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Location Estimation and Trajectory Prediction of Moving Lateral Vehicle Using Two Wheel Shapes Information in 2-D Lateral Vehicle Images by 3-D Computer Vision Techniques

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Abstract **– A novel approach to location estimation and trajectory prediction of moving lateral vehicle locations for driving assistance using wheels shape information in single 2-D vehicle images by 3-D computer vision techniques is proposed. The location scheme is supposed to be performed on a vehicle with a camera mounted on the front bumper. An analytical solution is applied to estimate the location of a lateral vehicle. Firstly, the normal vectors of the car wheels in the camera coordinate system as well as the projections of the wheel centers in the image plane are derived. Then orientation angle of the lateral vehicle with respect to the driving vehicle is derived from the two vectors. Next, the image projections of the contact points of the wheels with the ground are detected. Finally, the image positions of the contact points are used to determine the 3D relative position of the moving vehicle with respect to the camera coordinate system by the back-projection principle. After the location of a moving lateral vehicle is obtained, a new approach is proposed to predict the trajectory of a lateral vehicle by using the spatial information of the two wheels in a single image. Real images are tested and experimental results show the effectiveness of the proposed approaches. The results are useful for car driving assistance and vehicle collision avoidance.**

I. INTRODUCTION

A kind of import applications of computer vision techniques are the intelligent transportation systems (ITSs), for example, road following, vehicle detection and collision avoidance. Vehicle detection and location are important for collision avoidance in driving assistance applications. In a traffic environment, there are three views that a driver should take care of: the front view, the rear view, and the lateral view. In recent years, a lot of related studies about vehicle detection and location have been proposed[1-8]. Most researches [1-4] so far focus on vehicle or obstacle detection in the front view for collision avoidance. There are some studies [5-7] about observing the rear view of a vehicle. On the other hand, when driving a vehicle in a certain lane on a road, a driver should also notice the movement of a vehicle in a nearby lane, called a lateral vehicle in the sequel. Keeping alert on the lateral vehicles is necessary when the driver wants to drive into the nearby lane, or when a vehicle in a nearby lane is trying to get into the driver's lane. There are still very few studies on the detection and location of the lateral vehicle under such circumstances. For detection of lateral vehicles, we have proposed a method by using the rear wheel information [8]. In this study, the information of *both* the front and the rear wheels will be used, resulting in a more effective lateral vehicle location as well as trajectory prediction. It is proposed to employ the wheel shape information of the lateral vehicle to estimate the lateral vehicle location. The wheel shapes are captured with a camera mounted on the driver's vehicle. The proposed approach is based on computer vision techniques.

When a vehicle driver changes a driving path from a lane into a nearby one, collision with a lateral vehicle in the nearby lane should be avoided. Hence, the extraction of lateral vehicle information is emphasized in this study. To achieve this goal, the first task is to mount a camera on the right side of the front bumper of the vehicle. An illustration of this camera setup is shown in Fig. 1. The geometric relation of the vehicle being driven, called the *observing vehicle* in the sequel, and a lateral vehicle is described in Fig. 2.

To estimate the lateral vehicle location, an image of the lateral scene, which includes the lateral vehicle, is taken first. This is the start of an image processing stage. The Hough transform [12] is then applied to detect the projected shape of the front and the rear wheels. The equations of the detected ellipses are then computed and employed to infer the normal vectors of the wheels and the projections of the wheel centers. In addition, the normal of the ground is estimated by the cross product of two vectors mentioned above. Then the orientation angle of the lateral vehicle with respect to the observing vehicle is derived from those two vectors. By combining the projection of the wheel center and the ground normal, the contact points of the wheels and the ground are determined. By applying the back-projection rule, the relative positions of the contact points with respect to the observing vehicle are obtained. According to vehicle kinematics, the trajectory of the lateral vehicle can then be predicted from the orientation angles of the wheels and the distance of the front and the rear wheel centers.

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A major contribution of this study is that the wheels shape information of the lateral vehicle is utilized for lateral vehicle location estimation with no restrictive condition assumption. Neither the radius of the wheels need be known in advance, nor any special mark need be put on the vehicle. By the estimation of the vehicle location, a lot of useful information is gathered and can be used for lateral vehicle motion estimation, moving path prediction, collision avoidance, etc. These applications are important for autonomous vehicle navigation as well as car driving assistance, in the circumstances with multiple moving vehicles in multiple lanes.

In the remainder of this paper, after giving a more detailed definition of the lateral vehicle location estimation problem we are dealing with in Section II, we will derive the analytic equations for determining the orientation and the position of a lateral vehicle in Sections III and IV, respectively. In Section V, we will derive the equation for predicting the trajectory of the lateral vehicle using the information of the vehicle position and orientation. Finally, we give some experimental results in Section VI, and some concluding remarks in the last section.

II. PROBLEM DEFINITION

Given an image of a moving lateral vehicle, we want to derive the relative location of the vehicle with respect to the observing vehicle. As shown in Fig. 2, the relative location of a lateral vehicle can be described by the positions of two contact points, denoted as P_1 and P_2, of the wheels and the ground. More specifically, the coordinates of P_1 and P_2 in the observing vehicle coordinate system (VCS) are desired. Furthermore, the vehicle trajectory is also wanted for collision avoidance, which can be estimated by the normal vectors of the front and the rear wheels, denoted as $\overrightarrow{\mathbf{N_1}}$ and $\overrightarrow{\mathbf{N_2}}$, and the distance between P_1 and P_2. In the following, some terminologies and notations used in this study are introduced.

In a captured image of a lateral vehicle, the shapes of the two vehicle wheels are supposed to exist for use as the location marks. As shown in Fig. 3, the projections of the front wheel $W_1$and the rear one W_2 are both ellipses in shape, which are denoted as E_1 and E_2, respectively. The centers of W_1 and W_2 are denoted as C_1 and C_2, respectively. And the corresponding image points of C_1 and C_2 are denoted as c_1 and c_2, respectively. Moreover, assume that the contact points of W_1 and W_2 with the ground are P_1 and P_2, respectively. Let p_1 and p_2 be the image points of P_1 and P_2, respectively, and $\overrightarrow{\mathbf{N_1}}$ and $\overrightarrow{\mathbf{N_2}}$ be the normal vectors of W_1 and W_2, respectively. Then, the normal vector of the ground plane, denoted as $\overrightarrow{\mathbf{N_G}}$, can be figured out to be the cross product of $\overrightarrow{\mathbf{N_1}}$ and $\overrightarrow{\mathbf{N_2}}$, i.e., $\overrightarrow{\mathbf{N_G}} = \overrightarrow{\mathbf{N_1}} \times \overrightarrow{\mathbf{N_2}}$. In addition, three coordinate systems are used in this study, which are shown in Fig. 4. A camera coordinate system (CCS), denoted as *x-y-z*, is set up for the camera mounted on the observing vehicle. The z-axis is along the optical axis of the camera. The corresponding image coordinate system (ICS) is denoted as u-v. The observing vehicle coordinate system (VCS) mentioned previously is denoted as *X-Y-Z* and is set up in such a way that the origin V is located at the middle point of the line segment which connects the two contact points P_1 and P_2 of the two front wheels with the ground. The z-axis and the x-axis are on the ground and parallel to the long side and the short side of the vehicle body, respectively.

III. DETERMINING OF ORIENTAL PARAMETERS OF FRONT AND REAR WHEELS

From the projection of a circle, like the shape of a vehicle wheel, it is possible to infer the normal of a plane on which the circles lies [9]. As is well known, the projection of a circle is an ellipse. Suppose that the ellipse shape of a vehicle wheel has already been obtained by some image processing techniques, and let its equation be expressed as

$$Au^2 + 2Buv + Cv^2 + 2Du + 2Ev + F = 0 . \qquad (1)$$

Also, let **Q** be a matrix specified by

$$\mathbf{Q} = \begin{bmatrix} A & B & D/f \\ B & C & E/f \\ D/f & E/f & F/f^2 \end{bmatrix} \qquad (2)$$

where *f* denotes the focal length of the camera lens. Then, the approach proposed in Kanatani [9] to circle pose estimation is applied in this study to compute the normal vectors of the front and the rear wheels of the vehicle. Let the equations of the two elliptical shapes of the wheels be denoted by E_1 and E_2. By Kanatani's method, the normal vectors of W_1 and W_2, i.e., $\overrightarrow{\mathbf{N_1}}$ and $\overrightarrow{\mathbf{N_2}}$, in the CCS, can be obtained, and the coordinates of the image point of the circle center can be computed. Also, the two normal vectors can be used to estimate the lateral vehicle position. The details are described in the next section.

IV. DETERMINING OF POSITION PARAMETERS OF CONTACT POINTS OF WHEELS AND GROUND

After the normal vector of a wheel is found, the image point of the contact point of the wheel with the ground plane can be detected. Then, the back-projection principle is applied to get the 3D position of the contact point. The derivation is described as follows.

As shown in Fig. 3, the line segment $\overline{C_1P_1}$ passes W_1's center C_1 and the contact point P_1 and is vertical to the ground plane. Similar to $\overline{C_1P_1}$, $\overline{C_2P_2}$ passes W_2's center C_2 and the contact point P_2, and is vertical to the ground, too.

Hence, $\overline{C_1P_1}$ and $\overline{C_2P_2}$ are parallel. As is well known, the perspective projections of two parallel lines, i.e., $\overline{c_1p_1}$ and $\overline{c_2p_2}$, meet at a vanishing point a on an image plane. Let the direction of all parallel 3D lines be specified by a vector (g_1, g_2, g_3), which is just the normal vector of the ground. The image coordinates of the vanishing point a can then be expressed [10] as

$$(u_a, v_a) = (f\frac{g_1}{g_3}, f\frac{g_2}{g_3}), \tag{3}$$

where f is the camera focal length.

Because the wheel plane is vertical to the ground, the normal vector of the ground, $\overrightarrow{\mathbf{N_G}}$, can be computed, as mentioned previously, as the cross product of $\overrightarrow{\mathbf{N_1}}$ and $\overrightarrow{\mathbf{N_2}}$. In case $\overrightarrow{\mathbf{N_1}}$ and $\overrightarrow{\mathbf{N_2}}$ are almost parallel, (3) will fail to get the correct value of $\overrightarrow{\mathbf{N_G}}$. Nevertheless, the vector values of Y-axis of VCS in CCS can be an estimation of $\overrightarrow{\mathbf{N_G}}$, because the direction of Y-axis of VCS is the normal vector of the ground and the relations between CCS and VCS are calibrated in advance.

Substituting the resulting elements of $\overrightarrow{\mathbf{N_G}}$ into (3), the coordinates of the vanishing point a of $\overline{c_1p_1}$ and $\overline{c_2p_2}$ as shown in Fig. 3 are obtained. Moreover, the ICS coordinates of c_1 and c_2 can be computed by Kanatani's method [9] with the values of $\overrightarrow{\mathbf{N_1}}$ and $\overrightarrow{\mathbf{N_2}}$. After the coordinates of points c_1, c_2, and a are known, the line equations of $\overline{c_1a}$ and $\overline{c_2a}$ in the ICS can be obtained. Finally, the intersection points of the lines $\overline{c_1a}$ and $\overline{c_2a}$ with the bottom boundaries of the wheels are extracted as p_1 and p_2, which are the image point of P_1 and P_2, respectively.

Next, the 3D position of P_1 and P_2 can be derived by the back-projection principle. As shown in Fig. 5, after back-projecting the point P in the image into the CCS, we can get a line L which passes the lens center and P. The intersection point of the line L and the horizontal plane Π containing the corresponding space point of P is that we want. Denote this point as P'. Because Π is the ground plane, the equation of the horizontal plane Π can be written as $y = 0$. Assume that point P in the image plane has the CCS coordinates $(u_P, v_P, -f)$ where (u_P, v_P) is the position in the image and f is the focus length. Using transformation matrix between CCS to VCS, we get the VCS coordinate (X_P, Y_P, Z_P) of point P. The desired VCS coordinates $(X_{P'}, Y_{P'}, Z_{P'})$ of P', i.e. the contact point of the wheel with the ground, can be solved to be

$$\begin{aligned} X_{P'} &= X_C - \frac{Z_C}{Z_P - Z_C}(X_P - X_C), \\ Y_{P'} &= Y_C - \frac{Z_C}{Z_P - Z_C}(Y_P - Y_C), \\ Z_{P'} &= 0. \end{aligned} \tag{4}$$

Now, transform the CCS coordinates of p_1 and p_2 to the VCS coordinates. Substituting these VCS coordinates into (4), the 3D positions of P_1 and P_2 can be obtained, which are combined to be the desired lateral vehicle location.

V. PREDICTION OF MOVING LATERAL VEHICLE TRAJECTORIES USING SPATIAL INFORMATION IN SINGLE IMAGES

To extend the ability of the autonomous vehicle for driving assistance, it is desired to estimate the trajectory of a lateral vehicle. With the resulting capability of trajectory prediction, collisions with the lateral vehicle can be avoided. This is indeed a great goal of autonomous vehicle applications.

As described in the Section 3 and 4, the information of both front and rear wheel shapes of a lateral vehicle can be obtained from a single image. Let $\overrightarrow{\mathbf{N_1}}$ and $\overrightarrow{\mathbf{N_2}}$ be the normal vectors of two wheels $\overrightarrow{\mathbf{N_1}}$ and $\overrightarrow{\mathbf{N_2}}$ in the VCS. As shown in Fig. 6, the directions of the wheel shapes are important information to predict the lateral vehicle's trajectory according to the following vehicle kinematics.

Firstly, the turning angle δ can be computed as the angle between $\overrightarrow{\mathbf{N_1}}$ and $\overrightarrow{\mathbf{N_2}}$ as described in Section 3 Secondly, the distance between the front wheels and the rear wheels is derived as the length of $\overline{P_1P_2}$, denoted as d where P_1 and P_2 can be computed according to (3) and (4) in Section 4. By the basic kinematics of the vehicle, the rotation radius R can be found to be

$$R = \frac{d}{\sin\delta}. \tag{5}$$

Using the value of R and vector $\overrightarrow{\mathbf{N_1}}$, the VCS coordinates of the rotation center T of the lateral vehicle can be expressed as

$$\mathbf{T} = \mathbf{P_1} + R\overrightarrow{\mathbf{N_1}}, \tag{6}$$

where $\mathbf{T}$ and $\mathbf{P_1}$ represent the VCS vectors of T and P_1, respectively. Finally, the trajectory of the vehicle can be described as a circular path with a circle center T and a radius of R.

VI. EXPERIMENTAL RESULTS

A. Experimental Results of Vehicle Localization

Experiments on real images have been conducted with a Pentium III PC including a 640x480 Matrox imaging board connected to a CCD camera with 8mm lenses for taking pictures. A toy vehicle as shown in Fig. 7 was used as a test lateral vehicle. For each acquired vehicle image, ellipses are extracted by image-processing techniques. First, the sobel edge detector [11] is employed to find the edge points of the wheels. Hough transform for ellipse detection[12] is then applied to find the candidate ellipses. The detected ellipses are then merged if their axis lengths are similar and the center points of the corresponding ellipses on the acquired image are close enough. An example of the image processing result is shown in Fig. 8.

A sequence of images of a moving lateral vehicle was taken while a manually-simulated driving process was performed. The experimental results of image processing are shown in Fig. 9. The top view of the lateral vehicle locations is shown in Fig. 10. The numeric analysis of location results is listed in Table 1. The average error rate values are all smaller than 5%, so this proposed method is feasible for locating a lateral vehicle.

B. Experimental Results of Trajectory Prediction

Experiments on real world environment have been conducted based on the location results in Section VI.*A*, i.e. Fig. 10. A toy vehicle was droved manually from the font-right to the font with respect the observing vehicle.

The results of trajectory prediction of a vehicle by spatial information are listed in Table 2 The average error rate values of Table 2 are all smaller than 5%, so the proposed methods are feasible for locating a lateral vehicle.

VII. CONCLUSIONS

In this study, a new approaches to location estimation of a moving vehicle for driving assistance and collision avoidance using wheel shapes information by 3D computer vision techniques has been proposed. The proposed approach using single 2-D lateral vehicle images to avoid motion analysis, which is complicated and time consuming. The radius of a wheel need not be known in advance. No special mark on the vehicle is required. Only the front and rear wheel shapes information in a lateral vehicle image is used to detect the contact points of wheels with the ground in a systematic approach. Lower hardware cost is also guaranteed since only one camera is required. Serials of real images were tested and good experimental results prove the effectiveness of the proposed approach.

In addition, experiments on trajectory prediction have also been conducted to show the effectiveness of the proposed approach. With the trajectory information, a test of collision avoidance could be designed. When the lateral vehicle is close to the observing vehicle, a warning is alarmed to inform the driver watch out. It will also be interesting to combining the time interval of two consecutive images with the translation information of a vehicle. Hence, the velocity of moving vehicle can be estimated.

VIII. REFERENCES

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Table 1. Results of location for Figure 9.

front wheel	manual (cm) x	manual (cm) y	computed (cm) x	computed (cm) y	relative error of $\sqrt{x^2+y^2}$
(a)	104.5	224.7	104.28	230.43	2.30%
(b)	96.4	239.7	97.47	246.87	2.81%
(c)	87.3	255.0	86.54	264.98	3.71%
(d)	73.2	277.2	71.17	288.28	3.93%
(e)	53.0	306.4	49.33	319.33	4.32%
(f)	24.7	333.8	23.84	346.82	3.90%
				average	3.50%

rear wheel	manual (cm) x	manual (cm) y	computed (cm) x	computed (cm) y	relative error of $\sqrt{x^2+y^2}$
(a)	105.5	148.0	105.57	143.39	2.54%
(b)	101.4	160.1	101.99	156.57	1.89%
(c)	99.4	180.3	99.35	176.29	1.95%
(d)	93.4	204.5	93.97	207.02	1.14%
(e)	79.2	226.8	79.64	230.73	1.66%
(f)	65.1	255.0	65.61	260.24	1.98%
				average	1.86%

Table 2. Results of trajectory prediction by spatial approach for Figure 9, where dR is computed as $\sqrt{(x-Rx)^2+(y-Ry)^2}-R$.

Fig.9	front wheel (cm) x	front wheel (cm) y	rotation center (cm) Rx	rotation center (cm) Ry	radius (cm) R	distance (cm) dR	relative error eR=dR/R
(a)	104.28	230.43	-177.07	110.92	305.68		
(b)	97.47	246.87	-197.13	113.91	323.21	0.68	0.26%
(c)	86.54	264.98	-181.25	110.44	309.18	1.83	0.66%
(d)	71.17	288.28	-156.44	117.87	284.33	0.40	0.13%
(e)	49.33	319.33	-181.69	105.58	314.73	3.63	1.12%
(f)	23.84	346.82				2.19	0.63%
					average	1.75	0.56%

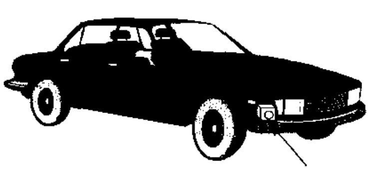


Figure 1. Illustration of a vehicle mounted with a camera for imaging lateral views.

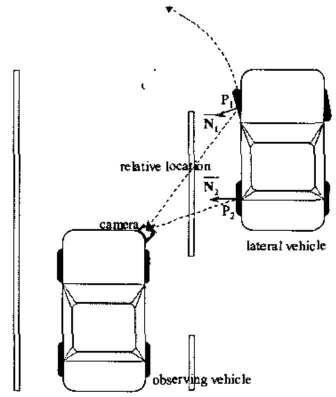


Figure 2. Top view of two vehicles in a neighborhood.

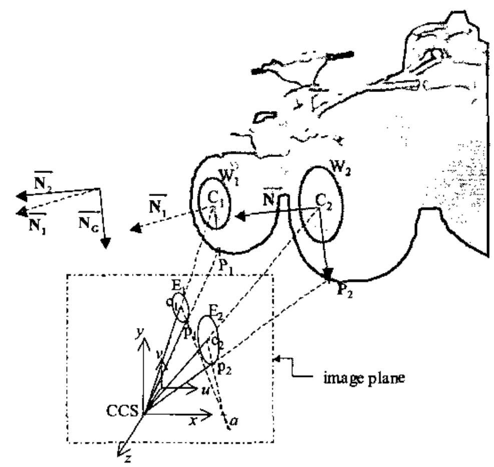


Figure 3. The projection of two wheels and the notations used in this study.

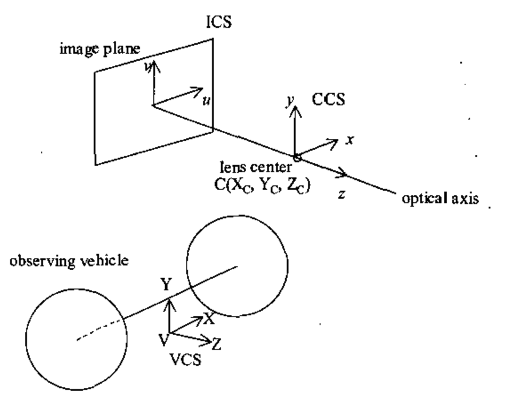


Figure 4. The coordinate systems used in this study, including the camera coordinate system *x-y-z*, the image coordinate system *u-v*, and the vehicle coordinate system *X-Y-Z*.

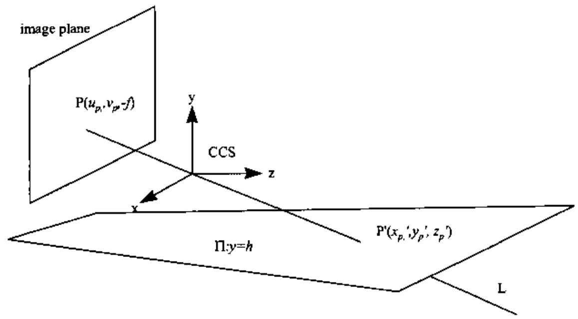


Figure 5. Illustration of the back-projection principle for finding the space point corresponding to an image pixel.

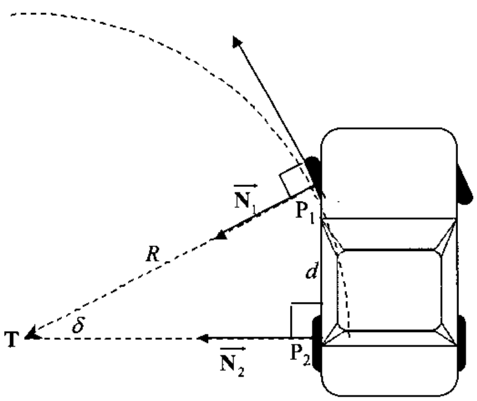


Figure 6. Vehicle Kinematics.

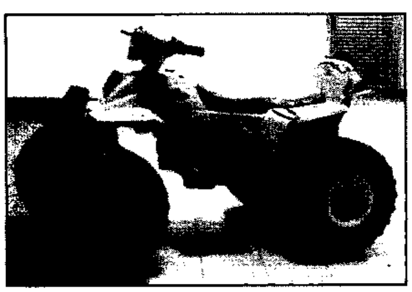
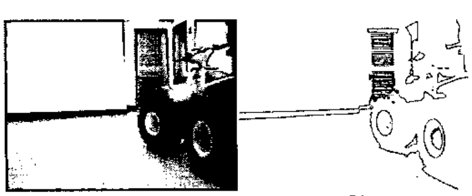
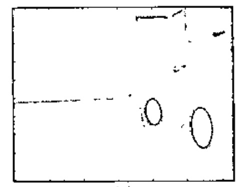


Figure 7. A toy vehicle as a test lateral vehicle.

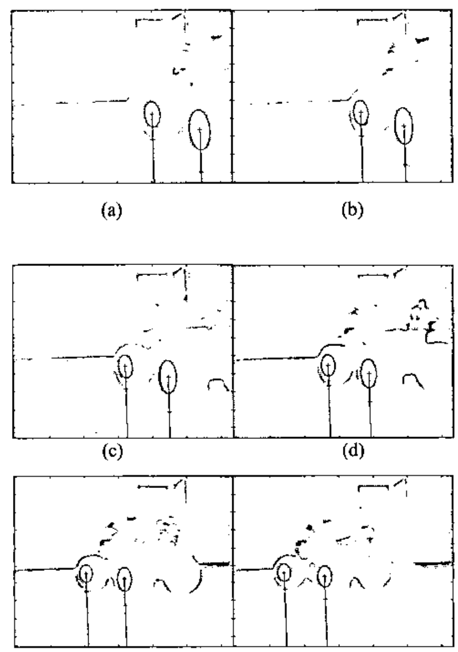


(a) (b)



(c)

Figure 8. An image processing result. (a) The original image. (b) The result of edge detection. (c) The result of ellipse detection of two wheels.



(a) (b) (c) (d) (e) (f)

Figure 9. A sequence of images a moving lateral vehicle detected.

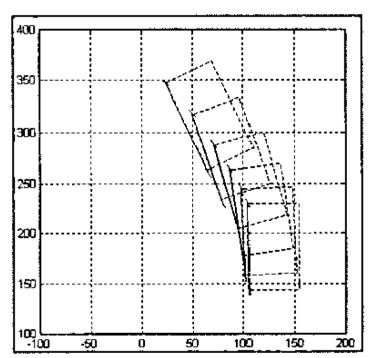


Figure 10. A top view of locations of a moving lateral vehicle according to the results of Figure 9.