# Efficient Path Planning for UAVs to Recognize Chimneys with Excessive Exhaust Emissions 

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#### Abstract

The severity of air pollution has prompted increased attention to air pollution monitoring. This paper uses unmanned aerial vehicles (UAVs) to discover chimneys with excessive exhaust emissions (called 3E-chimneys) in an industrial area. An efficient path-planning framework is proposed to find the flight path of a UAV for checking out chimneys and determine sampling points (SPs) to collect data. The framework builds the shortest path to visit chimneys. Four schemes are designed to select SPs around each chimney. The dual-location sampling (DLS) scheme chooses two SPs for each chimney according to its height. The upward spiral sampling (USS) scheme lets the UAV spiral upward at each chimney to gather data. In the inverted-U sampling (IUS) scheme, the UAV flies up to visit downwind SPs and then flies down to pass upwind SPs. The asynchronous isometric sampling (AIS) scheme picks an upwind SP and multiple downwind SPs. On doing the monitoring task, the UAV checks if some chimneys are skippable (i.e., not 3E-chimneys) using the industrial source complex (ISC3) model. In this way, the UAV can expedite the monitoring task and save energy. Simulation results reveal that the AIS scheme strikes a good balance between cost and performance for the monitoring task. The cost is defined by the length of the UAV's flight path. Four metrics are adopted to evaluate the performance: accuracy, recall, precision, and F1-score. Moreover, we make a prototype system to show the feasibility of our framework, which measures the concentration of $\mathrm{CO}_{2}$ gases emitted from small chimneys in a micro-field with a single wind direction to recognize 3E-chimneys.


Index Terms-Air pollution, chimney, industrial source complex (ISC3), path planning, unmanned aerial vehicle (UAV).

## 1 INTRODUCTION

AIR pollution is a global problem caused by air pollutants harmful to the ecosystem and people, like $\mathrm{CO}_{2}, \mathrm{NO}_{2}$, $\mathrm{SO}_{2}, \mathrm{O}_{3}$, and particulate matter (PM). Factories, automobiles, and forest fires are pollution sources. The World Health Organization indicates that $99 \%$ of the global population breathes air that exceeds guideline limits and contains high levels of pollutants [1]. Hence, people pay more and more attention to air pollution. How to efficiently collect data on air quality and monitor air pollution is a critical issue.

In the past, air pollution monitoring was done by installing expensive, static monitoring stations on dedicated sites, which provided a large-range assessment of air quality. However, this method has three drawbacks [2]. First, since only a few stations are employed, the spatial resolution of air pollution sampling would be poor. Second, using static stations lacks flexibility, as some sites to deploy stations may become superfluous (e.g., factories abandoned as a city grew) or be affected by weather (e.g., changes in seasonal winds). Third, it incurs a high cost to move the stations to support dynamic monitoring tasks, for example, detecting air pollution in a suspicious area.

Unmanned aerial vehicles (UAVs) are a mature technology that has been applied in many fields (e.g., search and rescue, precision agriculture, and surveillance) [3]. UAVs carry multiple sensing devices to collect data and wireless transceivers for communications. Compared to ground-based sensors, using UAVs as flying platforms for sensors brings many benefits, including offering 3D monitoring, adjusting altitude, bypassing obstacles, and quickly reaching destinations [4].

This paper locates air pollution sources via UAVs, focusing on chimneys with excessive exhaust emissions (referred to as

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3E-chimneys) in an industrial area. The ISC3 (industrial source complex, version 3) model [5] is applied to assess pollutant dispersion in the air. We develop a path-planning framework to schedule the UAV's flight path and select sampling points (SPs) for it to collect data at each chimney. The framework creates the shortest path to visit chimneys and then singles out SPs using four schemes that consider air pollutant dispersion. The dual-location sampling (DLS) scheme finds two SPs for each chimney based on its height. The upward spiral sampling (USS) scheme asks the UAV to fly along an upward spiral to gather data. The inverted-U sampling (IUS) scheme makes the UAV fly up to pass downwind (DW) SPs and fly down to visit upwind (UW) SPs. The asynchronous isometric sampling (AIS) scheme picks a UW SP and some DW SPs. With ISC3, the UAV checks if some chimneys can be skipped (i.e., not 3E-chimneys) to reduce the task execution time and save energy.

Through simulations, we measure the cost and performance with which the UAV conducts its monitoring task. The cost is defined as the length of the UAV's flight path. Four metrics are used to measure performance: accuracy, recall, precision, and F1-score. To show the feasibility of our pathplanning framework, we made a prototype by integrating sensing devices and a ZigBee module on an Arduino board. The prototype is used to monitor the concentration of $\mathrm{CO}_{2}$ gases emitted from small chimneys in a micro-field with one single wind direction for identifying 3E-chimneys.

Our contributions are threefold:

- We study the problem of searching for pollution sources (i.e., 3E-chimneys). This is important to pollution control (as we can stop 3E-chimneys from continuing to pollute) but has not been efficiently addressed yet.
- Our framework takes air pollutant dispersion into account (based on ISC3), so it can select suitable positions for a UAV to sample data to identify 3E-chimneys. Do-
ing so facilitates the UAV's execution of the monitoring task.
- Unlike existing solutions, which aim to reduce the UAV's flight path, our framework shortens the path while keeping the recognition accuracy of 3E-chimneys. Thus, it can strike a good balance between cost and performance.
The rest of this paper is organized as follows: Section 2 depicts ISC3, Section 3 surveys related work, and Section 4 defines the problem. We propose the path-planning framework in Section 5. Sections 6 and 7 give the simulation study and prototyping experience. Section 8 concludes this paper.


## 2 ISC3 Pollutant Dispersion Model

ISC3 is one variation of the Gaussian plume model used to formulate how pollutants diffuse and settle in the air. It takes account of various emission sources, such as point, line, area, and volume sources. ISC3 can analyze many characteristics of pollutants, including the downwash effect, both sedimentation and dry deposition of particles, and limited terrain adjustment.

The Pasquill stability category has seven classes that represent atmospheric turbulence [6]: (A) extremely unstable, (B) moderately unstable, (C) slightly unstable, (D) neutral, (E) slightly stable, (F) moderately stable, and (G) extremely stable. Here, we adopt class E for a general case.

Let an emission source of exhaust gas ( $E G$ ) be located at the origin. Then, the pollutant concentration at a position $(x, y, z)$ in the 3D space can be assessed as follows:

$$
\begin{equation*}
\vartheta(x, y, z)=\left(Q_{i} B \varphi_{y} \varphi_{z}\right) / \mu \tag{1}
\end{equation*}
$$

where $Q_{i}$ denotes the EG emission rate, $B$ is a coefficient for disintegration (used when the pollutant has a half-life period, like $\mathrm{SO}_{2}$ ), and $\mu$ is the wind speed. In Eq. (1), $\varphi_{y}$ and $\varphi_{z}$ are dilution factors in horizontal and vertical directions:

$$
\begin{align*}
& \varphi_{y}=\frac{\exp \left[-y^{2} / 2 \sigma_{y}^{2}\right]}{\sqrt{2 \pi} \sigma_{y}},  \tag{2}\\
& \varphi_{z}=\frac{\exp \left[-(z-h)^{2} / 2 \sigma_{z}^{2}\right]+\exp \left[-(z+h)^{2} / 2 \sigma_{z}^{2}\right]}{\sqrt{2 \pi} \sigma_{z}}
\end{align*}
$$

where $h$ is the height of a smoke plume. Moreover, $\sigma_{y}$ and $\sigma_{z}$ are dispersion parameters in horizontal and vertical directions:

$$
\begin{equation*}
\sigma_{y}=\frac{\varepsilon_{1} x}{\left(1+\varepsilon_{2} x\right)^{\varepsilon_{3}}} \quad \text { and } \quad \sigma_{z}=\frac{\varepsilon_{4} x}{\left(1+\varepsilon_{5} x\right)^{\varepsilon_{6}}} \tag{3}
\end{equation*}
$$

For class E , we set empirical constants as $\varepsilon_{1}=0.11, \varepsilon_{2}=0.004$, $\varepsilon_{3}=0.5, \varepsilon_{4}=0.08, \varepsilon_{5}=0.0015$, and $\varepsilon_{6}=0.5$.

The plume's height $h$ depends on the chimney's height $h_{i}$, the downwash effect $\Delta_{W}$, and the rise $\Delta_{\mathrm{R}}$ in the plume:

$$
\begin{equation*}
h=h_{i}+\Delta_{\mathrm{W}}+\Delta_{\mathrm{R}} \tag{4}
\end{equation*}
$$

Let $v_{i}$ be the gas exit velocity (GEV), the speed at which gases leave a chimney. If $v_{i}<1.5 \mu$, the downwash effect will occur, which can be estimated by $\Delta_{\mathrm{W}}=4 r_{i}\left(v_{i} / \mu-1.5\right)$, where $r_{i}$ is the chimney's radius.

Two methods are used to measure the rise in the plume. In the buoyancy-dominated method, the rise is estimated by

$$
\begin{equation*}
\Delta_{\mathrm{R}}=\sqrt[3]{\left(F_{\mathrm{b}} x^{2}\right) / \mu_{\mathrm{G}}^{3}} \tag{5}
\end{equation*}
$$

where $\mu_{\mathrm{G}}$ denotes the wind speed at ground level. In Eq. (5), $F_{\mathrm{b}}$ is a parameter for buoyancy flux. More specifically, $F_{\mathrm{b}}=$ $\left(1-\rho_{s} / \rho\right) g r_{i}^{2} v_{i}$, where $\rho_{s}$ is the density of target gas, $\rho$ is the
density of ambient air, and $g$ is the gravitational acceleration ( $g=9.8 \mathrm{~m} / \mathrm{s}^{2}$ ). In the momentum-dominated method, the rise is calculated by

$$
\begin{equation*}
\Delta_{\mathrm{R}}=\left(3 F_{\mathrm{m}} \times \sin (x \sqrt{\Omega} / \mu)\right) /(\lambda \mu \sqrt{\Omega}) \tag{6}
\end{equation*}
$$

Here, $\Omega$ is a stability parameter, $\lambda=1 / 3+\mu / v_{i}$, and $F_{\mathrm{m}}$ is a parameter for momentum flux, where $F_{\mathrm{m}}=\left(\rho_{s} / \rho\right) r_{i}^{2} v_{i}^{2}$.

Let $T_{s}$ and $T_{a}$ denote the temperatures of the target gas and ambient air, respectively. We compute their difference $\Delta T$ and the crossover temperature difference $T_{c}$ :

$$
\begin{equation*}
\Delta T=T_{s}-T_{a} \quad \text { and } \quad T_{c}=0.015982 \times T_{s} v_{i} \sqrt{\Omega} \tag{7}
\end{equation*}
$$

The stability parameter is derived by $\Omega=\varepsilon_{7} \times g / T_{a}$, where $\varepsilon_{7}$ is an empirical constant (usually set to 0.02 ). Based on $\Delta T$ and $T_{c}$, we can decide which method is used to measure rise $\Delta_{\mathrm{R}}$ in the plume. If $\Delta T>T_{c}$, the buoyancy-dominated method is used; otherwise, we adopt the momentum-dominated method.

## 3 Related Work

In this section, we divide related work into three categories: 1) deploying static sensor networks to monitor air quality; 2) performing air quality monitoring with the help of pedestrians and cars; and 3) using UAVs to detect air pollution.

### 3.1 Monitoring Using Static Sensor Networks

The study [7] measured PM concentration by sensors. Data from sensors was stored on a Raspberry Pi and sent to a server via Wi-Fi. Li et al. [8] developed a PM2.5 monitoring system. They placed sensors based on local weather, terrain, and land use. The work [9] investigated the optimal sensor deployment to capture pollution coverage with different weather conditions and ensure network connectivity. Sun et al. [10] placed multi-type sensors to monitor air quality in a Gaussian spatial field. They considered placing all types of sensors or at least one type of sensor on each chosen site.

Some studies predicted air quality. Four regression methods were employed in [11] for prediction: decision tree, random forest, gradient boosting, and neural network. With a machine learning method called CatBoost, the work [12] forecasted air quality and warned people of the upcoming severe pollution.

However, using static sensor networks to monitor air pollution has two disadvantages [13]. First, it requires deploying many sensors to collect data, incurring a high cost. Second, the sensor network cannot be directly reused when the monitoring task is moved to another place.

### 3.2 Monitoring with the Help of Pedestrians and Cars

Mobile crowdsensing relies on population density to monitor environments. Without deploying sensor networks, pedestrians carry mobile devices with sensors for monitoring [14]. Dutta et al. [15] designed a monitoring system to let people share collected data in the cloud, allowing them to view pollution footprints and air quality. In [16], pedestrians carried sensors to measure the temperature, humidity, and air pollutants. Sensing data was used to draw air quality and temperature maps. A personal monitoring system was used in [17] to observe polluted gases (e.g., $\mathrm{NO}_{2}$ ), ultraviolet intensity, and ambient noises. In [18], a set of locations were chosen for pedestrians to sample data. The concentration of air pollution in unselected locations was inferred by compressive sensing.

TABLE 1: Comparison between the prior work using UAVs for monitoring air pollution and our work (GP: Gaussian plume).

| work | dispersion <br> model | source <br> finding | SP <br> selection | path <br> reduction | monitoring <br> accuracy |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $[25]$ |  |  | $\checkmark$ |  |  |
| $[26]$ |  |  | $\checkmark$ |  | $\checkmark$ |
| $[27]$ | GP |  | $\checkmark$ |  | $\checkmark$ |
| $[28]$ |  | $\checkmark$ | $\checkmark$ |  | $\checkmark$ |
| $[29$ |  |  | $\checkmark$ |  |  |
| $[30]$ |  |  |  |  |  |
| $[31]$ | GP | $\sqrt{ }$ | $\checkmark$ | $\checkmark$ |  |
| ours | ISC3 | $\sqrt{ }$ | $\sqrt{ }$ | $\sqrt{ }$ | $\checkmark$ |

Equipping cars with sensors to monitor air quality has been discussed. The study [19] proposed two sensing scenarios: 1) buses moved on fixed routes to gather data, and 2) drivers put sensors on cars to monitor air quality. Hu et al. [20] used cars to carry sensors to monitor $\mathrm{CO}, \mathrm{NO}_{2}$, and $\mathrm{O}_{3}$ concentration. Roadside monitors helped calibrate the readings of sensors on cars. In [21], static sensors provided a whole view of air quality, while mobile sensors (by cars) gathered more accurate data on designated locations. Both [22] and [23] divided a monitoring area into grids and adjusted the reporting rates of cars in each grid to save on message costs and maintain monitoring accuracy. Chen et al. [24] exploited spatiotemporal correlation in the distribution of air pollutants to derive values at non-sensed locations in areas with sparse data.

However, the above methods have some limitations. First, the mobility of pedestrians and cars is uncontrollable. Asking them to move to specific locations to sample data is difficult. Second, some places are inaccessible to pedestrians and cars (e.g., hazardous or prohibited regions). Third, these methods provide only 2D monitoring of air quality. Some applications, like finding 3E-chimneys, require 3D monitoring.

### 3.3 Monitoring Using UAVs

Using UAVs to carry sensors for monitoring conquers the limitations mentioned in Section 3.2. They can fly to places not reachable by pedestrians or cars and offer 3D monitoring. In [25], a UAV carrying sensors flew to measure the concentration of air pollutants. The study [26] took aerial panoramic images using a UAV and estimated an air quality index (AQI) using the haze pictures. Yang et al. [27] designed a Gaussian plume model based on the neural network to describe particle dispersion and collected data via a UAV to draw AQI maps. Chhikara et al. [28] proposed a federated learning scheme for a swarm of UAVs to monitor air quality. Sensing data was fed to a convolutional neural network to predict AQI. Hu et al. [29] proposed an air quality monitoring system using ground and UAV sensors. Hoque et al. [30] put IoT devices on UAVs to support monitoring services (e.g., detecting air pollution).

Le et al. [31] adopted UAVs to find 3E-chimneys with the minimum search time. They picked two SPs for each chimney, whose heights were computed using a Gaussian plume model. To save the search time, an interference graph was employed to remove some chimneys from the UAV's visiting list. This method, called the interference graph-based algorithm (IGBA), has to know the GEV of each chimney to get the right height for its SPs. Nevertheless, GEVs cannot be known in practice, which may cause inaccurate monitoring results.

Table 1 compares the prior work using UAVs for monitoring air pollution with our work. To estimate air pollutant dispersion, both [27] and [31] employ Gaussian plume models,
while our work adopts the ISC3 model (suitable for industrial areas). For source finding, the work [28] identifies the region with the highest AQI. Both [31] and our work search for 3E-chimneys. Except for [30], others select SPs for UAVs to gather data. As can be seen, the prior work considers either reducing the UAV's flight path or raising monitoring accuracy. Compared to them, our work takes both path reduction and monitoring accuracy into consideration. This distinguishes our work from the prior work and presents the novelty of our work.

## 4 Problem Formulation

We are given a set $\hat{\mathcal{C}}$ of $n$ chimneys in the industrial area with no obstacles or buildings higher than the lowest chimney in $\hat{\mathcal{C}}$. Each chimney $c_{i} \in \hat{\mathcal{C}}$ is modeled as one cylinder with a height of $h_{i}$ and a radius of $r_{i}$. Moreover, $c_{i}$ 's location is a given parameter. The weather is sunny, and there is a stable and unidirectional wind source with a speed of $\mu$. Chimney $c_{i}$ emits EG at an unknown rate (denoted by $Q_{i}$ ). If $Q_{i}>Q_{\mathrm{th}}, c_{i}$ is a 3E-chimney, where $Q_{\mathrm{th}}$ is the statutory threshold.

To identify 3E-chimneys in $\hat{\mathcal{C}}$, a UAV $\psi$ carrying gaseous sensors flies to check out chimneys and sends the collected data to a nearby ground station for analysis ${ }^{1}$. As EG emitted from chimneys may be caustic or have a high temperature, $\psi$ cannot be right above or very close to a chimney. Instead, $\psi$ shall be away from each chimney at a minimum safe distance $d_{\text {MS }}$ to avoid damage.

Depending on the judgment and the true answer, the monitoring result for each chimney $c_{i} \in \hat{\mathcal{C}}$ has four combinations: 1) True positive: $c_{i}$ is judged as a 3E-chimney, and it indeed is. 2) True negative: $c_{i}$ is judged as not a 3E-chimney, and it indeed is not. 3) False positive: $c_{i}$ is judged as a 3E-chimney, but it actually is not. 4) False negative: $c_{i}$ is judged as not a 3E-chimney, but it actually is. Let $f_{\mathrm{TP}}, f_{\mathrm{TN}}, f_{\mathrm{FP}}, f_{\mathrm{FN}}$ be the number of true positives, true negatives, false positives, and false negatives for the monitoring results of all chimneys.

Then, our problem asks how to find a set $\hat{\mathcal{P}}_{i}$ of SPs for each chimney $c_{i}$ and decide a flight path $\Gamma$ for UAV $\psi$ such that the cost of the monitoring task is minimized and the performance is maximized. It can be expressed mathematically as follows:
minimize $\quad \tilde{L}(\Gamma)$,
maximize $\quad\left(f_{\mathrm{TP}}+f_{\mathrm{TN}}\right) /\left(f_{\mathrm{TP}}+f_{\mathrm{TN}}+f_{\mathrm{FP}}+f_{\mathrm{FN}}\right)$,
maximize $\quad \Lambda_{\mathbf{R}}=f_{\mathrm{TP}} /\left(f_{\mathrm{TP}}+f_{\mathrm{FN}}\right)$,
maximize $\quad \Lambda_{\mathbf{P}}=f_{\mathrm{TP}} /\left(f_{\mathrm{TP}}+f_{\mathrm{FP}}\right)$,
maximize $\quad 2\left(\Lambda_{\mathbf{R}} \times \Lambda_{\mathbf{P}}\right) /\left(\Lambda_{\mathbf{R}}+\Lambda_{\mathbf{P}}\right)$
subject to

$$
\begin{array}{ll}
h_{\min } \leq h_{i} \leq h_{\max }, & \forall c_{i} \in \hat{\mathcal{C}}, \\
r_{\min } \leq r_{i} \leq r_{\max } & \forall c_{i} \in \hat{\mathcal{C}}, \\
v_{\min } \leq v_{i} \leq v_{\max }, & \forall c_{i} \in \hat{\mathcal{C}}, \\
Q_{\min } \leq Q_{i} \leq Q_{\max } & \forall c_{i} \in \hat{\mathcal{C}}, \\
\text { distance }\left(\psi, c_{i}\right) \geq r_{i}+d_{\mathrm{MS}}, & \forall c_{i} \in \hat{\mathcal{C}}, \\
z_{i} \in\{0,1\}, & \forall c_{i} \in \hat{\mathcal{C}}, \\
\left|\hat{\mathcal{P}}_{i}\right| \geq 1 \text { if } z_{i}=1, & \forall c_{i} \in \hat{\mathcal{C}}, \\
\operatorname{distance}\left(p_{j}, c_{i}\right) \geq r_{i}+d_{\mathrm{MS}}, & \forall p_{j} \in \hat{\mathcal{P}}, \forall c_{i} \in \hat{\mathcal{C}} . \tag{20}
\end{array}
$$

1. Refer to [32] for how to offer reliable communications between the UAV and ground stations.

TABLE 2: Summary of acronyms and notations. (a) Summary of acronyms

|  | (a) Summary of acronyms |
| :---: | :---: |
| acronym | full name |
| 3E-chimney | chimney with excessive exhaust emissions |
| AIS | asynchronous isometric sampling |
| DLS | dual-location sampling |
| DW/UW | downwind/upwind |
| EG | exhaust gas |
| GEV | gas exit velocity |
| IGBA | interference graph-based algorithm |
| ISC3 | industrial source complex (version 3) |
| IUS | inverted-U sampling |
| SP | sampling point |
| TSP | traveling salesman problem |
| UAV | unmanned aerial vehicle |
| USS | upward spiral sampling |

(b) Summary of notations

| notation | definition |
| :---: | :--- |
| $\hat{\mathcal{C}}$ | set of chimneys in the industrial area $(\|\hat{\mathcal{C}}\|=n)$ |
| $\hat{\mathcal{P}}_{i}^{\mathrm{D}}, \hat{\mathcal{P}}_{i}^{\mathrm{U}}$ | sets of DW and UW SPs for chimney $c_{i}\left(\hat{\mathcal{P}}_{i}^{\mathrm{D}} \cup \hat{\mathcal{P}}_{i}^{\mathrm{U}}=\hat{\mathcal{P}}_{i}\right)$ |
| $h_{i}, r_{i}$ | height and radius of $c_{i}$ |
| $Q_{i}, Q_{\mathrm{th}}$ | $c_{i}$ 's EG emission rate and the statutory threshold |
| $d_{\mathrm{MS}}$ | minimum safe distance |
| $h_{s}$ | distance between two adjacent SPs in $\hat{\mathcal{P}}_{i}^{\mathrm{D}}$ or $\hat{\mathcal{P}}_{i}^{\mathrm{U}}$ |
| $\mu$ | wind speed |
| $m$ | number of pairs of DW/UW SPs in USS and IUS |
| $p_{0}$ | point of departure of UAV $\psi$ |
| $p_{x} \rightarrow p_{y}$ | straight line between two points $p_{x}$ and $p_{y}$ |
| $p_{x} \curvearrowright p_{y}$ | circular arc between $p_{x}$ and $p_{y}$ (radius: $\left.r_{i}+d_{\mathrm{MS}}\right)$ |
| $p_{x}$ 乌 $p_{y}$ | an upward spiral from $p_{x}$ to $p_{y}$ |
| $\tilde{L}(\cdot)$ | length of a path |
| $\Gamma_{\text {IC }}, \Gamma_{i}$ | inter-chimney path and the path to visit SPs in $\hat{\mathcal{P}}_{i}$ |
| $\tilde{h}$ | maximum height difference between two chimneys |
| $\tilde{d}$ | maximum distance between two adjacent chimneys |

In Eq. (8), the objective is to minimize $\psi$ 's flight path (i.e., cost), where $\tilde{L}(\cdot)$ signifies the length of a path. The objectives in Eqs. (9), (10), (11), and (12) are to maximize the accuracy, recall, precision, and F1-score (i.e., performance metrics). For constraints, Eqs. (13) and (14) put upper and lower bounds on the height and radius of each chimney $c_{i}$, where $h_{i}$ and $r_{i}$ are given parameters. Eqs. (15) and (16) impose upper and lower bounds on $c_{i}$ 's GEV and EG emission rate, where $v_{i}$ and $Q_{i}$ are unknown. Eq. (17) gives the constraint of a minimum safe distance. In Eq. (18), variable $z_{i}$ is used to indicate whether $c_{i}$ is scheduled to be visited by $\psi$. If so, $z_{i}=1$; otherwise, $z_{i}=0$. Eq. (19) means that if $c_{i}$ is visited by $\psi$, at least one SP shall be chosen. In Eq. (20), the choice of each SP must consider the constraint of a minimum safe distance.

Table 2 summarizes the acronyms and notations.
Remark 1 (Multiple UAVs). Though our discussion aims at one UAV, it can be extended to the case of multiple UAVs. To do so, we divide all chimneys in $\hat{\mathcal{C}}$ into groups using $K$-means, such that the number of groups is equal to the number of UAVs. Then, each UAV takes charge of checking out the chimneys in a group. Since the chimneys in each group would be close to each other (due to the property of $K$-means [33]), the flight path of each UAV can be reduced.

Remark 2 (Wardens). Some studies [34], [35] consider hindering wardens (i.e., adversaries) from detecting data transmissions of UAVs via the technique of covert communication. Our work aims to utilize UAVs to collect data for finding 3E-chimneys, not to provide secure communications for UAVs. Hence, how to resist the attacks of wardens is out of the paper's scope. We will leave this issue for future work.

## 5 The Proposed Path-Planning Framework

Given the point of departure $p_{0}$ of UAV $\psi$, the path-planning framework contains five steps:

1. Create the shortest path $\Gamma_{\mathrm{S}}$ for $\psi$ to start from $p_{0}$, visit all chimneys in $\hat{\mathcal{C}}$, and return to $p_{0}$.
2. For each chimney $c_{i} \in \hat{\mathcal{C}}$, we find a set $\hat{\mathcal{P}}_{i}$ of SPs for $\psi$ to collect data and decide a path $\Gamma_{i}$ to visit SPs in $\hat{\mathcal{P}}_{i}$.
3. UAV $\psi$ checks out chimneys based on the sequence on $\Gamma_{\mathrm{S}}$. On visiting a chimney $c_{i}, \psi$ flies on path $\Gamma_{i}$ and gathers data at each SP in $\hat{\mathcal{P}}_{i}$.
4. Let $c_{j}$ be the next chimney to be visited. UAV $\psi$ checks if $c_{j}$ is not a 3E-chimney using the ISC3 model. If so, $\psi$ skips $c_{j}$ and visits its next chimney. Otherwise, $\psi$ flies to visit $c_{j}$.
5. Repeat steps 3 and 4 until each chimney in $\hat{\mathcal{C}}$ has been visited or skipped.

In step 1, we can employ an approximation solution to the traveling salesman problem (TSP) [36] to compute path $\Gamma_{\mathrm{S}}$. For steps 2 and 3, we propose four sampling schemes to find set $\hat{\mathcal{P}}_{i}$ of SPs and decide path $\Gamma_{i}$, namely the DLS, USS, IUS, and AIS schemes, as detailed in Sections 5.1, 5.2, 5.3, and 5.4, respectively. Regarding step 4, we discuss how to judge whether a chimney can be skipped in Section 5.5.

Suppose that $\mathcal{C}^{\prime}$ is the set of chimneys skipped by step 4. The solution set of SPs is $\bigcup_{c_{i} \in \hat{\mathcal{C}} \backslash \hat{\mathcal{C}}^{\prime}} \hat{\mathcal{P}}_{i}$. Let $\Gamma_{\mathrm{S}}^{\prime}$ be the updated shortest path by removing the chimneys in $\hat{\mathcal{C}}^{\prime}$ from $\Gamma_{\mathrm{S}}$. The final flight path for UAV $\psi$ will be

$$
\begin{equation*}
\Gamma=\Gamma_{\text {IC }} \cup\left\{\Gamma_{i} \mid \forall c_{i} \in \hat{\mathcal{C}} \backslash \hat{\mathcal{C}}^{\prime}\right\} \tag{21}
\end{equation*}
$$

where $\Gamma_{\text {IC }}$ is the inter-chimney path. It is the union of 1) the path from $p_{0}$ to the first SP of the first chimney on $\Gamma_{S}^{\prime}, 2$ ) each path from the last SP of a chimney to the first SP of its next chimney on $\Gamma_{S^{\prime}}^{\prime}$, and 3) the path from the last SP of the last chimney on $\Gamma_{S}^{\prime}$ to $p_{0}$. Then, Section 5.6 discusses the design rationale and the computational complexity. In Section 5.7, we analyze the distance between each sampling scheme (i.e., a suboptimal solution) and the optimal solution.

### 5.1 Dual-Location Sampling (DLS) Scheme

Air pollutants spread from the UW side to the DW side, so UAV $\psi$ shall fly from the DW side to the UW side. Fig. 1(a) shows candidate SPs for $\psi$ to collect data around a chimney $c_{i}$. We select a set $\hat{\mathcal{P}}_{i}^{\mathrm{D}}$ of DW SPs, where each SP $p_{i, a}^{\mathrm{D}} \in \hat{\mathcal{P}}_{i}^{\mathrm{D}}$ is on $c_{i}$ 's DW side and has a distance $d_{\mathrm{MS}}$ from the boundary extension line of $c_{i}$, as shown by the yellow dots in Fig. 1(a). The distance between two adjacent SPs in $\hat{\mathcal{P}}_{i}^{\mathrm{D}}$ is $h_{s}$. Moreover, we choose a set $\hat{\mathcal{P}}_{i}^{\mathrm{U}}$ of UW SPs, where each SP $p_{i, b}^{\mathrm{U}} \in \hat{\mathcal{P}}_{i}^{\mathrm{U}}$ is on $c_{i}$ 's UW side and has a distance $d_{\mathrm{MS}}$ from the boundary extension line of $c_{i}$, as shown by the blue dots in Fig. 1(a). The distance between two adjacent SPs in $\hat{\mathcal{P}}_{i}^{\mathrm{U}}$ is $h_{s}$.

To avoid being damaged by EG, as UAV $\psi$ flies from a DW SP $p_{i, a}^{\mathrm{D}} \in \hat{\mathcal{P}}_{i}^{\mathrm{D}}$ to a UW SP $p_{i, b}^{\mathrm{U}} \in \hat{\mathcal{P}}_{i}^{\mathrm{U}}$, it follows a circular arc (denoted by $p_{i, a}^{\mathrm{D}} \curvearrowright p_{i, b}^{\mathrm{U}}$ ) whose radius is $r_{i}+d_{\mathrm{MS}}$. Otherwise, $\psi$ directly flies from a point $p_{x}$ to another point $p_{y}$ (as denoted by $p_{x} \rightarrow p_{y}$ ), where a point is an SP or $p_{0}$. If there are obstacles between $p_{x}$ and $p_{y}$, we can adopt the Dijkstra-based method in [37] to find the shortest path for $\psi$ to fly from $p_{x}$ to $p_{y}$ that bypasses obstacles.

For chimney $c_{i}$, DLS picks a DW SP $p_{i, 1}^{\mathrm{D}}$ and a UW SP $p_{i, 1}^{\mathrm{U}}$ with the same height of $h_{i}+\Delta_{\text {DLS }}$. Depending on EG, there are three cases to decide $\Delta_{\text {DLS }}$. In case 1 , the smoke plume trends up when EG is lighter than the air. Thus, we set $\Delta_{\text {DLS }}=h_{s}$ to


Fig. 1: Examples of our proposed sampling schemes.
make UAV $\psi$ fly higher than $c_{i}$ 's height. In this way, $\psi$ can be more likely to sample data in a region of the plume where EG concentration is high. In case 2, the plume trends down as EG is heavier than the air. Hence, we set $\Delta_{\text {DLS }}=-h_{s}$ to let $\psi$ fly lower than $c_{i}$ 's height to help it more likely sample data at a high-concentration region of EG in the plume. In case 3, we set $\Delta_{\text {DLS }}=0$, so $\psi$ 's flying altitude is equal to $c_{i}$ 's height.

Fig. 1(b) presents an example for DLS (case 2), where the visiting sequence of chimneys is $c_{1} \Rightarrow c_{2} \Rightarrow c_{3}$. The interchimney path is $\Gamma_{\text {IC }}=\left\{p_{0} \rightarrow p_{1,1}^{\mathrm{D}}\right\} \cup\left\{p_{1,1}^{\mathrm{U}} \rightarrow p_{2,1}^{\mathrm{D}}\right\} \cup\left\{p_{2,1}^{\mathrm{U}} \rightarrow\right.$ $\left.p_{3,1}^{\mathrm{D}}\right\} \cup\left\{p_{3,1}^{\mathrm{U}} \rightarrow p_{0}\right\}$. Based on Eq. (21), $\psi^{\prime}$ 's flight path is $p_{0} \rightarrow$ $p_{1,1}^{\mathrm{D}} \curvearrowright p_{1,1}^{\mathrm{U}} \rightarrow p_{2,1}^{\mathrm{D}} \curvearrowright p_{2,1}^{\mathrm{U}} \rightarrow p_{3,1}^{\mathrm{D}} \curvearrowright p_{3,1}^{\mathrm{U}} \rightarrow p_{0}$.

After collecting the EG concentration at SPs $p_{i, 1}^{\mathrm{D}}$ and $p_{i, 1}^{\mathrm{U}}$ for chimney $c_{i}$, we estimate $c_{i}$ 's EG emission rate $Q_{i}$. If $Q_{i}$ overtakes the statutory threshold $Q_{\mathrm{th}}, c_{i}$ is judged as a 3Echimney. The details will be discussed later in Section 5.5.

Suppose that UAV $\psi$ visits chimneys in $\hat{\mathcal{C}}$ by a sequence of $c_{x_{1}} \Rightarrow c_{x_{2}} \Rightarrow \cdots \Rightarrow c_{x_{n}}$. Lemma 1 gives the maximum length of inter-chimney path $\Gamma_{\text {IC }}$ in DLS, and Theorem 1 shows the maximum length of $\psi^{\prime}$ s flight path when using DLS.
Lemma 1. Let $\tilde{d}$ be the maximum distance between two adjacent chimneys on path $\Gamma_{S}$, and let $\tilde{h}$ be the maximum difference in height between two chimneys. The maximum length of $\Gamma_{I C}$ in DLS is $l_{D L S}^{I C}=(n-1) \sqrt{\left(\tilde{d}-2 d_{M S}\right)^{2}+\tilde{h}^{2}}+$ $\beta_{1}$, where $\beta_{1}$ is the length sum of paths $p_{0} \rightarrow p_{x_{1}, 1}^{D}$ and $p_{x_{n}, 1}^{U} \rightarrow p_{0}$.

Proof: Observing Fig. 1(b), the flight length between two adjacent chimneys $c_{i}$ and $c_{j}$ is $\tilde{L}\left(p_{i, 1}^{\mathrm{U}} \rightarrow p_{j, 1}^{\mathrm{D}}\right)=\sqrt{\alpha_{1}^{2}+\alpha_{2}^{2}}$. We have $\alpha_{1}=\tilde{d}_{i, j}-2 d_{\mathrm{MS}}$, where $\tilde{d}_{i, j}$ is the distance between $c_{i}$ and $c_{j}$. Since all chimneys have the same $\Delta_{\text {DLS }}$ value, $\alpha_{2}$ is the height difference between $c_{i}$ and $c_{j}$. The worst case is that no chimneys are skipped. Thus, we obtain that $\tilde{L}\left(\Gamma_{\text {IC }}\right)=$ $\sum_{k=1}^{n-1} \tilde{L}\left(p_{x_{k}, 1}^{\mathrm{U}} \rightarrow p_{x_{k+1}, 1}^{\mathrm{D}}\right)+\tilde{L}\left(p_{0} \rightarrow p_{x_{1}, 1}^{\mathrm{D}}\right)+\tilde{L}\left(p_{x_{n}, 1}^{\mathrm{U}} \rightarrow p_{0}\right) \leq$ $(n-1) \sqrt{\left(\tilde{d}-2 d_{\mathrm{MS}}\right)^{2}+\tilde{h}^{2}}+\beta_{1}$.

Theorem 1. With DLS, UAV $\psi$ 's flight path has a length no longer than $l_{D L S}^{I C}+n \pi\left(r_{\max }+d_{M S}\right)$.

Proof: The worst case is that $\hat{\mathcal{C}}^{\prime}=\emptyset$ in Eq. (21) (i.e., UAV $\psi$ has to check out every chimney). Thus, the flight length for $\psi$ in DLS is $\tilde{L}\left(\Gamma_{\text {IC }}\right)+\sum_{k=1}^{n} \tilde{L}\left(p_{x_{k}, 1}^{\mathrm{D}} \curvearrowright p_{x_{k}, 1}^{\mathrm{U}}\right)=\tilde{L}\left(\Gamma_{\text {IC }}\right)+$ $\sum_{k=1}^{n} \pi\left(r_{x_{k}}+d_{\mathrm{MS}}\right) \leq l_{\mathrm{DLS}}^{\mathrm{IC}}+n \pi\left(r_{\max }+d_{\mathrm{MS}}\right)$.

### 5.2 Upward Spiral Sampling (USS) Scheme

To improve monitoring performance, USS selects $m$ pairs of DW/UW SPs for each chimney $c_{i}$, where $m \geq 3$. The distance between two adjacent DW or UW SPs is equal. To do so, we use a vector $V=[\xi, \xi-1, \xi-2, \cdots, \xi-(m-1)]$ to determine the height of each SP, where $\xi \in \mathbb{Z}^{+}$and $\xi<m-1$. The heights of the $k$-th pair of DW/UW SPs (i.e., $p_{i, k}^{\mathrm{D}}$ and $p_{i, k}^{\mathrm{U}}$ ) are set to $h_{i}-h_{s} \times V[k]$. Since $\xi \geq 1$, we ensure that some SPs are lower than $c_{i}$. Besides, as $\xi<m-1$, some SPs must be higher than $c_{i}$. In this way, UAV $\psi$ can collect data in the smoke plume more comprehensively.

To gather $c_{i}$ 's data, beginning with SP $p_{i, 1}^{\mathrm{D}}$, UAV $\psi$ visits DW and UW SPs alternately. Hence, path $\Gamma_{i}$ is $p_{i, 1}^{\mathrm{D}} \curvearrowright p_{i, 1}^{\mathrm{U}} \curvearrowright$ $p_{i, 2}^{\mathrm{D}} \curvearrowright p_{i, 2}^{\mathrm{U}} \curvearrowright \cdots \curvearrowright p_{i, m}^{\mathrm{U}}$, which forms an upward spiral (as denoted by $p_{i, 1}^{\mathrm{D}} \leftrightarrow p_{i, m}^{\mathrm{U}}$ ). On flying along $\Gamma_{i}, \psi$ compares the EG concentration sampled at each pair of DW/UW SPs and employs the method in Section 5.5 to check if $c_{i}$ is a 3Echimney. If so, there is no need to further collect $c_{i}$ 's data, and $\psi$ can directly fly to the next chimney to reduce the flight path. Otherwise, $\psi$ flies to visit the next pair of $c_{i}$ 's DW/UW SPs to collect data for judging $c_{i}$.

Fig. 1(c) gives an example, where $m=4$ and $\xi=1$. Each chimney $c_{i}$ has four pairs of DW/UW SPs by default, whose heights are $h_{i}-h_{s}, h_{i}, h_{i}+h_{s}$, and $h_{i}+2 h_{s}$. Since UAV $\psi$ infers that $c_{2}$ is a 3 E -chimney after sampling data at $p_{2,3}^{\mathrm{D}}$ and $p_{2,3}^{\mathrm{U}}, \psi$ stops collecting $c_{2}$ 's data and flies to $c_{3}$. That is why $c_{2}$ has only three pairs of DW/UW SPs. Then, $\psi$ 's flight path is $p_{0} \rightarrow p_{1,1}^{\mathrm{D}} \rightarrow p_{1,4}^{\mathrm{U}} \rightarrow p_{2,1}^{\mathrm{D}} \rightarrow p_{2,3}^{\mathrm{U}} \rightarrow p_{3,1}^{\mathrm{D}} \rightarrow p_{3,4}^{\mathrm{U}} \rightarrow p_{0}$.

For USS, Lemma 2 analyzes the maximum length of interchimney path $\Gamma_{\text {IC }}$, and Theorem 2 shows the upper bound on the length of $\psi$ 's flight path.
Lemma 2. The maximum length of $\Gamma_{I C}$ in USS is $l_{U S S}^{I C}=(n-$ 1) $\sqrt{\left(\tilde{d}-2 d_{M S}\right)^{2}+\varrho^{2}}+\beta_{2}$, where $\varrho=\tilde{h}+(m-1) h_{s}$ and $\beta_{2}$ is the length sum of paths $p_{0} \rightarrow p_{x_{1}, 1}^{D}$ and $p_{x_{n}, m}^{U} \rightarrow p_{0}$.

Proof: In Fig. 1(c), the flight length of UAV $\psi$ between adjacent chimneys $c_{i}$ and $c_{j}$ is $\tilde{L}\left(p_{i, m}^{\mathrm{U}} \rightarrow p_{j, 1}^{\mathrm{D}}\right)=\sqrt{\alpha_{1}^{2}+\alpha_{3}^{2}}$, where $\alpha_{1}=\tilde{d}_{i, j}-2 d_{\text {MS }}$. Since each chimney has $m$ pairs of SPs and $\psi$ flies from $c_{i}$ 's last UW SP to ${\underset{\sim}{c}}_{j}$ 's first DW SP, we have $\alpha_{3}=\tilde{h}_{i, j}+(m-1) h_{s}$, where $\tilde{h}_{i, j}$ is the height difference between $c_{i}$ and $c_{j}$. Thus, $\Gamma_{\text {IC }}$ 's length is $\tilde{L}\left(\Gamma_{\text {IC }}\right)=$ $\sum_{k=1}^{n-1} \tilde{L}\left(p_{x_{k}, m}^{\mathrm{U}} \rightarrow p_{x_{k+1}, 1}^{\mathrm{D}}\right)+\tilde{L}\left(p_{0} \rightarrow p_{x_{1}, 1}^{\mathrm{D}}\right)+\tilde{L}\left(p_{x_{n}, m}^{\mathrm{U}} \rightarrow p_{0}\right) \leq$ $(n-1) \sqrt{\left(\tilde{d}-2 d_{\mathrm{MS}}\right)^{2}+\varrho^{2}}+\beta_{2}$.
Theorem 2. In USS, the length of UAV $\psi$ 's flight path is below
$l_{U S S}^{I C}+m n \sqrt{h_{s}^{2}+\left[2 \pi\left(r_{\max }+d_{M S}\right)\right]^{2}}$.
Proof: In USS, the worst case takes place when $\psi$ has to sample data at each pair of DW/UW SP for all chimneys in $\hat{\mathcal{C}}$. Thus, the flight path has a length of $\tilde{L}\left(\Gamma_{\text {IC }}\right)+\sum_{k=1}^{n} \tilde{L}\left(p_{x_{k}, 1}^{D} \rightarrow\right.$ $\left.p_{x_{k}, m}^{\mathrm{U}}\right)=\tilde{L}\left(\Gamma_{\text {IC }}\right)+\sum_{k=1}^{n} m \sqrt{h_{s}^{2}+\left[2 \pi\left(r_{x_{k}}+d_{\mathrm{MS}}\right)\right]^{2}} \leq l_{\mathrm{USS}}^{\mathrm{IC}}+$ $m n \sqrt{h_{s}^{2}+\left[2 \pi\left(r_{\max }+d_{\mathrm{MS}}\right)\right]^{2}}$.

### 5.3 Inverted-U Sampling (IUS) Scheme

Like USS, IUS selects $m$ pairs of DW /UW SPs for each chimney $c_{i}$. However, instead of flying in an upward spiral, UAV $\psi$ flies up to visit each DW SP and then flies down to visit each UW SP. In other words, path $\Gamma_{i}$ is $p_{i, 1}^{\mathrm{D}} \rightarrow p_{i, 2}^{\mathrm{D}} \rightarrow \cdots \rightarrow p_{i, m}^{\mathrm{D}} \curvearrowright p_{i, m}^{\mathrm{U}} \rightarrow$ $p_{i, m-1}^{\mathrm{U}} \rightarrow \cdots \rightarrow p_{i, 1}^{\mathrm{U}}$, which forms an inverted-U shape. In this way, we can save the time that $\psi$ spends hovering over each chimney and reduce its flight path.

Fig. 1(d) gives an example, where $m=4$. The flight path is $p_{0} \rightarrow p_{1,1_{\mathrm{U}}^{\mathrm{D}}}^{\mathrm{D}} \rightarrow \cdots \rightarrow p_{1,4}^{\mathrm{D}}{\underset{\mathrm{U}}{ }}_{\curvearrowright}^{p_{1,4}^{\mathrm{U}} \rightarrow \cdots \rightarrow p_{1,1}^{\mathrm{U}} \rightarrow p_{2,1}^{\mathrm{D}} \rightarrow \cdots \rightarrow}$ $p_{2,4}^{\mathrm{D}} \curvearrowright p_{2,4}^{\mathrm{U}} \rightarrow \cdots \rightarrow p_{2,1}^{\mathrm{U}} \rightarrow p_{3,1}^{\mathrm{D}} \rightarrow \cdots \rightarrow p_{3,4}^{\mathrm{D}} \curvearrowright p_{3,4}^{\mathrm{U}} \rightarrow$ $\cdots \rightarrow p_{3,1}^{\mathrm{U}} \rightarrow p_{0}$. Theorem 3 analyzes the maximum length of the flight path in IUS.
Theorem 3. In IUS, the length of the flight path is at most $l_{D L S}^{I C}+$ $n\left(\pi\left(r_{\max }+d_{\text {MS }}\right)+2(m-1) h_{s}\right)$.

Proof: Let us compare the flight paths in DLS and IUS, as shown in Fig. 1(b) and (d). They are almost the same, except that IUS makes UAV $\psi$ additionally fly on two paths $p_{i, 1}^{\mathrm{D}} \rightarrow$ $\cdots \rightarrow p_{i, m}^{\mathrm{D}}$ and $p_{i, m}^{\mathrm{U}} \rightarrow \cdots \rightarrow p_{i, 1}^{\mathrm{U}}$ for each chimney $c_{i}$. The length of each path is $(m-1) h_{s}$. Based on Theorem 1, the maximum length of the flight path in DLS is $l_{\mathrm{DLS}}^{\text {IC }}+n \pi\left(r_{\max }+\right.$ $d_{\text {MS }}$ ). Since $\hat{\mathcal{C}}$ has $n$ chimneys, the length of the flight path in IUS is at most $l_{\text {DIS }}^{\text {IC }}+n\left(\pi\left(r_{\text {max }}+d_{\text {MS }}\right)+2(m-1) h_{s}\right)$.

Note that UAV $\psi$ shall collect data at all DW SPs of each chimney $c_{i}$. If $\psi$ collects data at a UW SP $p_{i, k}^{\mathrm{U}}(k>1)$ and judges that $c_{i}$ is a 3E-chimney (by comparing the data collected at DW SP $p_{i, k}^{\mathrm{D}}$ ), residual UW SPs (i.e., $p_{i, k-1}^{\mathrm{U}}, \cdots, p_{i, 1}^{\mathrm{U}}$ ) can be skipped to reduce $\psi$ 's flight path.

### 5.4 Asynchronous Isometric Sampling (AIS) Scheme

Based on IUS, AIS makes two improvements: 1) decreasing SPs for each chimney without significantly degrading monitoring performance and 2) reducing the path for UAV $\psi$ to fly from one chimney to the next.

Improvement 1. After EG is discharged from a chimney, its spreading range expands as the distance from the chimney
increases. Thus, we shall choose more DW SPs to help UAV $\psi$ collect more data within the EG's spreading range. On the other hand, the number of UW SPs can be reduced to shorten the flight path. Hence, for each chimney $c_{i}$, AIS selects $m$ DW SPs $p_{i, 1}^{\mathrm{D}}, \cdots, p_{i, m}^{\mathrm{D}}$ and one UW SP $p_{i, y}^{\mathrm{U}}$, where $y=1$ or $y=m$ (depending on $\psi$ 's flight direction on the DW side, as discussed in Improvement 2). To check if $c_{i}$ is a 3E-chimney, $\psi$ compares the EG concentration sampled at $p_{i, k}^{\mathrm{D}}$ and $p_{i, y^{\prime}}^{\mathrm{U}}$ for $k=1 \ldots m$. In other words, the UW SP $p_{i, y}^{\mathrm{U}}$ is taken as the reference for comparison with each DW SP.

Improvement 2. In IUS, UAV $\psi$ always flies up on the DW side of each chimney. When there is only one UW SP, doing so may increase $\psi$ 's flight path. Take Fig. 1(e) as an example. When $\psi$ flies from $c_{1}$ to $c_{2}$, if IUS's policy is used, $\psi$ flies on path $p_{1,4}^{\mathrm{U}} \rightarrow p_{2,1}^{\mathrm{D}}$. In fact, there is a shorter path, namely $p_{1,4}^{\mathrm{U}} \rightarrow p_{2,4}^{\mathrm{D}}$. Hence, AIS makes two modifications:
(1) If $p_{i, 1}^{\mathrm{D}}$ is the first DW SP to be visited, $\psi$ flies up on $c_{i}{ }^{\text {'s }}$ DW side (i.e., $p_{i, 1}^{\mathrm{D}} \Rightarrow p_{i, m}^{\mathrm{D}}$ ), and we set $y=m$. Otherwise, $\psi$ flies down (i.e., $p_{i, m}^{\mathrm{D}} \Rightarrow p_{i, 1}^{\mathrm{D}}$ ), and we set $y=1$. For the first chimney on $\Gamma_{\mathrm{S}}, \psi$ flies up on its DW side.
(2) Suppose that the last SP of $c_{i}$ to be visited is $p_{a}$. When $\psi$ flies from $c_{i}$ to the next chimney, say, $c_{j}$, we pick an SP $p_{b}$ from four candidate SPs of $c_{j}$ by

$$
\begin{equation*}
p_{b}=\arg \min _{p_{b} \in\left\{p_{j, 1}^{\mathrm{D}}, p_{j, m}^{\mathrm{D}}, p_{j, 1}^{\mathrm{u}}, p_{j, m}^{\mathrm{u}}\right\}} \tilde{L}\left(p_{a}, p_{b}\right), \tag{22}
\end{equation*}
$$

to minimize the path for $\psi$ to fly from $c_{i}$ to $c_{j}$.
Fig. 1(e) shows an example, where $m=4$. The flight path is $p_{0} \rightarrow p_{1,1}^{\mathrm{D}} \rightarrow \cdots \rightarrow p_{1,4}^{\mathrm{D}} \curvearrowright p_{1,4}^{\mathrm{U}} \rightarrow p_{2,4}^{\mathrm{D}} \rightarrow \cdots \rightarrow p_{2,1}^{\mathrm{D}} \curvearrowright$ $p_{2,1}^{\mathrm{U}} \rightarrow p_{3,1}^{\mathrm{U}} \curvearrowright p_{3,1}^{\mathrm{D}} \rightarrow \cdots \rightarrow p_{3,4}^{\mathrm{D}} \rightarrow p_{0}$. Evidently, $\psi^{\prime}$ 's flight direction and the locations of UW SPs may adaptively change. We give the maximum inter-chimney path length and flight length in AIS in Lemma 3 and Theorem 4, respectively.
Lemma 3. The maximum inter-chimney path length in AIS is $l_{A I S}^{I C}=(n-1) \sqrt{\left(\tilde{d}-2 d_{M S}\right)^{2}+\tilde{h}^{2}}+\beta_{3}$, where $\beta_{3}$ is the length sum of paths $p_{0} \rightarrow p_{x_{1}, 1}^{D}$ and $p_{z} \rightarrow p_{0}$. Here, $p_{z}$ is the last SP to be visited at chimney $c_{x_{n}}$ (i.e., the last chimney on $\Gamma_{S P}$ ).

Proof: In Fig. 1(e), the flight length between chimneys $c_{i}$ and $c_{j}$ is $\sqrt{\alpha_{1}^{2}+\alpha_{2}^{2}}$, where $\alpha_{1}=\tilde{d}_{i, j}-2 d_{\mathrm{MS}}$. By Eq. (22), $p_{a}$ and $p_{b}$ have the same height index ${ }^{2}$, so their height difference (i.e., $\left.\alpha_{2}\right)$ is $\tilde{h}_{i, j}$. Like the proof in Lemma 1, the maximum length of $\Gamma_{\text {IC }}$ is $(n-1) \sqrt{\left(\tilde{d}-2 d_{\mathrm{MS}}\right)^{2}+\tilde{h}^{2}}+\beta_{3}$.
Theorem 4. In AIS, the maximum length of the flight path is $l_{A I S}^{I C}+n\left(\pi\left(r_{\max }+d_{M S}\right)+(m-1) h_{s}\right)$.

Proof: Let us compare the flight paths in IUS and AIS, as shown in Fig. 1(d) and (e). For each chimney $c_{i} \in \hat{\mathcal{C}}$, UAV $\psi$ does not need to fly on path $p_{i, m}^{\mathrm{U}} \rightarrow \cdots \rightarrow p_{i, 1}^{\mathrm{U}}$, as there is only one UW SP in AIS. Thus, compared to IUS, the length of $\Gamma_{i}$ is reduced by $(m-1) h_{s}$ in AIS. Based on Theorem 3 and Lemma 3, the maximum length of flight path in AIS is $l_{\text {AIS }}^{\text {IC }}+n\left(\pi\left(r_{\max }+d_{\text {MS }}\right)+(m-1) h_{s}\right)$.

### 5.5 Judgment on 3E-Chimneys

To check if a chimney $c_{i}$ is a 3E-chimney, UAV $\psi$ compares the EG concentration sampled at a DW SP $p_{d}$ (denoted by $\vartheta_{d}$ ) and that at a UW SP $p_{u}$ (denoted by $\vartheta_{u}$ ) for $c_{i}$. Specifically, $\vartheta_{u}$ can be viewed as $c_{i}$ 's ambient EG concentration (without $c_{i}$ 's effect), and $\vartheta_{d}$ is the sum of $c_{i}$ 's ambient EG concentration

$$
\text { 2. If } p_{a}=p_{i, 1}^{\mathrm{U}}, p_{b}=p_{j, 1}^{\mathrm{D}} \text { or } p_{j, 1}^{\mathrm{U}} \text {. If } p_{a}=p_{i, m^{\prime}}^{\mathrm{U}}, p_{b}=p_{j, m}^{\mathrm{D}} \text { or } p_{j, m}^{\mathrm{U}} \text {. }
$$

and the concentration of EG emitted from $c_{i}$. Hence, the EG concentration caused by $c_{i}$ is $\vartheta_{d}-\vartheta_{u}$. Based on Eq. (1), we can deduce $c_{i}$ 's EG emission rate by

$$
\begin{equation*}
Q_{i}=\mu\left(\vartheta_{d}-\vartheta_{u}\right) /\left(\varphi_{y} \varphi_{z} B\right) . \tag{23}
\end{equation*}
$$

To calculate dilution factors $\varphi_{y}$ and $\varphi_{z}$ in Eq. (2), we employ $p_{d}$ 's coordinates as the reference and set the plume's height to $h=h_{i}+h_{s}$. Then, $c_{i}$ is judged as a 3E-chimney if $Q_{i}>Q_{\mathrm{th}}$, where $Q_{\mathrm{th}}$ is the statutory threshold.

Generally speaking, a nearby chimney $c_{j}$ on $c_{i}$ 's UW side can affect the EG concentration at $c_{i}{ }^{\prime}$ s UW SPs. If $c_{j}$ emits much EG (i.e., 3E-chimney), the EG concentration sampled at $c_{i}$ 's UW SPs would be unusually high. On the contrary, if the difference between the EG concentration at $c_{i}{ }^{\prime}$ s UW SPs and the environmental EG concentration is small, $c_{j}$ is very likely not a 3E-chimney. In this case, UAV $\psi$ can skip $c_{j}$ (for collecting its data) to reduce the flight path.

Suppose that $c_{j}$ is the next chimney to $c_{i}$ on path $\Gamma_{\mathrm{S}}$. If $c_{j}$ is on $c_{i}$ 's UW side and $\tilde{d}_{i, j} \leq D_{\mathrm{th}}$, UAV $\psi$ estimates $c_{j}$ 's EG emission rate after collecting data at all of $c_{i}$ 's UW SPs. Here, the condition $\tilde{d}_{i, j} \leq D_{\mathrm{th}}$ indicates that $c_{j}$ is close enough to $c_{i}$, so the emission of EG by $c_{j}$ has a great impact on the EG concentration at $c_{i}{ }^{\prime}$ s UW SPs. Given set $\hat{\mathcal{P}}_{i}^{\mathrm{U}}$ of $c_{i}{ }^{\prime}$ s UW SPs, let $\vartheta_{k}$ be the EG concentration sampled at each SP $p_{i, k}^{\mathrm{U}} \in \hat{\mathcal{P}}_{i}^{\mathrm{U}}$. Then, we can estimate $c_{j}$ 's EG emission rate as follows:

$$
\begin{equation*}
Q_{j}^{\mathrm{E}}=\min _{\forall p_{i, k}^{\mathrm{U}} \in \hat{\mathcal{P}}_{i}^{\mathrm{U}}}\left\{\mu\left(\vartheta_{k}-\bar{\vartheta}\right) /\left(\varphi_{y} \varphi_{z} B\right) \mid \varphi_{y} \varphi_{z} B>0\right\} \tag{24}
\end{equation*}
$$

where $\bar{\vartheta}$ is the environmental EG concentration ${ }^{3}$. When using $\vartheta_{k}$ to do calculations, we take $p_{i, k}^{\mathrm{U}}$ 's coordinates as the reference for computing dilution factors $\varphi_{y}$ and $\varphi_{z}$. In Eq. (24), the condition $\varphi_{y} \varphi_{z} B>0$ ensures that the emission of EG by $c_{j}$ is correlated with the EG concentration at $\mathrm{SP} p_{i, k}^{\mathrm{U}}$. However, if no SP in $\hat{\mathcal{P}}_{i}^{\mathrm{U}}$ fulfills this condition, we set $Q_{j}^{\mathrm{E}}=\infty$, which means that $c_{j}$ 's EG emission rate cannot be estimated by using the EG concentration sampled at $c_{i}{ }^{\prime}$ s UW SPs. Then, UAV $\psi$ infers that $c_{j}$ is not a 3E-chimney if

$$
\begin{equation*}
Q_{j}^{\mathrm{E}} \leq Q_{\mathrm{th}}-Q_{\mathrm{adj}} \tag{25}
\end{equation*}
$$

where $Q_{\text {adj }}$ is used to reflect the error of estimation in Eq. (24) and $Q_{\mathrm{adj}}<Q_{\mathrm{th}} / 2$.

### 5.6 Design Rationale and Computational Complexity

Let us discuss the rationale of our path-planning framework. It creates path $\Gamma_{\mathrm{S}}$ via a TSP approximation solution to decide the sequence for UAV $\psi$ to visit all chimneys in $\hat{\mathcal{C}}$. Then, we design four schemes to select SPs for each chimney, which is the framework's core. DLS aims to minimize $\psi$ 's flight path by picking two SPs for each chimney. Due to only a few SPs, the monitoring performance in DLS may be low. Thus, USS selects $m$ pairs of DW/UW SPs for each chimney and lets $\psi$ alternately collect data at DW and UW SPs, so the flight path around each chimney forms an upward spiral. However, these spirals lead to a long flight path. Hence, IUS asks $\psi$ to visit DW SPs and then UW SPs to reduce the path length. AIS improves IUS by reducing UW SPs and shortening the path as $\psi$ flies between chimneys. Hence, AIS can shorten the flight path without significantly degrading performance.
3. To obtain $\bar{\vartheta}$, UAV $\psi$ can sample data at a location where there is no chimney nearby on the UW side of the industrial area.

When executing the monitoring task on a chimney, UAV $\psi$ checks if the next chimney is obviously not a 3E-chimney (using the ISC3 model) at the same time. In this way, some chimneys can be removed from $\psi^{\prime}$ s visiting list, which further reduces the flight path. Theorem 5 analyzes the time complexity of our path-planning framework.

Theorem 5. Let $T_{\text {TSP }}(n)$ be the amount of time taken by a TSP approximation solution to find the shortest path to visit $n$ nodes. The worst-case time complexity of our pathplanning framework is $T_{\mathrm{TSP}}(n)+T_{\mathrm{SM}}$, where $T_{\mathrm{SM}}=O(n)$ if the DLS scheme is used to find SPs and $T_{\mathrm{SM}}=O(n m)$ otherwise.

Proof: In the path-planning framework, step 1 uses a TSP approximation solution to find path $\Gamma_{\mathrm{S}}$ to visit $n$ chimneys in $\hat{\mathcal{C}}$, which takes $T_{\text {TSP }}(n)$ time. In steps 2 and 3 , UAV $\psi$ flies to sample data at the SPs for each chimney $c_{i}$ and uses Eq. (23) to compute its EG emission rate. Since DLS picks a pair of DW/UW SPs for $c_{i}$, it spends $O(1)$ time. On the other hand, USS, IUS, and AIS select $m$ DW SPs for $c_{i}$, so they take $O(m)$ time to run steps 2 and 3 . For step $4, \psi$ checks if the next chimney, say, $c_{j}$ is not a 3E-chimney based on the data collected at $c_{i}$ 's UW SPs (i.e., $\hat{\mathcal{P}}_{i}^{\mathrm{U}}$ ). This is done by estimating $c_{j}$ 's EG emission rate $Q_{j}^{\mathrm{E}}$ using Eq. (24). Because DLS, USS, IUS, and AIS pick $1, m, m$, and 1 UW SPs for $c_{i}$, they take $O(1), O(m), O(m)$, and $O(1)$ time to execute step 4 . As $\hat{\mathcal{C}}$ contains $n$ chimneys, $T_{\mathrm{SM}}$ is equal to $n(O(1)+O(1))=O(n)$, $n(O(m)+O(m))=O(n m), n(O(m)+O(m))=O(n m)$, $n(O(m)+O(1))=O(n m)$ when DLS, USS, IUS, and AIS are respectively used. Hence, the theorem is proven.

### 5.7 Distance to the Optimal Solution

We investigate the distance between each sampling scheme (i.e., a suboptimal solution) and the optimal solution. Let $\mathbf{L}_{\mathrm{x}}$ and $\mathbf{L}_{\mathrm{opt}}$ denote the flight lengths of a sampling scheme X and the optimal solution. The distance is defined as the maximum difference between $\mathbf{L}_{\mathrm{x}}$ and $\mathbf{L}_{\text {opt }}$.

Like DLS, the optimal solution selects a pair of DW/UW SPs (with the same height) for each chimney $c_{i} \in \hat{\mathcal{C}}$. Suppose that no chimneys are excluded by step 4 in the path-planning framework. Then, the optimal solution has the same interchimney path (i.e, $\Gamma_{\text {IC }}$ ) as DLS. Based on Eq. (4), the height of an SP for $c_{i}$ in the optimal solution is $h_{i}+\Delta_{\mathrm{W}}+\Delta_{\mathrm{R}}$ (i.e., the plume's height). On the other hand, the height of an SP for $c_{i}$ in DLS is $h_{i}+\Delta_{\text {DLS }}$, where $\Delta_{\text {DLS }}=h_{s},-h_{s}$, or 0 . Hence, to sample data at two SPs $p_{i, 1}^{\mathrm{D}}$ and $p_{i, 1}^{\mathrm{U}}$ for a chimney $c_{i}$, the maximum difference between the flight lengths in the optimal solution and DLS is $2\left(\Delta_{\mathrm{W}}+\Delta_{\mathrm{R}}+h_{s}\right)$. Since $\hat{\mathcal{C}}$ has $n$ chimneys, the distance between DLS and the optimal solution is $2 n\left(\gamma+h_{s}\right)$, where $\gamma=\max \left\{\Delta_{\mathrm{W}}+\Delta_{\mathrm{R}} \mid \forall c_{i} \in \hat{\mathcal{C}}\right\}$.

To obtain the distance for USS, IUS, and AIS, we compute the maximum gap between the flight lengths in each sampling scheme and DLS. For convenience, we denote this as $\mathbf{L}_{\mathrm{x}}-\mathbf{L}_{\text {DLS }}$. Besides, we set $\pi\left(r_{\max }+d_{\text {MS }}\right)=\Phi$.

For USS, based on Theorems 2 and 1, we obtain that $\mathbf{L}_{\text {USS }}-$ $\mathbf{L}_{\text {DLS }}=\left(l_{\text {USS }}^{\text {IC }}+m n \sqrt{h_{s}^{2}+4 \Phi^{2}}\right)-\left(l_{\text {DLS }}^{\text {IC }}+n \Phi\right)=l_{\text {USS }}^{\text {IC }}-l_{\text {DLS }}^{\text {IC }}+$ $n\left(m \sqrt{h_{s}^{2}+4 \Phi^{2}}-\Phi\right)$. Thus, the distance between USS and the optimal solution is $l_{\mathrm{USS}}^{\mathrm{IC}}-l_{\mathrm{DLS}}^{\text {IC }}+n\left(m \sqrt{h_{s}^{2}+4 \Phi^{2}}-\Phi\right)+2 n(\gamma+$ $h_{s}$ ).

For IUS, according to Theorems 3 and 1, we can derive that $\mathbf{L}_{\text {IUS }}-\mathbf{L}_{\text {DLS }}=\left(l_{\text {DLS }}^{\text {IC }}+n\left(\Phi+2(m-1) h_{s}\right)\right)-\left(l_{\text {DLS }}^{\text {IC }}+n \Phi\right)=$

TABLE 3: Simulation parameters.
Environmental parameters:

| area size | $2 \mathrm{~km} \times 2 \mathrm{~km}$ |
| :--- | :--- |
| ambient temperature $\left(T_{a}\right)$ | $25^{\circ} \mathrm{C}$ |
| wind speed $(\mu)$ | $1 \mathrm{~m} / \mathrm{s}$ |
| EG type | $\mathrm{CO}_{2}($ concentration: $\bar{\vartheta}=400 \mathrm{ppm})$ |
| molecular weight | air: $29 \mathrm{~g} / \mathrm{m}^{3}, \mathrm{EG}: 44 \mathrm{~g} / \mathrm{m}^{3}$ |
| Chimney-related parameters: |  |
| number | $20,40,60,80,100$ |
| height $\left(h_{i}\right)$ | 25 m or 30 m |
| radius $\left(r_{i}\right)$ | 0.5 m or 1 m |
| EG temperature $\left(T_{s}\right)$ | $30^{\circ} \mathrm{C}$ |
| GEV $\left(v_{i}\right)$ | $2 \mathrm{~m} / \mathrm{s}$ or $3 \mathrm{~m} / \mathrm{s}$ |
| EG emission rate $\left(Q_{i}\right)$ | $50,000 \sim 500,000 \mathrm{mg} / \mathrm{s}$ |
| Parameters of sampling schemes: |  |
| minimum safe distance $\left(d_{\mathrm{MS}}\right)$ | 5 m |
| interval between SPs $\left(h_{s}\right)$ | 1 m |
| number of SP pairs $(m)$ | 20 |
| statutory threshold $\left(Q_{\mathrm{th}}\right)$ | $400,000 \mathrm{mg} / \mathrm{s}$ |
| $Q_{\text {adj }}$ in Eq. $(25)$ | $130,000 \mathrm{mg} / \mathrm{s}$ |

$2 n(m-1) h_{s}$. Thus, the distance between IUS and the optimal solution is $2 n(m-1) h_{s}+2 n\left(\gamma+h_{s}\right)=2 n\left(m h_{s}+\gamma\right)$.

For AIS, with both Theorems 4 and 1, we derive that $\mathbf{L}_{\text {AIS }}-$ $\mathbf{L}_{\mathrm{DLS}}=\left(l_{\mathrm{AIS}}^{\text {IC }}+n\left(\Phi+(m-1) h_{s}\right)\right)-\left(l_{\mathrm{DLS}}^{\text {IC }}+n \Phi\right)=l_{\mathrm{AIS}}^{\text {ICS }}-$ $l_{\text {DLS }}^{\text {IC }}+n(m-1) h_{s}$. Hence, the distance between AIS and the optimal solution is $l_{\mathrm{AIS}}^{\text {IC }}-l_{\mathrm{DLS}}^{\text {IC }}+n(m-1) h_{s}+2 n\left(\gamma+h_{s}\right)=$ $l_{\text {AIS }}^{\text {IC }}-l_{\text {DLS }}^{\text {IC }}+n\left(2 \gamma+(m+1) h_{s}\right)$.

## 6 Simulation Study

Our simulation is written in C++. Table 3 lists parameters. We consider a $2 \mathrm{~km} \times 2 \mathrm{~km}$ industrial area with some chimneys randomly distributed. For the Pasquill stability category, class E (i.e., slightly stable) is used, where the ambient temperature is $25^{\circ} \mathrm{C}$. Wind blows along the X -axis with a speed of $1 \mathrm{~m} / \mathrm{s}$. We choose $\mathrm{CO}_{2}$ as EG, whose environmental concentration is 400 ppm . The molecular weights of air and EG are $29 \mathrm{~g} / \mathrm{m}^{3}$ and $44 \mathrm{~g} / \mathrm{m}^{3}$, respectively.

To study the effect of chimney density, we raise the number of chimneys from 20 to 100 . For each chimney, its height is 25 m or 30 m , and its radius is 0.5 m or 1 m . Hence, there are four types of chimneys with different heights and radii. The temperature of EG emitted from chimneys is $30^{\circ} \mathrm{C}$. Though both GEV (affecting the plume's height) and EG emission rate (used for 3E-chimney judgment) are given in Table 3, these two parameters are unknown when scheduling the flight path and finding SPs for a UAV.

To avoid being damaged, the UAV has to keep a chimney away from at least 5 m (i.e., the minimum safe distance $d_{\mathrm{MS}}$ ). For the USS, IUS, and AIS schemes, we set $m=20$ for each chimney (i.e., 20 pairs of DW/UW SPs), where the interval $h_{s}$ between SPs is 1 m . The statutory threshold (i.e., $Q_{\mathrm{th}}$ ) is set at $400,000 \mathrm{mg} / \mathrm{s}$. If a chimney's EG emission rate exceeds $Q_{\mathrm{th}}$, it is a 3E-chimney. Moreover, $Q_{\text {adj }}$ is set to $130,000 \mathrm{mg} / \mathrm{s}$ to reflect the error of estimation in Eq. (24).

The TSP approximation solution in [38] is used to find the shortest path $\Gamma_{S}$ to visit all chimneys. We evaluate the cost and performance of different sampling schemes. As mentioned in Section 4, the cost is defined as the length of the flight path, and the performance is assessed via the metrics of accuracy, recall, precision, and F1-score. We also choose IGBA [31], a cutting-edge solution to find 3E-chimneys, for comparison. As discussed in Section 3.3, IGBA relies on the knowledge of the GEV (i.e., $v_{i}$ ) of each chimney to get the correct heights of SPs. In practice, GEV is an unknown parameter, so we set $v_{i}$ to
$2.5 \mathrm{~m} / \mathrm{s}$ (i.e., the average GEV of chimneys). Besides, we give the optimal solution as a reference for comparison. It picks two SPs for each chimney based on the plume's height. Evidently, the optimal solution is theoretical, as the plume's height cannot be known. In addition to cost and performance, we also study the effects of the mechanism to skip chimneys (discussed in Section 5.5) and some parameters. In each experiment, we repeat the simulation 100 times and take the average.

### 6.1 Comparison of Cost

Fig. 2(a) gives the path length using different schemes. The path length increases with more chimneys since the UAV has to collect data at more SPs. IGBA and DLS have a path length similar to the optimal solution, as they pick two SPs for each chimney. USS results in a much longer path length than others, and its length increases rapidly when the number of chimneys grows. That is because USS asks the UAV to spiral upward at each chimney to sample data. As analyzed in Theorem 2, doing so significantly lengthens the flight path. IUS lets the UAV fly in an inverted-U shape to avoid spirals, thereby greatly reducing path length. As can be seen, the difference between IUS's length and the lengths of IGBA, DLS, and the optimal solution is small. By decreasing UW SPs, AIS can reduce the path length as compared to IUS.

Using the optimal solution as a benchmark, the percentage increase in path length by a scheme $X$ is $\frac{\mathbf{L}_{x}-\mathbf{L}_{\text {opt }}}{\mathbf{L}_{\text {opt }}} \times 100 \%$, where $\mathbf{L}_{\mathrm{x}}$ and $\mathbf{L}_{\text {opt }}$ denote the path lengths of scheme x and the optimal solution. A lower percentage increase means that scheme x requires a lower cost. On average, IGBA, DLS, USS, IUS, and AIS have percentage increases of $0.07 \%, 0.09 \%$, $279.37 \%, 18.43 \%$, and $3.68 \%$. This result reveals that IGBA, DLS, and AIS can efficiently save on monitoring costs.

Fig. 2(b) shows the number of SPs chosen to collect data. The more chimneys there are, the more SPs are selected. In general, IGBA, DLS, and the optimal solution choose two SPs for each chimney (excluding those chimneys skipped), so they have a similar number of SPs. USS and IUS select $m$ pairs of DW/UW SPs for each chimney. However, USS lets the UAV check if the current chimney $c_{i}$ is a 3E-chimney whenever it has sampled data at each pair of DW/UW SPs. If so, the UAV leaves $c_{i}$ and flies to the next chimney. In IUS, the UAV has to visit all DW SPs of each chimney (that is, only some UW SPs can be skipped). Hence, IUS picks more SPs than USS. Despite this, IUS has a much shorter path than USS (referring to Fig. 2(a)), as the spiral flight in USS makes the flight path pretty long. AIS selects $m$ DW SPs and one UW SP for each chimney, so it has fewer SPs than USS and IUS.

### 6.2 Comparison of Performance

Fig. 2(c) gives the accuracy of each scheme, which indicates the proportion of true positives and true negatives (i.e., right answers) as defined in Eq. (9). Both IGBA and DLS select only two SPs for each chimney. Without knowing the correct plume's height at each chimney, their accuracy is below 0.83 . On the other hand, USS, IUS, and AIS perform almost as well in accuracy. Since they pick enough DW SPs for sampling data, even if the plume's height cannot be known, the accuracy of these three schemes can stay above 0.90 .

In Fig. 2(d), we measure the recall of each scheme. Based on Eq. (10), the recall cares about whether there is a miss without considering false alarms. The recall of IGBA and DLS is lower


Fig. 2: Comparison of the monitoring cost and performance.

| TABLE 4: Performance reduction percentage for each scheme. |  |  |  |  |  |
| :---: | ---: | ---: | ---: | ---: | ---: |
| metric | IGBA | DLS | USS | IUS | AIS |
| accuracy | $17.50 \%$ | $18.06 \%$ | $9.20 \%$ | $8.97 \%$ | $8.92 \%$ |
| recall | $75.59 \%$ | $76.26 \%$ | $30.50 \%$ | $29.18 \%$ | $29.42 \%$ |
| precision | $23.84 \%$ | $24.02 \%$ | $14.05 \%$ | $13.73 \%$ | $13.65 \%$ |
| F1-score | $63.24 \%$ | $64.03 \%$ | $23.21 \%$ | $22.28 \%$ | $22.40 \%$ |

than 0.25 , meaning that many 3E-chimneys are missed. On the contrary, the recall of IUS, USS, and AIS is close to 0.70 . The result reveals that they can identify much more 3E-chimneys than IGBA and DLS.

Then, Fig. 2(e) compares the precision of different schemes. Unlike the recall, the precision pays attention to false positives (referring to Eq. (11)). In IGBA and DLS, the precision rises as the number of chimneys grows, where the maximum precision is 0.836 . On the other hand, the precision of USS, IUS, and AIS is stable and stays above 0.85 . This implies that USS, IUS, and AIS can efficiently reduce false positives, as compared to IGBA and DLS, especially with fewer chimneys.

Fig. 2(f) gives the F1-score of each scheme. According to Eq. (12), the F1-score considers both recall and precision. The maximum F1-score in IGBA and DLS is 0.3826 (due to their low recall). For USS, IUS, and AIS, their average F1-scores are $0.7593,0.7686$, and 0.7674 , respectively. Evidently, they perform much better than IGBA and DLS.

In Fig. 2(c)-(f), the accuracy, recall, precision, and F1-score of the optimal solution are close to one but lower than one. These slight differences are caused by skipping chimneys. The details will be discussed later in Section 6.3.

Let $\mathbf{M}_{\mathrm{x}}$ and $\mathbf{M}_{\text {opt }}$ denote the values of a performance metric (i.e., accuracy, recall, precision, and F1-score) in a scheme x and the optimal solution. Taking the optimal solution as a benchmark, the performance reduction percentage of scheme X
is $\frac{\mathbf{M}_{\text {opt }}-\mathbf{M}_{\mathbf{x}}}{\mathbf{M}_{\text {opt }}} \times 100 \%$. A lower performance reduction percentage implies that scheme x performs better in that metric. Table 4 presents the performance reduction percentages of different schemes. Since IGBA and DLS let the UAV sample data at only two SPs for each chimney, they perform the worst in all metrics. The performance of AIS is not much different from that of USS and IUS, which means that it is safe for AIS to reduce UW SPs. According to the discussion in Section 6.1 and the result in Table 4, we conclude that AIS can strike a good balance between monitoring cost and performance as compared to other schemes.

### 6.3 Effect of Skipping Chimneys

As mentioned in Section 5.5, after the UAV samples data at UW SPs of a chimney $c_{i}$, it also estimates the EG emission rate $Q_{j}^{\mathrm{E}}$ of another chimney $c_{j}$ using Eq. (24), where $c_{j}$ meets three conditions: 1) $c_{j}$ is the next chimney on path $\Gamma_{s}$;2) $c_{j}$ is located on $c_{i}$ 's UW side; and 3 ) the distance between $c_{i}$ and $c_{j}$ is below $D_{\text {th }}$ (i.e., $\tilde{d}_{i, j} \leq D_{\text {th }}$ ). If $Q_{j}^{\mathrm{E}}$ satisfies the condition in Eq. (25), the UAV infers that $c_{j}$ is not a 3E-chimney and thereby skips $c_{j}$. In the simulation, we set $D_{\text {th }}$ to 300 m .

Fig. 3(a) shows the percentage of chimneys skipped. When the density of chimneys is low (i.e., 20 chimneys), the distance between two chimneys is large. So, the condition $\tilde{d}_{i, j} \leq D_{\text {th }}$ is relatively difficult to achieve. On the other hand, when the density of chimneys is high (e.g., 80 chimneys or more), the EG concentration sampled at $c_{i}$ 's UW SPs is more susceptible to other 3E-chimneys. In other words, $c_{j}$ becomes less likely to be judged as not a 3E-chimney. Hence, this percentage first increases and then decreases as the number of chimneys grows. The peak of each scheme occurs with 40 and 60 chimneys.

In Fig. 3(a), since the optimal solution, DLS, and AIS let the UAV sample data at only one UW SP for each chimney, their


Fig. 3: Effects of skipping chimneys and parameters.
percentages are high (i.e., $12.6 \%-16.8 \%$ ). Both USS and IUS choose $m$ UW SPs for each chimney. Due to the calculation of $Q_{j}^{\mathrm{E}}$ by Eq. (24), it becomes relatively difficult for $Q_{j}^{\mathrm{E}}$ to meet the condition in Eq. (25). That explains why both USS and IUS have lower percentages. Moreover, USS lets the UAV sample data at fewer UW SPs than IUS (referring to Fig. 2(b)). Hence, more chimneys are skipped in USS than in IUS. Specifically, USS skips $10.5 \%-14.9 \%$ of chimneys, while IUS skips only $9.4 \%-13.8 \%$ of chimneys.

In the path-planning framework, only when $c_{j}$ is obviously not a 3E-chimney can $c_{j}$ be skipped. Hence, we are concerned about false negatives, where a false negative occurs when $c_{j}$ is judged as not a 3E-chimney, but $c_{j}$ actually is. Fig. 3(b) shows the false negative rate (FNR) of skipped chimneys, whose trend is similar to that in Fig. 3(a). When more chimneys are skipped, the FNR increases accordingly. Observing Fig. 3(b), the FNR stays below 0.05 for all schemes. This result reveals that our proposed ISC3-based mechanism to skip chimneys in Section 5.5 is accurate.

### 6.4 Effect of Parameters

For each chimney, USS and IUS select $m$ pairs of DW/UW SPs and AIS picks $m$ UW SPs. Besides, the distance between two adjacent SPs is $h_{s}$. Hence, we study the effect of $m$ and $h_{s}$ on the cost and performance of USS, IUS, and AIS. Three combinations are considered: $\left(m, h_{s}\right)=(20,1),(10,2)$, and $(5,4)$. Therefore, the sampling altitude range for each chimney can be kept at $m \times h_{s}=20$.

Fig. 3(c) shows the effect of $m$ and $h_{s}$ on path length. In IUS and AIS, the UAV flies along a beeline to visit all DW (or UW) SPs of each chimney, as shown in Fig. 1(d) and (e).

Since the sampling altitude range stays the same, changing $m$ and $h_{s}$ has no effect on path length in IUS and AIS. Thus, in Fig. 3(c), we only show the result of IUS and AIS by setting $\left(m, h_{s}\right)=(20,1)$. On the contrary, changing $m$ and $h_{s}$ has a great impact on path length in USS, because USS makes the UAV fly on an upward spiral to collect data. Based on Theorem 2, the length of a spiral can be significantly decreased by reducing $m$, thereby shortening the UAV's flight path. Despite this, USS's path length is still longer than that of IUS and AIS.

In Fig. 3(d), (e), and (f), we present the effect of $m$ and $h_{s}$ on the F1-scores of USS, IUS, and AIS, respectively. As can be seen, reducing $m$ (and raising $h_{s}$ ) decreases F1-scores, as the UAV samples data at fewer SPs for each chimney. This result shows that using a large $m$ value (i.e., increasing the number SPs) can help improve monitoring performance.

## 7 Prototyping Experience

To show the feasibility of our path-planning framework, we made a prototype system to measure the concentration of $\mathrm{CO}_{2}$ gases emitted from small chimneys in a micro-field, as shown in Fig. 4(a). Nine 12-inch computer fans are used to generate a steady airflow, where the wind blows along the X -axis and has a speed of $1 \mathrm{~m} / \mathrm{s}$. Acrylic tubes are used to simulate chimneys. The radius of a chimney is 0.5 cm , and its height can be 20 cm or 30 cm . Each chimney is connected to the $\mathrm{CO}_{2}$ gas cylinder (i.e., the EG source) through an air pipe, where its EG emission can be adjusted using a control valve.

To implement the UAV, one Arduino board [39] is used to integrate a $\mathrm{CO}_{2}$ sensor and an XBee module [40], as shown in Fig. 4(b). The $\mathrm{CO}_{2}$ sensor uses non-dispersive infrared and


Fig. 4: A prototype system for identifying 3E-chimneys.
has an effective range of $0-50000 \mathrm{ppm}$. The XBee module offers low-powered ZigBee communications, and its communication range is at most 120 m . The UAV glides along tracks to check out each chimney. Besides, the $\mathrm{CO}_{2}$ sensor can be hung down to collect data at different heights.

Fig. 4(c) gives the micro-field and the coordinates of chimneys. The micro-field is a $40 \mathrm{~cm} \times 40 \mathrm{~cm}$ square where nine locations are placed with outlets for air pipes. We select three

TABLE 5: Performance of different schemes using the prototype.

| metric | IGBA | DLS | USS | IUS | AIS |
| :---: | :---: | :---: | :---: | :---: | :---: |
| accuracy | 0.9444 | 0.93333 | 0.9704 | 0.9733 | 0.9726 |
| recall | 0.9000 | 0.86667 | 1.0000 | 1.0000 | 1.0000 |
| precision | 0.9310 | 0.92857 | 0.9184 | 0.9259 | 0.9240 |
| F1-score | 0.9153 | 0.89655 | 0.9574 | 0.9615 | 0.9605 |

locations from them to place chimneys. There are five types of chimney placement, as Fig. 4(d) shows. Chimney C1 is a 3E-chimney whose GEV is $2 \mathrm{~m} / \mathrm{s}$. The GEV of chimney C2 is $0.1 \mathrm{~m} / \mathrm{s}$. Chimney C 3 does not discharge $\mathrm{CO}_{2}$. In other words, C2 and C3 are not 3E-chimneys. Moreover, we consider three cases for chimney height: 1) the height of every chimney is 30 cm ; 2) C1's height is 30 cm and other chimneys have a height of 20 cm ; and 3) C1's height is 20 cm and other chimneys have a height of 30 cm . Hence, there are 5 (placement) $\times$ 3 (height) combinations. We conduct 15 experiments, where each experiment uses a different combination of placement and height. Each experiment is repeated 30 times.

Since the ISC3 model considers large-scale fields, we cannot use Eq. (23) to deduce a chimney's EG emission rate based on the EG concentration sampled at its DW and UW SPs (i.e., $\vartheta_{d}$ and $\vartheta_{u}$ ). Instead, we check if $\vartheta_{d}-\vartheta_{u}>5000 \mathrm{ppm}$. If so, the chimney is inferred to be a 3E-chimney. Besides, we do not use the mechanism in Section 5.5 to skip chimneys. Hence, the UAV collects data at every chimney for judgment. For the USS, IUS, and AIS schemes, we set $m=3$ and $h_{s}=2 \mathrm{~cm}$. In other words, each chimney has three pairs of DW/UW SPs and the distance between two adjacent SPs is 2 cm .

Table 5 compares the performance of IGBA and our four proposed sampling schemes (i.e., DLS, USS, IUS, and AIS) by conducting experiments using the prototype. In IGBA, we set $v_{i}=2 \mathrm{~m} / \mathrm{s}$ (i.e., the GEV of a 3E-chimney). Since IGBA guesses chimney C1's GEV correctly, it performs better than DLS. However, IGBA and DLS pick only a pair of DW/UW SPs to collect data for each chimney, so their performance is not good compared to other schemes. On the other hand, USS, IUS, and AIS perform similarly, where they can recognize 3Echimneys more efficiently, especially in recall and F1-score. This phenomenon illustrates the need to select multiple pairs of SPs for each chimney, which helps reduce false positives and false negatives.

## 8 Conclusion

Air pollution is one serious problem, and finding pollution sources is a challenge. This paper searched 3E-chimneys in an industrial area using UAVs. We proposed the path-planning framework to schedule a UAV's flight path and choose SPs around each chimney for the UAV to collect data. Specifically, the shortest path was created to visit chimneys. We designed the DLS, USS, IUS, and AIS schemes to select suitable SPs. On executing the monitoring task, the UAV checked if the next chimney was not a 3E-chimney by the ISC3 model. If so, the UAV skipped that chimney to reduce the flight path. Through simulations, we showed that USS, IUS, and AIS performed better than IGBA, a cutting-edge method to find 3E-chimneys, especially in recall and F1-score. AIS had a shorter flight path than USS and IUS, so it better balanced cost and performance. In addition, we developed a prototype system to measure the concentration of $\mathrm{CO}_{2}$ gases emitted from small chimneys in a micro-field to demonstrate the viability of our framework.

For future work, the warden issue mentioned in Remark 2 deserves further investigation. UAVs conduct covert communications based on the partial location information of a warden. In this case, the flight path of a UAV needs to be adjusted to not only collect data from the selected SPs but also prevent the warden from detecting its data transmissions.

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