# A Unified Approach to Unwarping of Omni-Images into Panoramic and Perspective-View Images Using Pano-Mapping Tables 

S. W. Jeng and W. H. Tsai


#### Abstract

A unified approach to unwarping of omni-images into panoramic or perspective-view images is proposed. The approach does not adopt the conventional technique of calibrating the related parameters of an omni-camera. Instead, it is based on a new concept of pano-mapping table, which is created once forever by a simple learning process for an omni-camera of any kind as a summary of the information conveyed by all the camera parameters. The learning process takes as input a set of landmark point pairs in the world space and in a given image. With the help of the pano-mapping table, any panoramic or perspective-view image can be created from an input omni-image taken by the omni-camera according to an analytic computation process proposed in this study. Experimental results show the feasibility of the proposed approach.


## I. INTRODUCTION

Omni-cameras are used in many applications for taking omni-images of camera surroundings. Usually, there exists an extra need to create perspective-view images from omni-images for human comprehensibility. This work is called unwarping, which usually is a complicated work. Fig. 1 shows an example where Fig. 1(a) is an omni-image and Fig. 1(b) a perspective-view image obtained from unwarping the image of Fig. 1(a) by a method proposed by the authors [5].

Omni-cameras can be categorized into two types according to the involved optics, namely, dioptric and catadioptric. A dioptric omni-camera captures incoming light going directly into the imaging sensor to form omni-images. An example of dioptric omni-cameras is the fish-eye camera. A catadioptric omni-camera captures incoming light reflected by a mirror to form omni-images. The mirror surface may be in various shapes, like conic, hyperbolic, etc. If all the reflected light rays pass through a common point, the camera is said additionally to be of the single-view-point (SVP) type [1]; otherwise, of the non-single-view-point (non-SVP) type.

The method for unwarping omni-images is different for each distinct type of omni-camera. Generally speaking, omni-images taken by the SVP catadioptric camera [2][3] as well as the dioptric camera are easier to unwarp than those

Manuscript received June 29, 2006. This work was supported by Ministry of Economic Affairs under Project No. MOEA 94-EC-17-A-02-S1-032 in Technology Development Program for Academia.

Sheng-Wen Jeng is with the Department of Computer Science, National Chiao Tung University, Hsinchu 30010, Taiwan (phone: 886-6-3847167; fax: 886-6-3847297; e-mail: sunny@itri.org.tw).

Wen-Hsiang Tsai is with the Department of Computer Science and Information Engineering, Asia University, Taichung, 41354, Taiwan(email: whtsai@asia.edu.tw).
taken by the non-SVP catadioptric camera [4][5]. Conventional methods for unwarping omni-images require the knowledge of certain camera parameters, like the focal length of the lens, the coefficients of the mirror surface shape equation, etc. But in some situations, we cannot get the complete information of the omni-camera parameters. Then the unwarping work cannot be conducted. It is desired to have a more convenient way to deal with this problem.


Fig. 1. An example image unwarping. (a) An omni-image. (b) A perspective-view image resulting from unwarping a portion of the omni-image in (a).

In this paper, we propose a unified approach to unwarping of omni-images taken by all kinds of omni-cameras. It is unnecessary to know the camera parameters in advance. This is made possible by the use of a pano-mapping table, which essentially is a summary of the information conveyed by all the camera parameters. The table is created once forever for each camera. And given an image taken by the camera, the table may be utilized to unwarp the image in an analytic way to create panoramic or perspective-view images from any viewpoints. The table is invariant with respective to the camera position, that is, it is not changed even when the camera is moved around. The table is created by a calibration process making use of certain selected points in the world space with known coordinates and their corresponding pixels in an omni-image.

In the remainder of this paper, the proposed approach is described in detail in Section 2. Some experimental results are shown in Section 3, and conclusions are made in Section 4.

## II. Proposed Method Using Pano-Mapping Table

The proposed method includes three major steps: landmark learning, table creation, and image unwarping.

## A. Landmark learning

The first step, landmark learning, is a procedure in which some pairs of selected world space points with known positions and their corresponding pixels in a taken
omni-image are set up. More specifically, the coordinates of at least five points, called landmark points hereafter, which are easy to identify in the world space (for example, a corner in a room), are measured manually with respect to a selected origin in the world space. Then the corresponding pixels of such landmark points in the taken omni-image are segmented out. A world space point and its corresponding image pixel so selected together are said to form a landmark point pair.

## B. Table creation

The second step, table creation, is a procedure in which a pano-mapping table is built using the coordinate data of the landmark point pairs. The table is 2-D in nature with the horizontal and vertical axes specifying respectively the range of the azimuth angle $\theta$ as well as that of the elevation angle $\rho$ of all possible incident light rays going through the mirror center. An illustration is shown in Fig. 2, and an example of the pano-mapping table of size $M \times N$ is shown in Table 1.


Fig. 2. System configuration.
Each entry $E_{i j}$ with indices $(i, j)$ in the pano-mapping table specifies an azimuth-elevation angle pair ( $\theta_{i}, \rho_{j}$ ), which represent an infinite set $S_{i j}$ of world space points passing through by the light ray with azimuth angle $\theta_{i}$ and elevation angle $\rho_{j}$. These world space points in $S_{i \mathrm{j}}$ are all projected onto an identical pixel $p_{i j}$ in any omni-image taken by the camera, forming a pano-mapping $f_{\mathrm{pm}}$ from $S_{i j}$ to $p_{i j}$. An illustration is shown in Fig. 3. This mapping is shown in the table by filling entry $E_{i j}$ with the coordinates ( $u_{i j}, v_{i j}$ ) of pixel $p_{i j}$ in the omni-image. The table as a whole specifies the nature of the omni-camera, and may be used to create any panoramic or perspective-view images, as described subsequently.

## C. Image unwarping

The third step, image unwarping, is a procedure in which the pano-mapping table of an omni-camera is used as a media to construct a panoramic or perspective-view image Q of any size and any viewpoint from a given input omni-image I taken
by the omni-camera. The basic concept in the procedure is to map each pixel q in Q to an entry $\mathrm{E}_{\mathrm{ij}}$ with entry values $\left(u_{\mathrm{ij}}, v_{\mathrm{ij}}\right)$ in the pano-mapping table and to take the pixel value at coordinates $\left(u_{\mathrm{i},}, v_{\mathrm{ij}}\right)$ of image I for use as that of q .

The detail of each step is elaborated in the following.
Table 1. An example of pano-mapping table of size $M \times N$.

|  | $\theta_{1}$ | $\theta_{2}$ | $\theta_{3}$ | $\ldots$ | $\theta_{M}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\rho_{1}$ | $\left(u_{11}, v_{11}\right)$ | $\left(u_{21}, v_{21}\right)$ | $\left(u_{31}, v_{31}\right)$ | $\ldots$ | $\left(u_{M 1}, v_{M 1}\right)$ |
| $\rho_{2}$ | $\left(u_{12}, v_{12}\right)$ | $\left(u_{22}, v_{22}\right)$ | $\left(u_{32}, v_{32}\right)$ | $\ldots$ | $\left(u_{M 2}, v_{M 2}\right)$ |
| $\rho_{3}$ | $\left(u_{13}, v_{13}\right)$ | $\left(u_{23}, v_{23}\right)$ | $\left(u_{33}, v_{33}\right)$ | $\ldots$ | $\left(u_{M 3}, v_{M 3}\right)$ |
| $\rho_{4}$ | $\left(u_{14}, v_{14}\right)$ | $\left(u_{24}, v_{24}\right)$ | $\left(u_{34}, v_{34}\right)$ | $\ldots$ | $\left(u_{M 4}, v_{M 4}\right)$ |
|  | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ |
| $\rho_{N}$ | $\left(u_{1 N}, v_{1 N}\right)$ | $\left(u_{2 N}, v_{2 N}\right)$ | $\left(u_{3 N}, v_{3 N}\right)$ | $\ldots$ | $\left(u_{M N}, v_{M N}\right)$ |



Fig. 3. Mapping between Pano-mapping table and omni-image.

### 2.1 Landmark Learning Procedure

Before describing the landmark learning procedure, we briefly explain the system configuration as shown in Fig. 2. A "downward-looking" omni-camera is attached "horizontally" at the ceiling center of a room with both its mirror base plane and omni-image plane parallel to the floor, which is just the $X-Y$ plane of the world coordinate system with its coordinates denoted by $(X, Y, Z)$ and its origin by $\mathrm{O}_{\mathrm{w}}$. The mirror center of the omni-camera, denoted as $\mathrm{O}_{\mathrm{m}}$, is located at $(-D, 0, H)$ with respect to $\mathrm{O}_{\mathrm{w}}$ in the world coordinate system. A camera coordinate system is set up at $\mathrm{O}_{\mathrm{m}}$ with its coordinates denoted by $(x, y, z)$ and its three axes all parallel to those of the world coordinate system, respectively. Also shown in the figure as an illustration of the definitions of the azimuth and elevation angles are two points $P_{1}$ and $P_{2}$ in the world space with corresponding image points $p_{1}$ and $p_{2}$ in the omni-image. The elevation angles of $P_{1}$ and $P_{2}$ with respect to the horizontal base plane of the mirror are $\rho_{1}$ and $\rho_{2}$, respectively, and their azimuth angles with respect to the $x$-axis of the camera coordinate system are $\theta_{1}$ and $\theta_{2}$, respectively.

The landmark learning procedure proceeds at first by selecting a sufficient number ( $\geq 5$ ) of landmark point pairs with the world space points being easy to identify. The coordinates of the world space points are then measured. Fig. 4 shows the interface we have designed for acquiring the data of the landmark point pairs easily. Especially, note that in Fig. 3 the mirror center $\mathrm{O}_{\mathrm{m}}$ of the camera with known world coordinates $\left(X_{0}, Y_{0}, Z_{0}\right)$ just appears to be the image center $\mathrm{O}_{\mathrm{c}}$
with known coordinates ( $u_{0}, v_{0}$ ). This image center can be automatically extracted by a simple image analysis scheme. We skip the detail here, and take the coordinate data of the point pair $\left(\mathrm{O}_{\mathrm{c}}, \mathrm{O}_{\mathrm{m}}\right)$ as the first set of the learned data. After learning, assume that we have $n$ sets of landmark point pair data, each set including the coordinates $\left(u_{k}, v_{k}\right)$ and $\left(X_{k}, Y_{k}, Z_{k}\right)$ of the image point and the corresponding world space point,


Fig. 4. Landmark learning interface.
respectively, where $k=0,1, \ldots, n-1$.

### 2.2 Table Creation procedure

The pano-mapping table is a $2-\mathrm{D}$ array for use as a media for unwarping omni-images after it is constructed. We may imagine the table as a longitude and latitude system with a horizontal $\theta$-axis and a vertical $\rho$-axis, specifying the azimuth and elevation angles of incident light rays through the mirror center, as mentioned previously. We divide the range $2 \pi$ of the azimuth angles equally into $M$ units, and the range of the elevation angles, say from $\rho_{\mathrm{s}}$ to $\rho_{\mathrm{c}}$, into $N$ units, to create a table $T_{\mathrm{pm}}$ of $M \times N$ entries. Each entry $E$ with corresponding angle pair $(\theta, \rho)$ in $T_{\mathrm{pm}}$ maps to a pixel $p$ with coordinates ( $u$, $v$ ) in the input omni-image $I$. This mapping $f_{\mathrm{pm}}$ may be decomposed into two mappings, one in the azimuth direction and the other in the radial direction, called azimuth-directional mapping and radial-directional mapping, respectively.

Because of the rotation-invariant property of the omni-camera, the azimuth angle $\phi$ of each world space point through which the light ray passes is actually identical to the angle $\theta$ of the corresponding pixel $p$ with respect to the $u$-axis in the input image $I$. That is, the azimuth-directional mapping is just an identity function $f_{\mathrm{a}}$ such that $f_{\mathrm{a}}(\boldsymbol{\theta})=\boldsymbol{\phi}=\boldsymbol{\theta}$.

On the other hand, because of the nonlinear property of the mirror surface shape, the radial-directional mapping should be specified by a nonlinear function $f_{\mathrm{r}}$ such that the radial distance $r$ from each image pixel $p$ with coordinates ( $u$, $v$ ) in $I$ to the image center $\mathrm{O}_{\mathrm{c}}$ at $\left(u_{0}, v_{0}\right)$ may be computed by $r$ $=f_{\mathrm{r}}(\rho)$. And based on the mappings $f_{\mathrm{a}}$ and $f_{\mathrm{r}}$, we can regard the
pairs $(r, \phi)=\left(f_{\mathrm{r}}(\rho), \theta\right)$ of all the image pixels to form a polar coordinate system with image coordinates $(u, v)$ specified by

$$
\begin{align*}
& u=r \times \cos \phi=f_{\mathrm{r}}(\rho) \times \cos \theta, \\
& v=r \times \sin \phi=f_{\mathrm{r}}(\rho) \times \sin \theta . \tag{1}
\end{align*}
$$

In this study, we also call $f_{\mathrm{r}}$ a radial stretching function. And we propose to describe it by the following 4th-degree polynomial function:

$$
\begin{equation*}
r=f_{\mathrm{r}}(\rho)=a_{0}+a_{1} \times \rho^{1}+a_{2} \times \rho^{2}+a_{3} \times \rho^{3}+a_{4} \rho^{4}, \tag{2}
\end{equation*}
$$

where $a_{0}$ through $a_{4}$ are coefficients to be estimated using the data of the landmark point pairs, as described in the following algorithm. A similar idea of approximation can be found in Scotti, et al. [6]. Let the data of the $n$ selected landmark point pairs be denoted as $\left(P_{0}, p_{0}\right),\left(P_{1}, p_{1}\right), \ldots,\left(P_{n-1}, p_{n-1}\right), n \geq 5$.

## Algorithm 1. Estimation of coefficients of radial stretching

 function.Step 1. Transform the world coordinates $\left(X_{k}, Y_{k}, Z_{k}\right)$ of each selected landmark point $P_{k}, k=1,2, \ldots, n-1$, with respect to $\mathrm{O}_{\mathrm{w}}$ into coordinates with respect to $\mathrm{O}_{\mathrm{m}}$ by subtracting from $\left(X_{k}, Y_{k}, Z_{k}\right)$ the coordinate values $\left(X_{0}, Y_{0}, Z_{0}\right)=(-D, 0$, $H)$ of $\mathrm{O}_{\mathrm{m}}$. Thus, the origin of the world coordinate system will be assumed to be at $\mathrm{O}_{\mathrm{m}}$ hereafter.
Step 2. Use the coordinate data of each landmark point pair $\left(P_{k}, p_{k}\right)$, i. e., the world coordinates $\left(X_{k}, Y_{k}, Z_{k}\right)$ and the image coordinates $\left(u_{k}, v_{k}\right)$, to calculate the elevation angle $\rho_{k}$ of $P_{k}$ in the world space and the radial distance $r_{k}$ of $p_{k}$ in the image space by the following equations:

$$
\begin{align*}
& \rho_{k}=\tan ^{-1}\left[\frac{Z_{k}}{D_{k}}\right]  \tag{3}\\
& r_{k}^{2}=u_{k}^{2}+v_{k}^{2} \tag{4}
\end{align*}
$$

where $D_{k}$ is the measured distance in the $X-Y$ plane of the world coordinate system from the landmark point $P_{k}$ to the mirror center $\mathrm{O}_{\mathrm{m}}$, computed by $D_{k}=\sqrt{X_{k}^{2}+Y_{k}^{2}}$.
Step 3. Substitute all the data $\rho_{0}, \rho_{2}, \ldots, \rho_{n-1}$ and $r_{1}, r_{2}, \ldots, r_{n-1}$ computed in the last step into Eq. (2) to get $n$ simultaneous equations:

$$
\begin{aligned}
& r_{0}=f_{\mathrm{r}}\left(\rho_{0}\right)=a_{0}+a_{1} \times \rho_{0}{ }^{1}+a_{2} \times \rho_{0}^{2}+a_{3} \times \rho_{0}^{3}+a_{4} \rho_{0}^{4} ; \\
& r_{1}=f_{\mathrm{r}}\left(\rho_{1}\right)=a_{0}+a_{1} \times \rho_{1}^{1}+a_{2} \times \rho_{1}^{2}+a_{3} \times \rho_{1}^{3}+a_{4} \rho_{1}^{4} ;
\end{aligned}
$$

$r_{n-1}=f_{\mathrm{r}}\left(\rho_{n-1}\right)=a_{0}+a_{1} \times \rho_{n-1}{ }^{1}+a_{2} \times \rho_{n-1}{ }^{2}+a_{3} \times \rho_{n-1}{ }^{3}+a_{4} \rho_{n-1}{ }^{4}$, and solve them to get the desired coefficients $\left(a_{0}, a_{1}, a_{2}, a_{3}\right.$, $a_{4}$ ) of the radial stretching function $f_{\mathrm{r}}$ by a numerical analysis method.

Now, the entries of the pano-mapping table can be filled with the corresponding image coordinates using Eqs. (1) and (2) by the following algorithm. Note that the table is $M \times N$.


Fig. 5. Lateral-view configuration for generating $a$ panoramic image.


Fig. 6. Top-view configuration for generating a perspective-view image.

## Algorithm 2. Filling entries of pano-mapping table.

Step 1. Divide the range $2 \pi$ of the azimuth angles into $M$ intervals, and compute the $i$ th azimuth angle $\theta_{i}$ by

$$
\begin{equation*}
\theta_{i}=i \times(2 \pi / M) \text {, for } i=0,1, \ldots, M-1 . \tag{5}
\end{equation*}
$$

Step 2. Divide the range $\rho_{e}$ to $\rho_{s}$ of the elevation angles into $N$ intervals, and compute the $j$-th elevation angle $\rho_{j}$ by

$$
\begin{equation*}
\rho_{j}=j \times\left[\left(\rho_{e}-\rho_{s}\right) / N\right]+\rho_{s}, \text { for } j=0,1, \ldots, N-1 . \tag{6}
\end{equation*}
$$

Step 3. Fill the entry $E_{i j}$ with the corresponding image coordinates ( $u_{i j}, v_{i j}$ ) computed according to Eqs. (1) and (2) as follows:

$$
\begin{equation*}
u_{i j}=r_{i j} \times \cos \theta_{i} ; v_{i j}=r_{i j} \times \sin \theta_{i} \tag{7}
\end{equation*}
$$

where $r_{i j}$ is computed by

$$
\begin{equation*}
r_{i j}=f_{\mathbf{r}}\left(\rho_{j}\right)=a_{0}+a_{1} \times \rho_{j}^{1}+a_{2} \times \rho_{j}^{2}+a_{3} \times \rho_{j}^{3}+a_{4} \rho_{j}^{4} \tag{8}
\end{equation*}
$$

with $a_{0}$ thru $a_{4}$ being those computed by Algorithm 1 .

### 2.3 Unwarping procedure

Now, we are ready to show how to reconstruct a panoramic or perspective-view image from an omni-image with the aid of the pano-mapping table. Three cases can be identified.

## A. Generation of a generic panoramic image

Given an input omni-image $G$ and a pano-mapping table $T_{\mathrm{pm}}$, we may generate from $G$ a generic panoramic image $Q$ which is exactly of the same size $M \times N$ of $T_{\mathrm{pm}}$. The steps are as follows. First, for each entry $E_{i j}$ of the pano-mapping table with azimuth angle $\theta_{i}$ and elevation angle $\rho_{j}$, take out the coordinates $\left(u_{i j}, v_{i j}\right)$ filled in $E_{i j}$. Then, assign the color values of the pixel $p_{i j}$ of $G$ at coordinates $\left(u_{i j}, v_{i j}\right)$ to the pixel $q_{i j}$ of $Q$ at coordinates $(i, j)$. After all entries of the table are processed, the final $Q$ becomes a generic panoramic image which we want. In this process, we may regard $Q$ as the output of the pano-mapping $f_{\mathrm{pm}}$ described by the pano-mapping table $T_{\mathrm{pm}}$ with $G$ as the input, i. e., $f_{\mathrm{pm}}(G)=Q$.

## B. Generation of a specific panoramic image

With the aid of a pano-mapping table $T_{\mathrm{pm}}$ with $M \times N$ entries, we may also generate from a given omni-image $G$ a panoramic image $Q$ of any size, say $M_{Q} \times N_{Q}$, which is the panoramic projection of the original scene appearing in $G$ at any distance $D$ with respect to the mirror center $\mathrm{O}_{\mathrm{m}}$ with a projection band of any height $H$. An illustration of such an imaging configuration from a lateral view is shown in Fig. 5.

The process for generating such an image is similar to that for generating the generic panoramic image described previously. First, we map each image pixel $q_{k l}$ in $Q$ at coordinates ( $k, l$ ) to an entry $E_{i j}$ in $T_{\mathrm{pm}}$ filled with coordinates $\left(u_{i j}, v_{i j}\right)$. Then, we assign the color value of the pixel $p_{i j}$ of $G$ at ( $u_{i j}, v_{i j}$ ) to $q_{k l}$. Mapping of $q_{k l}$ to $E_{i j}$ is based on the use of the knowledge of the parameters $M_{Q}, N_{Q}, D$, and $H$ as well as some related principles like triangulation, proportionality, etc.

In more details, since the azimuth angle range $2 \pi$ is divided into $M_{Q}$ intervals in image $Q$, the image pixel $q_{k l}$ at coordinates ( $k, l$ ) is presumably the projection result of a light ray $R_{q}$ with an azimuth angle $\theta_{q}=k \times\left(2 \pi / M_{Q}\right)$ by linear proportionality. Since each azimuth angle interval in $T_{\mathrm{pm}}$ is $2 \pi / M$, the index $i$ of the corresponding entry $E_{i j}$ in $T_{\mathrm{pm}}$ with the azimuth angle of $\theta_{q}$ is just

$$
\begin{equation*}
i=\theta_{q} /(2 \pi / M)=\left[k \times\left(2 \pi / M_{Q}\right)\right] /(2 \pi / M)=k \times \frac{M}{M_{Q}} \tag{9}
\end{equation*}
$$

where we assume $M$ is a multiple of $M_{Q}$. If not, the right-hand side of (9) should be replaced with its integer floor value.

Next, we compute the elevation angle $\rho_{q}$ of the light ray $R_{q}$ projecting onto pixel $q_{k l}$ to decide the index $j$ of $E_{i j}$. For this, since the height of the projection band is $H$ and the image
$Q$ is divided into $N_{Q}$ intervals, by linear proportionality again,
we may compute the height of $R_{q}$ at $D$ as

$$
\begin{equation*}
H_{q}=l \times \frac{H}{N_{Q}} . \tag{10}
\end{equation*}
$$

Then, by trigonometry, we have the elevation angle $\rho_{q}$ as

$$
\begin{equation*}
\rho_{q}=\tan ^{-1}\left(\frac{H_{q}}{D}\right) \tag{11}
\end{equation*}
$$

Therefore, we can compute the index $j$ of $E_{i j}$ by proportionality again as

$$
\begin{equation*}
j=\left(\rho_{q}-\rho_{s}\right) /\left[\left(\rho_{e}-\rho_{s}\right) / N\right]=\frac{\left(\rho_{q}-\rho_{s}\right) \times N}{\left(\rho_{e}-\rho_{s}\right)} \tag{12}
\end{equation*}
$$

since the elevation angle range $\rho_{e}-\rho_{s}$ is divided into $N$ intervals.

With the indices $(i, j)$ of $E_{i j}$ available, the content of $E_{i j}$, i. e., the coordinates $\left(u_{i j}, v_{i j}\right)$, may be obtained. And finally the color value of the omni-image $G$ at $\left(u_{i j}, v_{i j}\right)$ is assigned to the pixel $q_{k l}$ at coordinates $(k, l)$ of $Q$. After all pixels of $Q$ are processed in the above way, the final content of $Q$ is just the desired panoramic image.

## C. Generation of a specific perspective-view image

Given an omni-image $G$ and a pano-mapping table $T_{\mathrm{pm}}$ with $M \times N$ entries, we may also generate from $G$ a perspective-view image $Q$ of any size $M_{Q} \times N_{Q}$, which is the perspective projection of the original scene appearing in $G$ onto a planar rectangular region $A_{\mathrm{P}}$ of any size $W \times H$ at any distance $D$ with respect to the mirror center $\mathrm{O}_{\mathrm{m}}$. A top-view of the configuration for such an image generation process is shown in Fig. 6. The idea again is to map each image pixel $q_{k l}$ in $Q$ at coordinates ( $k, l$ ) to an entry $E_{i j}$ in $T_{\mathrm{pm}}$ filled with coordinates $\left(u_{i j}, v_{i j}\right)$, and then to assign the color value of the pixel $p_{i j}$ of $G$ at $\left(u_{i j}, v_{i j}\right)$ to $q_{k l}$. Mapping of $q_{k l}$ to $E_{i j}$ is accomplished by computing the azimuth and the elevation angles $\theta_{q}$ and $\rho_{q}$ associated with $E_{i j}$ and corresponding to $q_{k l}$.

Referring to Fig. 6, we first compute the angle $\phi$. By trigonometry, we have $W^{2}=D^{2}+D^{2}-2 \times D \times D \times \cos \phi$ from which $\phi$ may be solved to be

$$
\begin{equation*}
\phi=\cos ^{-1}\left[1-\frac{W^{2}}{2 \times D^{2}}\right] \tag{13}
\end{equation*}
$$

Also, it is easy to see from the figure that

$$
\begin{equation*}
\beta=\frac{\pi-\phi}{2} . \tag{14}
\end{equation*}
$$

Next, we compute the index $i$ of entry $E_{i j}$ of table $T_{\mathrm{pm}}$ corresponding to pixel $q_{k l}$ in image $Q$. First, let $p_{i j}$ denote the intersection point of the light ray $R_{q}$ projecting onto $q_{k l}$ and the planar projection region $A_{\mathrm{p}}$. Note that each entry $E_{i j}$ has a corresponding $p_{i j}$. Then, we compute the distance $\ell$ between point $p_{i j}$ and the border point $P_{r}$ shown in Fig. 6 by linear proportionality as

$$
\begin{equation*}
\ell=k \times \frac{W}{M_{Q}} \tag{15}
\end{equation*}
$$

since the projection region $A_{\mathrm{p}}$ has a width of $W$, the image $Q$ has a width of $M_{Q}$ pixels, and pixel $q_{k l}$ has an index of $k$ in the horizontal direction. Also, by trigonometry we can compute the distance $L$ between point $p_{i j}$ and the mirror center $\mathrm{O}_{\mathrm{m}}$ as

$$
\begin{equation*}
L=\sqrt{D^{2}+\ell^{2}-2 \times \ell \times D \times \cos \beta} \tag{16}
\end{equation*}
$$

and then the distance $h$ from point $p_{i j}$ to the line segment $\overline{\mathrm{O}_{\mathrm{m}} P_{\mathrm{r}}}$ connecting $\mathrm{O}_{\mathrm{m}}$ and $P_{\mathrm{r}}$ as

$$
\begin{equation*}
h=L \times \sin \beta \tag{17}
\end{equation*}
$$

So, the azimuth angle $\theta_{q}$ of point $p_{i j}$ with respect to $\overline{\mathrm{O}_{\mathrm{m}} P_{\mathrm{r}}}$ satisfies

$$
\sin \theta_{q}=\frac{h}{L}=\frac{\ell \times \sin \beta}{\sqrt{D^{2}+\ell^{2}-2 \times \ell \times D \times \cos \beta}}
$$

which leads to

$$
\begin{equation*}
\theta_{q}=\sin ^{-1}\left[\frac{\ell \times \sin \beta}{\sqrt{D^{2}+\ell^{2}-2 \times \ell \times D \times \cos \beta}}\right] \tag{18}
\end{equation*}
$$

Finally, the index $i$ of entry $E_{i j}$ may computed by linear proportionality as

$$
\begin{equation*}
i=\left[\frac{\theta_{q}}{2 \pi}\right] \times M \tag{19}
\end{equation*}
$$

where we assume the right side of (19) is an integer. In case not, it should be replaced by its integer floor value.

As to the index $j$ of $E_{i j}$, it can be computed in a way similar to that for deriving Eqs. (10), (11) and (12) as follows:

$$
\begin{equation*}
H_{q}=l \times \frac{H}{N_{Q}} ; \rho_{q}=\tan ^{-1}\left[\frac{H_{q}}{L}\right] ; j=\frac{\left(\rho_{q}-\rho_{s}\right) \times N}{\left(\rho_{e}-\rho_{s}\right)} . \tag{20}
\end{equation*}
$$

With the indices $(i, j)$ of $E_{i j}$ ready, finally we can take the coordinates $\left(u_{i j}, v_{i j}\right)$ in $E_{i j}$ out and assign the color value of the image pixel of $G$ at coordinates $\left(u_{i j}, v_{i j}\right)$ to pixel $q_{k l}$ of $Q$ at coordinates $(k, l)$. After all pixels of $Q$ are processed, the final result of $Q$ is just the desired perspective-view image.

## III. Experimental Results

When we conducted landmark learning and table creation procedures, we used the interface shown in Fig. 4 to identify landmark pairs. Fig. 7(a) shows the result of learning with ten landmark point pairs identified. Also, the coordinates of the image center, and the length of the "cut-off radius" within which the image is invisible because of camera self-occlusion, are extracted automatically in the learning procedure using an algorithm developed in this study. The region within the cut-off radius is marked by a circle with the coordinate values of the image center and the radius length printed at the bottom of Fig. 7(a). We used the cut-off radius to calculate one end $\rho_{e}$ of the full range of the elevation angles. The landmark point pairs were used to estimate the coefficients of the radial stretching function of Eq. (2). Fig. 7 (b) shows the fitted curve with the 10 learned landmark points superimposed on the drawing (marked with " + ").

With the coefficients estimated, Eqs. (1) and (2) were used to construct a pano-mapping table, as described in Section 2.2. The table can be used to unwarp an input omni-image into a panoramic or perspective-view image of any viewpoint, as described in Section 2.3. Fig. 8 shows some examples of images obtained from unwarping the omni-image shown in Fig. 8(c). Fig. 8(a) is a generated generic panoramic image. Note that this image is unique for an omni-camera. Fig. 8(b) is a panoramic image viewed at distance 184.1 cm with respect to the mirror center $\mathrm{O}_{\mathrm{m}}$ of the omni-camera with a projection height of 208.5 cm . Fig. 8(d) is a perspective-view image viewed at distance 184.1 cm with respect to the camera in a projection region of $216.3 \times 208.5$ $\mathrm{cm}^{2}$. Note that Figs. 8(b) and $8(\mathrm{~d})$ will look different by changing the relative positions with respect to the omni-camera. Fig. 9 shows some perspective-view images generated from an omni-image video sequence at different projection distances

## IV. Conclusions and Discussions

A new approach to unwarping of omni-images taken by all kinds of omni-cameras has been proposed. The approach is based on the use of pano-mapping tables proposed in this study, which may be regarded as a summary of the information conveyed by all the parameters of an omni-camera. The pano-mapping table is created once forever for each omni-camera, and can be used to construct panoramic or perspective-view images of any viewpoint in the world space. This is a simple and efficient way to unwarping an omni-image taken by any omni-camera, when some of the physical parameters of the omni-camera cannot be obtained.

A possible application of this approach is panoramic imaging in a visual surveillance system in a room. We tried this idea in this study, and the experimental results shown previously come from this test. Fig. 9 shows some perspective-view snapshots generated from the taken videos of the surveillance system, which are useful for specific person or activity monitoring. Because the snapshots are unwarping results from a portion of an omni-image, their image qualities are not so good, compared with images taken by a normal camera. If this issue is to be solved, we may add a pan-tilt-zoom camera into the surveillance system [6] to capture images of higher quality corresponding to the tracked versions of the perspective-view snapshots.

## References

[1] S. Baker and S. K. Nayar, "A theory of single-viewpoint catadioptric image formation," International Journal of Computer Vision, Vol. 35, no. 2, pp. 175-196, 1999.
[2] S. K. Nayar, "Catadioptric omni-directional camera," Proceedings of IEEE Conference on Computer Vision and Pattern Recognition, San-Juan, Puerto Rico, pp. 482-488, June 1997.
[3] Y. Onoe, N. Yokoya, K. Yamazawa, and H. Takemura, "Visual surveillance and monitoring system using an omni-directional video camera," Proceedings of 1 4th International Conference on

Pattern Recognition, Brisbane, Australia, vol. 1, pp. 588-592, August 1998.
[4] S. W. Jeng and W. H. Tsai, "Precise image unwarping of omni-directional cameras with hyperbolic-shaped mirrors," Proceedings of 16th IPPR Conference on Computer Vision, Graphics and Image Processing, Kinmen, Taiwan, August, 2003, pp. 414-422.
[5] S. W. Jeng and W. H. Tsai, "Construction of perspective and panoramic images from omni-images taken from hypercatadioptric cameras for visual surveillance," Proceedings of 2004 IEEE International Conference on Networking, Sensing, and Control, Taipei, Taiwan, pp. 204-209, March 2004.
[6] G. Scotti, L. Marcenaro, C. Coelho, F. Selvaggi and C. S. Regazzoni, "Dual camera intelligent sensor for high definition 360 degrees surveillance," IEE Proceeding on Vision Image Signal Processing, pp. 250-257, Vol. 152, No. 2, April 2005.


Fig. 7. Landmark calibration. (a) Landmark pairs,
(b) Fitted radial direction curve.


Fig. 8. Examples of image unwarping. (a) Generated generic panoramic image. (b) A generated panoramic image at distance 184.1 cm . (c) Original omni-image. (d) A generated perspective-view image.


Fig. 9. Perspective-view images generated from an omni-image video sequence of taken by a tracking system. (These images can be considered as some snapshots of a video.)

