Thick Circle Detection by Hough Transform

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Abstract

A new extension of the Hough transform for detecting thick circles, called circular bands, in images is proposed in this study. The conventional method to detect a circular band in an image using the Hough transform usually requires the preprocessing step of edge detection or thinning before the transform can be performed on the image. This causes loss of useful position relationship existing among the pixels of a circular band, and requires certain costly postprocessing steps to recover the band in the original image. The proposed method with an image as the direct input removes this shortcoming, requiring neither preprocessing nor postprocessing. Experimental results, showing the feasibility of the proposed approach, are also included.

Key Words: Extension of Hough Transform, Thick Circles, Circular Band, Edge Detection, Gray-scale Hough Transform .

1. Introduction

The Hough transform (HT) [1-3] is suitable for detecting and locating analytic curves such as circle, ellipse, etc. in images. It transforms a binary image into a Hough parameter counting space (HPCS). An HPCS consists of an array of cells, called accumulators, which are the functions of the parameters of analytic curves. For example, the parameters of a circle are the coordinates (a, b) of the center and the length r of the radius, of the circle. In the HT process, co-curvilinear points that have the same set of parameters map to the same accumulator and each point contributes to the corresponding accumulator value by an increment of one to form a local maximum value in the HPCS. By the way of detecting each local maximum value from the HPCS, a set of corresponding parameters can be found and an analytic curve can be detected and located with the set of parameters. A main advantage of the transform is its robustness to noise as well as discontinuities in images. In spite of this advantage it requires the preprocessing step of edge detection or thinning for an input image to become a point pattern with curves of one-pixel width.

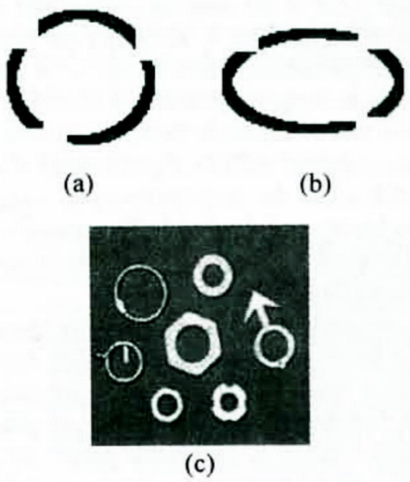

Figure 1. Several kinds of analytic band shapes existing in images. (a) A circular band shape. (b) An elliptical band shape. (c) A real image including hex nuts, washers, etc.

However, in many common images like Fig. 1, there exist shapes consisting of *band-type* arcs, circles, ellipses, etc. Here, a curve segment is said to be of the band-type if the width of the segment is larger than one pixel. For simplification while not losing generality, we focus discussions on detection of circular bands and the results also can be applied to the other analytic curve bands. During edge detection or thinning, a lot of useful position relationship among the pixels of circular bands is lost. In order to recognize the original band shape after the HT, *a prior* knowledge of knowing where the band will appear or the use of a certain costly region growing operation is required. So, the HT is not suitable for direct use to detect circular band shapes in images.

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To improve these shortages of the HT, a new extension of the HT for circular band shape detection is proposed. Both the width and useful position relationship among the pixels of a circular band are considered in the extended HT (EHT). A circular band shape can be considered as a set of connected concentric circles with an identical center point. All the pixels of each circle map to an identical accumulator to form a local maximum value in the 3-dimensional (3D) HPCS, and the set of all concentric circles map to a cluster of connected accumulators with the identical center but successively varying radii to form a *local maximum plateau* in the HPCS. On the other hand, two features may be used to describe a circular band existing in an image: the total number of pixels and the width, of the circular band. The values of these features can be calculated from the local maximum plateau to detect the circular band in the original image. The details are described in Section 3. Although the proposed EHT can detect circular bands directly, the order of memory space complexity and time complexity are the same as that of the circle detection process using the conventional HT (CHT). For the sake of clarity and simplification, the discussions in this study are limited to binary circular band. The results can be extended to gray-scale circular band detection using the gray-scale Hough transform [11].

The remainder of this paper is organized as follows. In Section 2 the use of the CHT to detect the circle is first reviewed. In Section 3 the proposed EHT for detecting circular bands in an image is described in detailed. Experimental results are given in Section 4. Some conclusions are given in Section 5.

2. Review of Conventional Hough Transform to Detect Circles

The HT was originally described in Hough [1]. Duda et al. [2, 3] used it to detect straight lines and curves in pictures. It also has received a lot of improvements [4-14].

The idea of using the CHT to detect a circle can be illustrated by considering a set of co-curvilinear points forming a circle in a binary image. Three parameters which may be used to describe the circle are the coordinates (a, b) of the center and the length r of the radius, of the circle. Accordingly, a circle can be defined by a relation f with the following normal form:

$$f(((a,\ b)\ \ r)\ (x, y)) = (x-a)^2 + (y-b)^2 - r^2 = 0\ . \qquad (1)$$

From Eq. (1), it is easy to show that each point (x_i, y_i) in the image coordinate system is mapped to a series of accumulators $A((a_i,\ b_i),\ r_i)$ in the HPCS. For each pixel (x_i, y_i) satisfying Eq. (1), the content of each of all the mapped accumulators is incremented by one according to the CHT. If the input image includes a circle, a local maximum value in one of the accumulators can be detected from the HPCS. Accordingly, three parameters, $((a_i,\ b_i),\ r_i)$, corresponding to the local maximum value can be found from the HPCS, from which the desired circle can be detected in the input image.

Although the CHT is a powerful method for detecting the presence of circles in binary images, it is not suitable to use it directly to detect circular bands in images.

3. Proposed Extension of Hough Transform for Detecting Circular Bands in Binary Images

In this section, the proposed EHT for circular band detection in images is described in detail. For the sake of clarity and simplification, the discussions are focused on binary circular band shapes. It is possible to extend the result to detect gray-scale circular bands in gray-scale images [11].

3.1. Extension of Hough transform for circular band detection without edge detection or thinning

In conventional use of the Hough transform to detect a circle in a binary image, it needs to perform edge detection or thinning in advance, and so a lot of position information among pixels is lost. The proposed EHT instead detects linear bands directly in binary images without performing the preprocessing step of edge detection or thinning. The algorithm is described as follows.

Algorithm 1. Extension of Hough transform for circular band detection without edge detection or thinning.

Input. A binary image containing white circular bands on black background, on which no edge detection or thinning has been performed.

Output. The values of an array of accumulators $A((a,\ b),\ r)$ in the HPCS.

Steps.

1. Initialize each accumulator content $A((a_j, b_j), r_j)$ to 0.

2. For each white pixel (x_i, y_i),

3. for a_j = 0 to 127 with increment of 1,

4. for b_j = 0 to 127 with increment of 1,

5 set $r_j = \left((x_i - a_j)^2 + (y_i - b_j)^2\right)^{1/2}$, and

6. set accumulator value $A((a_j, b_j), r_j) = A((a_j, b_j), r_j) + 1$.

7. End.

Algorithm 1 directly maps the input binary image into the HPCS without edge detection or thinning. Therefore, the HPCS has more information than that obtained from the process of edge detection or thinning followed by the CHT. To detect a circle, the two methods are the same, but to detect a circular band, the former is more suitable than the latter. Take Fig. 2 as an illustration. First, the use of the EHT to detect a circle is discussed. For each white pixel (x_i, y_i) in the image of Fig. 2(a), Algorithm 1 adds 1 to all mapped accumulators that form the HPCS. All co-curvilinear pixels forming a circle in the original image have common triple parameters, $((a_j, b_j) r_j)$, which represent the coordinates (a_j, b_j) of the center and the length r_j of the radius, of the circle, and each of these pixels increments by one the value of an identical accumulator $A((a_j, b_j), r_j)$ as shown in Fig. 2(b). The final value of $A((a_j, b_j), r_j)$ is equal to the total number of pixels of the circle in the original image and represents the length of the circumference of the circle (assuming that the circle has no gap and that the length of the circumference is measured by the number of points on the circle). It forms a local maximum peak in the 3D HPCS. To make the HPCS clear, three cross sections of the HPCS along the a-axis are shown as 2D (b-r) images in Fig. 2(c). In the second sectional image, a local maximum peak exists at ((*a*, *b*), *r*) = ((63, 63), 20). Using the parameters ((*a*, *b*), *r*), the circle can be detected and located. From the viewpoint of geometric structures, a circular band is composed of many connective concentric circles that have identical coordinates (*a*, *b*) of the center, and successively varying radii, *r*, as shown in Fig. 2(d). After the EHT is performed on the circular band, each concentric circle of the circular band maps to an accumulator whose value is a local maximum, and all of the concentric circles map to a row of successive maximum values in the HPCS, called local maximum plateau, as shown in Fig. 2(e). Then, the sum of all the values of the row of accumulators corresponding to the concentric circles is equal to the total number of pixels in the circular band. On the other hand, each of other possible circular bands which do not fully contain the detected circular band has fewer pixels and the sum of all the values of its corresponding accumulators is smaller.

As an illustration, for the circular band shown in Fig. 2(f), the mapped 3D local maximum plateau is shown in Fig. 2(g) or 2(h), where Fig. 2(g) includes three cross-sectional images along the a-axis, and Fig. 2(h) shows the thresholded images obtained from Fig. 2(g).

3.2. Detecting circular bands in a binary image from HPCS

For each circular band, a local maximum plateau exists in the HPCS from which the circular band can be detected. It is assumed that each circular band existing in the image has a band width larger than one pixel and that the length of the circumference of each concentric circle is larger than a certain value *t*. After the EHT is performed on the band, the local maximum plateau in the HPCS is detected by discarding the accumulators whose values are smaller than the threshold value *t*. For each circular band, the thresholded local maximum plateau forms a 3D region in the HPCS. See Fig. 2(h) for an illustration. As a matter of fact, the total number of pixels of any shape in an image is proportional to its occupied area in the image. So, the total number of pixels of a circular band is proportional to the area S of the circular band in the image (assume that the band has no gap). The value S can be computed as follows:

$$S = \pi * r_2^{\,2} - \pi * r_1^{\,2}, \qquad (2)$$

where r_2 is the outer radius and r_1 is the inner radius, of the circular band. The value S can be used to decide the amount of gaps existing in the circular band. If the ratio of the total number of pixels of the band to the value S is larger, it means that the circular band has fewer gaps. Then, it is more possible that a circular band is detected.

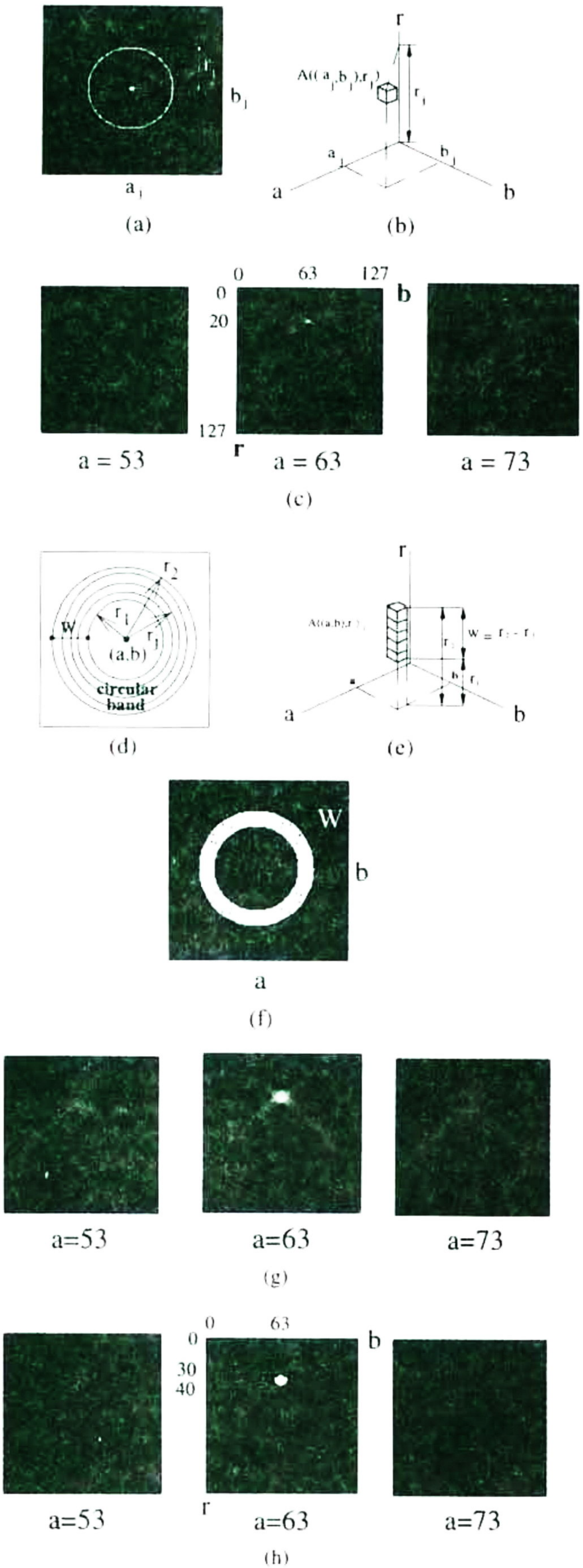

Figure 2. Illustration of using proposed EHT to detect a circular band in a binary image. (a) A circle with center at (a_j, b_j) = (63, 63) and radius r_j = 20 in x-y space. (b) Resulting accumulator $A((a_j, b_j), r_j)$ in the 3D HPCS after the CHT is applied. (c) 3D HPCS shown with multiple 2D images after the CHT is performed on (a); three 2D (b-r) sectional images with a=53, 63 and 73, respectively, are shown; a local maximum value at A((63, 63), 20) in HPCS is found. (d) A circular band, shown as a connective concentric circles, with center at (a, b), inner radius r_1 and outer radius r_2. (e) After the EHT is performed on (d) without edge detection or thinning, all the concentric circles map to a row of connective accumulators in the 3D HPCS. (f) Real image of (d) with (a, b) = (63, 63), r_1 = 30 and r_2 = 40 (i.e.,width = 10). (g) HPCS shown with three 2D (b-r) cross-sectional images with a = 53, 63 and 73, respectively; a cluster of connective accumulators exists in HPCS. (h) Local maximum plateau in HPCS obtained with threshold value 195; in the second image, a row of local maximum values exists at A((63, 63), r) where $30 \leq r \leq 40$.

Now, to detect the possible candidates of the circular band from the information of the accumulators in the thresholded plateau, two features are used to describe a circular band existing in an image: the total number of pixels and the area S (that is the function of the band width), of the circular band. For each thresholded plateau region in the HPCS like the one shown in Fig. 2(h), each row of successive local accumulators along r at each coordinates (a, b) in the plateau region corresponds to a possible band candidate. As discussed previously, the sum of the values of the row of the successive accumulators $A((a, b), r_i)$, where $r_1 \leq r_i \leq r_2$ with r_i and r_2 specifying two endpoints of the plateau region along r, is equal to the total number of pixels of the circular band candidate. During checking the values of all the sums, if a sum is found to be the maximum and greater than a fraction of the value S, it is then decided that a circular band is detected. The coordinates (a, b) specify the center, and r_i and r_2 are taken to be the inner and outer radii, of the detected circular band. The detailed steps for detecting circular bands in a binary image using the HPCS are described in the following algorithm.

Algorithm 2. Detecting circular bands in a binary image using the HPCS.

Input. A binary image including possibly more than one circular bands, a threshold value t (minimum length of the circumference of each circle), and a fractional constant k.

Output. The outer radius r_2, the inner radius r_1, the width $|r_2 - r_1|$, and the center coordinates (a,b), of each detected circular band.

Steps.

1. Perform the EHT of Algorithm 1.
2. Detect all the local maxima $A((a,b),r)$ in the HPCS whose values are larger than t.
3. Perform region growing [15] to find all plateau regions.
4. Check each region. In each region, for each center coordinates (a,b), compute the sum, denoted as m, of the values of the row of the successive accumulators $A((a, b), r_j)$ along the r-axis, where $r_1 \le r_j \le r_2$, and r_1 and r_2 are the two endpoints of the region along the r-axis.
5. For each region, search the local maximum value of m computed in Step 4;

if the local maximum value of m is larger than the fraction k of the area S of the circular band as computed in Eq. (2), then a band is found and go to Step 6; else no band is found and go to Step 7.

6. Output the values of r_2, r_1, $|r_2 - r_1|$, and (a,b) corresponding to the local maximum value m as the desired parameters of the circular band.
7. End.

4. Experimental Results

The EHT algorithm has been implemented on a DIGITAL DEC 3000 Alpha workstation and a set of images has been tested. Due to the limitation of memory size, the experimental images are limited to be binary with a size of 128 x 128 pixels. Several results are shown in Fig. 3 through Fig. 5. Fig. 3(a) is an input image. Fig. 3(b) shows the mapped 3D HPCS viewed as mutilple 2D (b-r) images including two local maximum plateaus (marked by 1 and 2). Fig. 3(c) shows two detected band candidates. Fig. 4(a) is a real image including hex nuts, washers, etc. Fig. 4(b) is the resulting binary image after performing thresholding on Fig. 4(a). Fig. 4(c) shows all the detected bands. Fig. 5(a) is a real image including several kinds of rings. Fig. 5(b) is the resulting binary image after performing thresholding on Fig. 5(a). Fig. 5(c) shows all the detected rings.

5. Conclusions

A new approach to circular band detection using a new extension of the CHT without the preprocessing step edge detection or thinning has been proposed. The approach is robust to handle noise and discontinuity in binary images. The space complexity and the time complexity of the proposed approach are the same as that of the CHT. The CHT can be viewed as a special case of the approach with input circular shapes being of one-pixel width. From the experimental results it was seen that the proposed approach has high potential for applications. For the sake of clarity and simplification, the tested images used in the experiments are limited to be binary images. It is possible to extend the proposed approach to handle the gray-scale images directly [11].

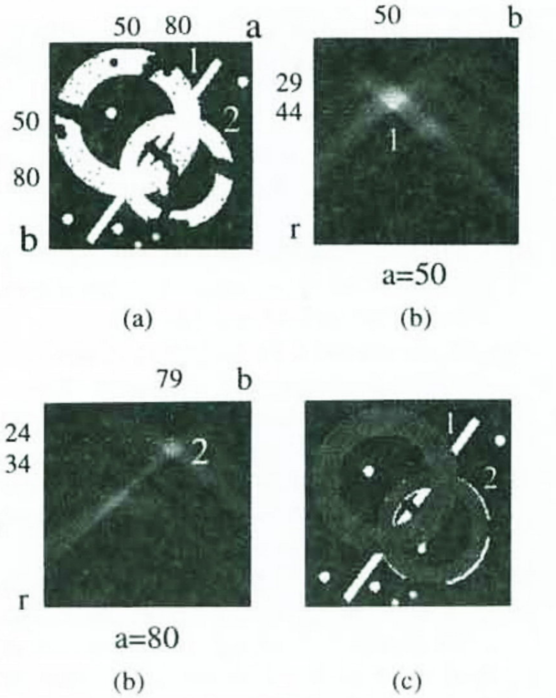

Figure 3. Experimental results of circular band detection in a binary image. (a) Input image including two binary circular bands marked with 1 and 2. (b) After performing the EHT on (a), two clusters of accumulators in the 3D HPCS (a-b-r) are shown with two 2D (b-r) cross-sectional images with a = 50 and 80, respectively. (c) Two circular bands are detected.

(a) (b) (c)

Figure 4. Experimental results of hex nut and washer detection in a real image. (a) Input real image. (b) Resulting binary image after performing thresholding on (a). (c) All detected circular bands.

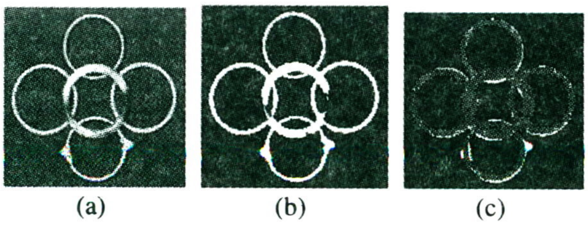

Figure 5. Experimental results of ring detection in a real image. (a) Input real image. (b) Resulting binary image after performing thresholding on (a). (c) All detected circular bands.

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