with the mirror axis. Here, the mirror axis is defined as the line going through the mirror surface center and perpendicular to the mirror base plane. The middle point between the perspective camera's lens center $\mathrm{O}_{1}$ and the mirror's focus point $\mathrm{O}_{\mathrm{m}}$ is taken to define the origin $\mathrm{O}_{\mathrm{a}}$ of the CCS. The hyperboloidal mirror shape so may be described by

$$
\frac{R^{2}}{a^{2}}-\frac{Z^{2}}{b^{2}}=-1, \quad R=\sqrt{X^{2}+Y^{2}}, \quad c=\sqrt{a^{2}+b^{2}}
$$

where $a, b$ are two parameters, and $\mathrm{O}_{\mathrm{m}}$ is located at $(0,0$,
$+c)$ and $\mathrm{O}_{1}$ at $(0,0,-c)$ in the CCS.


Figure 3. Camera and image coordinate systems in this study.
According to [8, 9], the relation between the camera coordinates $(X, Y, Z)$ of a space point $P$ and the image coordinates ( $u, v$ ) of its corresponding image point $p$ may be described by the following equalities, where $\alpha$ and $\beta$ are two space angles as illustrated in Fig. 4:

$$
\begin{align*}
& \tan \alpha=\frac{\left(b^{2}+c^{2}\right) \sin \beta-2 b c}{\left(b^{2}-c^{2}\right) \cos \beta}  \tag{1}\\
& \cos \beta=r / \sqrt{r^{2}+f^{2}}  \tag{2}\\
& \sin \beta=f / \sqrt{r^{2}+f^{2}}  \tag{3}\\
& \tan \alpha=(Z-c) / \sqrt{X^{2}+Y^{2}} \tag{4}
\end{align*}
$$

where $r=\sqrt{u^{2}+v^{2}}$ and $f$ is the camera's focal length. We assume that $a, b, c$, and $f$ are known in advance, which may be obtained by proper camera calibration. Also, according to the rotational invariance property of the omni-camera system [10], we have

$$
\begin{align*}
& \cos \theta=X / \sqrt{X^{2}+Y^{2}}=u / \sqrt{u^{2}+v^{2}}  \tag{5}\\
& \sin \theta=Y / \sqrt{X^{2}+Y^{2}}=v / \sqrt{u^{2}+v^{2}} \tag{6}
\end{align*}
$$

where $\theta$ is the angle of space point $P$ with respect to the $X$-axis and is also that of image point $p$ with respective to the $u$-axis. The above equations may be used to derive two equalities describing direct relation between $(u, v)$ and $(X, Y$, $Z$ ) as follows [8-10]:

$$
\begin{align*}
& u=\frac{X f\left(b^{2}-c^{2}\right)}{\left(b^{2}+c^{2}\right)(Z-c)-2 b c \sqrt{(Z-c)^{2}+X^{2}+Y^{2}}}  \tag{7}\\
& v=\frac{Y f\left(b^{2}-c^{2}\right)}{\left(b^{2}+c^{2}\right)(Z-c)-2 b c \sqrt{(Z-c)^{2}+X^{2}+Y^{2}}} \tag{8}
\end{align*}
$$

When the helicopter is not too close to the helipad, according to Wu and Tsai [7], the circle $S$ enclosing the H-shape, though irregular in shape when appearing in a given omni-image $I$, may be approximated well as an ellipse $S^{\prime}$, and the length of the major axis of the extracted ellipse gives a hint for computing the distance of the helicopter to the H-shape. Accordingly, in this study we use the Hough transform technique to detect $S^{\prime}$ in $I$ after $I$ is processed into an edge-point image using the Sobel edge detection operator. Specifically, the coordinates ( $u_{C}, v_{C}$ ) of the center $C^{\prime}$ of the detected ellipse $S^{\prime}$ as well as the values $U$ and $V$ of the major and minor axes of $S^{\prime}$ may be obtained as the output of the Hough transform process for use in helicopter orientation and distance computations described next.
(B) Task B - orientation and distance computation

While referring to Fig. 4 which is a top view of the omni-camera system and the circular shape $S$, let the known radius of $S$ be denoted by $R_{S}$ and let the camera coordinates of the center $C$ of $S$ be denoted by $\left(X_{C}, Y_{C}, Z_{C}\right)$. In the landing process, it is assumed that the omni-camera is looking downward, so that the normal vector of the helipad is parallel to the optical axis of the omni-camera. We will now derive the distance and orientation of the helicopter to the helipad, or more precisely in a reverse way, the distance $d_{1}$ and orientation $\theta_{1}$ of the center $C$ of the circle with respect to the CCS on the helicopter. $d_{1}$ and $\theta_{1}$ can then be used for maneuvering the helicopter to approach $S$ (Step 4 of Algorithm 1).

First, we know that the projection of the center $C$ of $S$ on an image $I$ is just the center $C^{\prime}$ of the detected ellipse $S^{\prime}$ whose coordinates $\left(u_{C}, v_{C}\right)$ can be obtained by the Hough transform process as mentioned previously. Then, according to the rotational invariance property described by (5) and (6) we can get the desired parameter $\theta_{1}$, which is both the angle of $S^{\prime}$ with respect to the $u$-axis and the angle of $S$ with respect to the $X$-axis, as:

$$
\begin{align*}
\theta_{1} & =\cos ^{-1}\left[X_{C} / \sqrt{X_{C}{ }^{2}+Y_{C}^{2}}\right]
\end{align*}=\cos ^{-1}\left[u_{C} / \sqrt{u_{C}^{2}+v_{C}{ }^{2}}\right] ; \text { or (9) }
$$

As to $d_{1}$, to make the derivation of it easier, we rotate $S^{\prime}$ by the angle of $\theta_{1}$, with the rotation result being shown in the image plane depicted in Fig. 5. Then, the rotated $S^{\prime}$ may be described by the following equation:

$$
\begin{equation*}
\frac{\left(u^{\prime}-u_{C}{ }^{\prime}\right)^{2}}{U^{2}}+\frac{v^{\prime 2}}{V^{2}}=1 \tag{11}
\end{equation*}
$$

where the new image coordinates are described by ( $u^{\prime}, v^{\prime}$ ) and the new ellipse center is located at $\left(u_{C}{ }^{\prime}, 0\right)$. Furthermore, let the two endpoints of the major axis of $S^{\prime}$ be denoted as $p_{\alpha}$ and $p_{\beta}$, and their corresponding space points be denoted as $P_{\alpha}$ and $P_{\beta}$, respectively. Then, by the rotational invariance property again, it can be figured out that the following side proportionality property is truth as illustrated by Fig. 5:

$$
\begin{equation*}
\left\|\overline{\mathrm{O}_{\mathrm{i}} \mathrm{C}}\right\| /\left\|\overline{\mathrm{O}_{\mathrm{i}} C^{\prime}}\right\|=\left\|\overrightarrow{P_{\alpha} P_{\beta}}\right\| /\left\|\overline{p_{\alpha} p_{\beta}}\right\| \tag{12}
\end{equation*}
$$

where $\mathrm{O}_{\mathrm{i}}$ is the origin of the ICS (also the origin of the CCS seen from the top view), $\left\|\overline{\mathrm{O}_{\mathrm{i}}}\right\|$ is the desired real-world distance $d_{1}$ on the floor from $C$ to the camera, $\left\|\overline{\mathrm{O}_{i} C^{\prime}}\right\|$ is the image distance from $C^{\prime}$ to $\mathrm{O}_{\mathrm{i}}$ which equals $\sqrt{u_{c}{ }^{2}+v_{c}{ }^{2}}$, $\left\|\overline{P_{\alpha} P_{\beta}}\right\|$ is the diameter value $2 R_{s}$ of $S$, and $\left\|\overline{p_{\alpha} p_{\beta}}\right\|$ is the length $U$ of the major axis of $S^{\prime}$. Accordingly, $d_{1}$ can be computed finally from (12) as:

$$
\begin{equation*}
d_{1}=\frac{2 R_{s}}{U} \sqrt{u_{c}^{2}+v_{c}^{2}} . \tag{13}
\end{equation*}
$$

### 3.2 Proposed Techniques for Aligning Stage

The major steps in the second stage for the helicopter to align with the outmost boundary lines of the H -shape include the following three tasks.

## (A) Task A - Boundary line detection

Based on some optics and geometry of the omni-camera, detection of an outmost boundary line $L$ (either of $L_{\ell}$ or $L_{r}$ ) of the H -shape in an omni-image $I$ is accomplished in this study
first by deriving an equation to describe the projection $L^{\prime}$ of $L$ in $I$, which is a conic section, as will proved later. The derivation is conducted in a novel way in this study so that the resulting equation becomes simple and analytic, facilitating design of an uncomplicated Hough transform algorithm to extract $L^{\prime}$ and computation of the helicopter location in a faster speed for practical applications.


Figure 4. Top view of image plane and circular shape S illustrating side proportionality relation between approximating ellipse $S^{\prime}$ and circular shape $S$.


Figure 5.Illustration of a space line projected onto the image plane.
In more detail, as shown in Fig. 6, suppose that the boundary line $L$ has a specific point $P_{0}$ located at camera coordinates ( $X_{0}, Y_{0}, Z_{0}$ ), and let $P$ be an arbitrary point on $L$ with camera coordinates $(X, Y, Z)$. Then $P$ and $P_{0}$ together form a vector $V_{0}=\left(X-X_{0}, Y-Y_{0}, Z-Z_{0}\right)$. Also, let the direction vector of $L$ be denoted as $V_{L}=\left(d_{X}, d_{Y}, d_{Z}\right)$. Then, by the fact that $V_{0}$ and $V_{L}$ are parallel, we get the equality $V_{0}=$ $\lambda V_{L}$ which leads to

$$
\begin{equation*}
(X, Y, Z)=\left(X_{0}+\lambda d_{X}, Y_{0}+\lambda d_{Y}, Z_{0}+\lambda d_{Z}\right) \tag{14}
\end{equation*}
$$

where $\lambda$ is a parameter.
Also, let $Q$ be the space plane going through both the line $L$ and the mirror base center $\mathrm{O}_{\mathrm{m}}$ which is located at camera coordinates $(0,0,+c) ; N_{q}$ be the normal vector of $Q$ with components $(l, m, n)$; and $P^{\prime}$ be an arbitrary point on $Q$ with camera coordinates $(X, Y, Z)$. Then, $P^{\prime}$ and $\mathrm{O}_{\mathrm{m}}$ together form a vector $V_{\mathrm{m}}=(X-0, Y-0, Z-c)=(X, Y, Z-c)$ which is perpendicular to $N_{q}$, so that the inner product of $V_{\mathrm{m}}$ and $N_{q}$ becomes zero, leading to the following equality:

$$
\begin{equation*}
l X+m Y+n(Z-c)=0 \tag{15}
\end{equation*}
$$

which is equivalent to

$$
\begin{equation*}
Z-c=-(l X+m Y) / n . \tag{16}
\end{equation*}
$$

Now we want to derive an equation for describing the projection of the space line $L$ on the image, which also expresses the relation between the camera coordinates ( $X, Y, Z$ ) of a space point $P$ on $L$ and the image coordinates $(u, v)$ of the image point $p$ corresponding to $P$. Note that $P$ is also on plane $Q$. Combining (4) through (6) and (16), we get

$$
\begin{align*}
& \tan \alpha=\frac{Z-c}{\sqrt{X^{2}+Y^{2}}}=-\left[\frac{l X}{\sqrt{X^{2}+Y^{2}}}+\frac{m Y}{\sqrt{X^{2}+Y^{2}}}\right] / n \\
= & -\left[\frac{l u}{\sqrt{u^{2}+v^{2}}}+\frac{m v}{\sqrt{u^{2}+v^{2}}}\right] / n=-\frac{(l u / n)+(l v / n)}{\sqrt{u^{2}+v^{2}}} . \tag{17}
\end{align*}
$$

Also, substituting (2) and (3) into (1), we can get

$$
\begin{equation*}
\tan \alpha=\frac{\left(b^{2}+c^{2}\right) f-2 b c \sqrt{f^{2}+u^{2}+v^{2}}}{\left(b^{2}-c^{2}\right) \sqrt{u^{2}+v^{2}}} \tag{18}
\end{equation*}
$$

Equating (17) and (18) followed by some reductions leads to
$\left(A^{2}-D^{2}\right) u^{2}+2 A B u v+\left(B^{2}-D^{2}\right) v^{2}+2 A C u+2 B C v+E=0(19)$ where

$$
\begin{equation*}
A=\frac{l}{n}, B=\frac{m}{n}, C=\frac{\left(b^{2}+c^{2}\right) f}{\left(b^{2}-c^{2}\right)}, D=\frac{2 b c}{\left(b^{2}-c^{2}\right)}, E=C^{2}-D^{2} f . \tag{20}
\end{equation*}
$$

Multiplying (19) by $n^{2}$, we get:
$\left(l^{2}-n^{2} D^{2}\right) u^{2}+2 l m u v+\left(m^{2}-n^{2} D^{2}\right) v^{2}+2 \ln C u+2 m n C v+n^{2} E=0$.(21)
Eq. (20) or (21) shows that the projection of a space line on an image is a conic section curve. And (20) shows that the coefficients of the equation may be described indirectly in terms of the parameters of the normal $N_{q}=(l, m, n)$ of plane $Q$. These coefficients actually are related to the elements of the direction vector $V_{L}=\left(d_{X}, d_{Y}, d_{Z}\right)$ of $L$ by the equality

$$
\begin{equation*}
N_{q} \cdot V_{L}=(l, m, n) \cdot\left(d_{X}, d_{Y}, d_{Z}\right)=l d_{X}+m d_{Y}+n d_{Z}=0 \tag{22}
\end{equation*}
$$

because $N_{q}$ and $V_{L}$ are perpendicular, where "." means the inner product for vectors. Furthermore, Eq. (19) has a good property that the parameters $l, m$, and $n$ are confined to appear in just two variables $A$ and $B$, as shown by (20). This facilitates the extraction of the conic section curve by a simple technique using a 2D Hough transform described in the following.
Algorithm 2. Extraction of conic-section projection of a space line on an omni-image by Hough transform.
Input: an omni-image $I$ which includes the projection $L^{\prime}$ of a space line $L$ on $I$.
Output: the two parameter $A$ and $B$ in the conic-section description, Eq. (19), of $L^{\prime}$.

## Steps:

1. Extract the points of $L^{\prime}$ out of $I$ by thresholding and edge detection to form a new edge-point image $I^{\prime}$.
2. Set up a 2D Hough space with parameters $A$ and $B$ and set all cell values in the space to be zero.
3. For each point in $I^{\prime}$ at coordinates $(u, v)$ and for each cell at parameters $(A, B)$, if $u, v, A$, and $B$ satisfy Eq. (19), then increment the cell value by one.
4. Detect the peak cell value in the Hough space and take the parameters $(A, B)$ of the cell with the peak value as output to draw a conic-section curve described by (19).

In the sequel, whenever a conic section described by (19) is mentioned, we assume that the two parameter values $A$ and $B$ in (19) have been obtained by Algorithm 2 above. This means that the ratios $l / n$ and $m / n$ are also known because $l / n$ $=A$ and $m / n=B$ according to (20).

## (B) Task B - Height computation

Suppose we are dealing with one of the two outmost boundary lines $L_{\ell}$ and $L_{r}$ of the H -shape and let the line be denoted as $L$. We want to find a reference point, which we propose to be the minimum-distance point $P_{\min }$ on $L$ to the
origin $\mathrm{O}_{\mathrm{a}}$ of the camera coordinate system, as illustrated by Fig. 7. This point $P_{\min }$ will be used later for the purpose of helicopter height computation here.

To find $P_{\text {min }}$ on $L$, we know that $L$ is on the floor, so $L$ is parallel to the $X-Y$ plane of the camera coordinate system. Consequently, the component $d_{Z}$ of the direction vector $V_{L}=$ ( $d_{X}, d_{Y}, d_{Z}$ ) of $L$ is zero, and Eq. (22), which is $l d_{X}+m d_{Y}+$ $n d_{Z}=0$, may be transformed into

$$
\begin{equation*}
d_{X} /(m / n)=-d_{Y} /(l / n)=s \tag{23}
\end{equation*}
$$

where $s$ is a new parameter. Combining (14) and (23), we get a parametric equation for $L$ as

$$
\begin{equation*}
(X, Y, Z)=\left(X_{0}+\frac{m}{n} s \lambda, Y_{0}-\frac{l}{n} s \lambda, Z_{0}+d_{Z} \lambda\right) \tag{24}
\end{equation*}
$$

where $\left(X_{0}, Y_{0}, Z_{0}\right)$ specify the coordinates of point $P_{0}$ on $L$ yet to be determined. Since $L$ is on the landing floor with the helicopter being at a certain height $h$ which we want to find out here, we have $Z=-h$ for all points on $L$, and so (24) above may be rewritten as

$$
\begin{equation*}
(X, Y, Z)=\left(X_{0}+m t / n, Y_{0}-l t / n,-h\right) \tag{25}
\end{equation*}
$$

where $t=s \lambda$. The distance from the camera coordinate system origin $\mathrm{O}_{\mathrm{a}}$ at $(0,0,0)$ to an arbitrary point on $L$ at $(X, Y, Z)$ therefore is

$$
d(t)=\|(X, Y, Z)-(0,0,0)\|=\sqrt{\left(X_{0}+m t / n\right)^{2}+\left(Y_{0}-l t / n\right)^{2}+h^{2}} .
$$

And the minimum distance $d_{\text {min }}$ from $\mathrm{O}_{\mathrm{a}}$ to $L$ may be obtained by taking the derivative of the square of $d(t)$ above, setting it to be zero, and solving the resulting equation which is of the following form:

$$
\begin{equation*}
2\left(X_{0}+m t / n\right)(m / n)+2\left(Y_{0}-l t / n\right)(-l / n)=0 . \tag{26}
\end{equation*}
$$

The solution $t_{\min }$ of (26) above is

$$
\begin{equation*}
t_{\min }=\left(-m X_{0} / n+l Y_{0} / n\right) /\left[(m / n)^{2}+(l / n)^{2}\right] . \tag{27}
\end{equation*}
$$



Figure 6. Finding minimum-distance point $P_{\min }$ on a boundary line.
To decide the values of $X_{0}$ and $Y_{0}$ in the above equality, we may regard $L$ as a line with an infinite length and going through the space plane $Q_{0}$ described geometrically by the equation $X=0$. We then take the intersection point of $L$ and $Q_{0}$ to be the point $P_{0}$, which has coordinates ( $X_{0}, Y_{0}, Z_{0}$ ) with $X_{0}=0$ and $Z_{0}=-h$. Accordingly, (16) may be reduced to be

$$
\begin{equation*}
Y_{0}=n(h+c) / m, \tag{28}
\end{equation*}
$$

so that (27) becomes

$$
\begin{equation*}
t_{\min }=(l / m)(h+c) /\left[(m / n)^{2}+(l / n)^{2}\right] \tag{29}
\end{equation*}
$$

Therefore, the coordinates $\left(X_{\text {min }}, Y_{\text {min }}, Z_{\text {min }}\right)$ of the mini-mum-distance point $P_{\text {min }}$ on $L$ to $\mathrm{O}_{\mathrm{a}}$ may now be calculated from (26) and (29) to be

$$
\begin{align*}
\left(X_{\min }, Y_{\min }, Z_{\min }\right) & =\left(\frac{(l / n)(h+c)}{(m / n)^{2}+(l / n)^{2}}, \frac{(m / n)(h+c)}{(m / n)^{2}+(l / n)^{2}},-h\right) \\
& =\left(\frac{A(h+c)}{A^{2}+B^{2}}, \frac{B(h+c)}{A^{2}+B^{2}},-h\right) \tag{30}
\end{align*}
$$

where $A=l / n, B=m / n$ are known values as mentioned previously, of the coefficients of the projection $L^{\prime}$ of $L$ on the image described by (19) and (20). Accordingly, we can get from (30) the following values for the coordinates of the minimum-distance points $P_{\min }{ }^{1}$ and $P_{\min }{ }^{2}$ of $L_{1}=L_{\ell}$ and $L_{2}=$ $L_{r}$, respectively, with respect to $\mathrm{O}_{\mathrm{a}}$ :

$$
\begin{align*}
& \left(X_{\min }^{1}, Y_{\min }^{1}, Z_{\min }^{1}\right)=\left(\frac{A_{1}(h+c)}{A_{1}^{2}+B_{1}^{2}}, \frac{B_{1}(h+c)}{A_{1}^{2}+B_{1}^{2}},-h\right)  \tag{31}\\
& \left(X_{\min }^{2}, Y_{\min }^{2}, Z_{\min }^{2}\right)=\left(\frac{A_{2}(h+c)}{A_{2}^{2}+B_{2}^{2}}, \frac{B_{2}(h+c)}{A_{2}^{2}+B_{2}^{2}},-h\right), \tag{32}
\end{align*}
$$

where $A_{1}$ and $B_{1}$ are the coefficients of the projection $L_{1}{ }^{\prime}=$ $L_{\ell}^{\prime}$ of $L_{1}=L_{\ell}$ on the image described by (19) and (20), and $A_{2}$ and $B_{2}$ are those of the projection $L_{2}{ }^{\prime}=L_{r}{ }^{\prime}$ of $L_{2}=L_{r}$. By the fact that $L_{\ell}$ and $L_{r}$ are parallel and are separated with a known distance $d_{\mathrm{m}}$, it is not difficult to figure out that the points $P_{\min }{ }^{1}, P_{\min }{ }^{2}$, and the projection point of $\mathrm{O}_{\mathrm{a}}$ on the landing floor are all on an identical line which is perpendicular both to $L_{\ell}$ and to $L_{r}$, so that the distance between $P_{\text {min }}{ }^{1}$ and $P_{\text {min }}{ }^{2}$ on the landing floor is just $d_{\mathrm{m}}$, leading to the following equality:

$$
\left(\frac{A_{1}(h+c)}{A_{1}^{2}+B_{1}^{2}}-\frac{A_{2}(h+c)}{A_{2}{ }^{2}+B_{2}^{2}}\right)^{2}+\left(\frac{B_{1}(h+c)}{A_{1}^{2}+B_{1}^{2}}-\frac{B_{2}(h+c)}{A_{2}{ }^{2}+B_{2}^{2}}\right)^{2}=d_{\mathrm{m}}^{2}(33)
$$

Furthermore, parallelism of $L_{\ell}$ and $L_{r}$ means that their directional vectors are identical, which we assume to be ( $d_{\mathrm{X}}, d_{\mathrm{Y}}$, $d_{\mathrm{Z}}$ ) with $d_{\mathrm{Z}}=0$. So, if ( $l_{1}, m_{1}, n_{1}$ ) is the normal vector of the space plane including $L_{1}=L_{\ell}$ and $\mathrm{O}_{\mathrm{m}}$ and ( $l_{2}, m_{2}, n_{2}$ ) is that for the plane including $L_{2}=L_{r}$ and $\mathrm{O}_{\mathrm{m}}$, then according to (23) we get

$$
\begin{equation*}
\left(l_{1} / n_{1}\right) /\left(l_{2} / n_{2}\right)=\left(m_{1} / n_{1}\right) /\left(m_{2} / n_{2}\right) \tag{34}
\end{equation*}
$$

or $A_{1} / A_{2}=B_{1} / B_{2}$ because $A_{1}=l_{1} / n_{1}, A_{2}=l_{2} / n_{2}, B_{1}=m_{1} / n_{1}, B_{2}$ $=m_{2} / n_{2}$ according to (20). Define $\lambda$ as

$$
\begin{equation*}
\lambda=A_{1} / A_{2}=B_{1} / B_{2} \tag{35}
\end{equation*}
$$

which is equivalent to

$$
\begin{equation*}
A_{1}=\lambda A_{2}, B_{1}=\lambda B_{2} \tag{36}
\end{equation*}
$$

Then, substituting (36) into (33) to eliminate the terms of $A_{1}$ and $B_{1}$, we get

$$
\begin{equation*}
h=\lambda d_{\mathrm{m}} /\left[(1-\lambda) \sqrt{A_{2}^{2}+B_{2}^{2}}\right]-c \tag{37}
\end{equation*}
$$

And using (36) again, we can rewrite the above equation in terms of the known values of $A_{1}, B_{1}, A_{2}$, and $B_{2}$ in two ways to describe finally the desired helicopter height $h$ as
$h=\frac{A_{1} d_{\mathrm{m}}}{A_{2}-A_{1}} \sqrt{A_{2}{ }^{2}+B_{2}{ }^{2}}-c=\frac{B_{1} d_{\mathrm{m}}}{B_{2}-B_{1}} \sqrt{A_{2}{ }^{2}+B_{2}{ }^{2}}-c$.

## (C) Task $\mathbf{C}-$ Orientation and height computation

From (31), we know that the helicopter position on the landing floor with respect to the reference point $P_{\min }$ on $L_{1}=$ $L_{\ell}$ is described by $\left(X_{\min }{ }^{1}, Y_{\min }{ }^{1}\right)=\left(A_{1}(h+c) /\left(A_{1}{ }^{2}+B_{1}^{2}\right)\right.$, $\left.B_{1}(h+c) /\left(A_{1}^{2}+B_{1}^{2}\right)\right)$. And the desired distance $d_{\ell}$ of the helicopter with respect to $L_{1}=L_{\ell}$ on the landing floor is just

$$
\begin{equation*}
d_{\ell}=\sqrt{\left(X_{\min }^{1}\right)^{2}+\left(Y_{\min }^{1}\right)^{2}}=(h+\mathrm{c}) / \sqrt{A_{1}^{2}+B_{1}^{2}} \tag{39}
\end{equation*}
$$

Similarly, the desired distance of the helicopter to $L_{2}=L_{r}$ on the landing floor is

$$
\begin{equation*}
d_{r}=\sqrt{\left(X_{\min }^{2}\right)^{2}+\left(Y_{\min }^{2}\right)^{2}}=(h+\mathrm{c}) / \sqrt{A_{2}^{2}+B_{2}^{2}} \tag{40}
\end{equation*}
$$

As to the helicopter orientation seen from the top view, it is taken to be the angle $\theta_{2}$ of line $L=L_{\ell}$ with respect to the
$X$-axis of the CCS, which may be computed by $\cos \theta_{2}=$ $\left(V_{L} \cdot V_{X}\right) /\left(\left\|V_{L}\right\| \times\left\|V_{X}\right\|\right)$ where $V_{L}$ and $V_{X}$ are the direction vectors of $L=L_{\ell}$ and the $X$-axis, respectively. We know that $V_{X}=(1,0,0)$. And $V_{L}=\left(d_{X}, d_{Y}, d_{Z}\right)$ may be computed to be the unit vector from the point $P_{0}$ found previously on $L$ (see discussions in Section B above) to the point $P_{\text {min }}$ also on $L$ (see Fig. 7) in the following way:

$$
V_{L}=\frac{\left(X_{\min }^{1}-X_{0}, Y_{\min }^{1}-Y_{0}, Z_{\min }^{1}-Z_{0}\right)}{\left\|\left(X_{\min }^{1}-X_{0}, Y_{\min }^{1}-Y_{0}, Z_{\min }^{1}-Z_{0}\right)\right\|}
$$

where

$$
\begin{gathered}
\left(X_{0}, Y_{0}, Z_{0}\right)=(0, n(h+c) / m, h)=\left(0, \quad(h+c) / B_{2},-h\right) \\
\left(X_{\min }{ }^{2}, Y_{\min }{ }^{2}, Z_{\min }{ }^{2}\right)=\left(\frac{A_{2}(h+c)}{A_{2}{ }^{2}+B_{2}{ }^{2}}, \frac{B_{2}(h+c)}{A_{2}{ }^{2}+B_{2}{ }^{2}},-h\right)
\end{gathered}
$$

Consequently, we get, after some reductions using the above formulas, the desired $\theta$ as

$$
\begin{equation*}
\theta_{2}=\cos ^{-1}\left(B / \sqrt{A^{2}+B^{2}}\right) . \tag{41}
\end{equation*}
$$

## IV. EXPERIMENTAL RESULTS

Experiments in a small-scaled simulation environment as shown in Fig. 9 have been conducted on a simulated helipad with a reduced scale of $1 / 100$. A series of images of the helipad with different postures was taken. The real distance and orientation of each posture were measured manually as reference data. Then, each image was processed to extract the circular shape and the derived formulas were used to compute the distance and orientation of the H-shape center with respect to the helicopter, which is regarded to be located at the origin of the CCS. Some results are shown in Fig. 8 with 8 (a) being the original image and $8(b)$ the result of circular shape extraction. The computation results together with the reference data are shown in Figs. 9 and 10. The error of a computed orientation is defined as the difference between the real and the computed ones, and the error ratio of $a$ computed distance as the ratio of the absolute difference between the real and the computed distances over the real one. From the figures, we can see that all the computed orientations have errors smaller than 2.5 degree and that all the computed distances have error ratios smaller than \%. Such results may be considered to be within allowable tolerance for the helipad approaching stage in which the helicopter is maneuvered at a farther distance from the helipad.

## V. Conclusions

An omni-vision localization method for assisting automatic helicopter landing on a helipad with a circled H-shape has been proposed. The proposed automatic landing process is divided into two stages: helipad approaching, aligning with the outmost boundary lines of the H -shape, and docking on the center of the H -shape. In each stage, proper geometric features of point, line, and circle on the landmark are used as hints for deriving formulas for computing the location (including the height, distance, or/and orientation) of the helicopter with respect to the landing site. The proposed techniques are all based on the use of single-view omni-images taken by a hyperboloidal omni-camera, in contrast with traditional methods using multiple views taken by projective cameras. Experimental results with good location estimation precisions have also been shown.

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Figure 7.A view of simulated helipad used in experiments.

(a)


Figure 8. An example of experimental results. (a) An omni-image taken in the first stage. (b) Detected helipad shape. (c) Extracted bpundary lines of helipad.


Figure 9. Error ratios of estimated helicopter distances $d_{1}, d_{r}, d_{l}$, and heights $h$ with respect to real helicopter distances.


Figure 10. Error ratios of estimated helicopter orientations $\theta_{1}$ and $\theta_{2}$ with respect to real helicopter distances.


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