



# Accelerator Architectures for Machine Learning (AAML)

## Lecture 4: Model Pruning

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# Acknowledgements and Disclaimer

- Slides was developed in the reference with  
Joel Emer, Vivienne Sze, Yu-Hsin Chen, Tien-Ju Yang, ISCA 2019 tutorial  
Efficient Processing of Deep Neural Network, Vivienne Sze, Yu-Hsin Chen,  
Tien-Ju Yang, Joel Emer, Morgan and Claypool Publisher, 2020  
Yakun Sophia Shao, EE290-2: Hardware for Machine Learning, UC  
Berkeley, 2020  
CS231n Convolutional Neural Networks for Visual Recognition, Stanford  
University, 2020
- 6.5940, TinyML and Efficient Deep Learning Computing, MIT
- NVIDIA, Precision and performance: Floating point and IEEE 754  
Compliance for NVIDIA GPUs, TB-06711-001\_v8.0, 2017



# Outline

- Neural Network Pruning
- Pruning granularity
- Pruning criterion
- Pruning ratio
- Fine-tune/train pruned neural network



# Pruning Happens in Human Brain

- **Neural Network Pruning**

- Reduce the network connections
- Small weight while maintaining training accuracy

50 Trillion Synapses



New born

1000 Trillion Synapses



1 year old

500 Trillion Synapses



Teenager

Christopher A Walsh, Peter Huttenlocher (1931 - 2013). Nature, 502(7470), 2013



# Approaches to Reduce Model Sizes

- **Weight sharing**

- Trained quantization

- **Quantization**

- Quantizing the weight and activation
- Fine-tune in floating-point format
- Reduce to fixed-point format

~~2.03, 2.11, 1.98, 1.94~~



2.0

32 bit

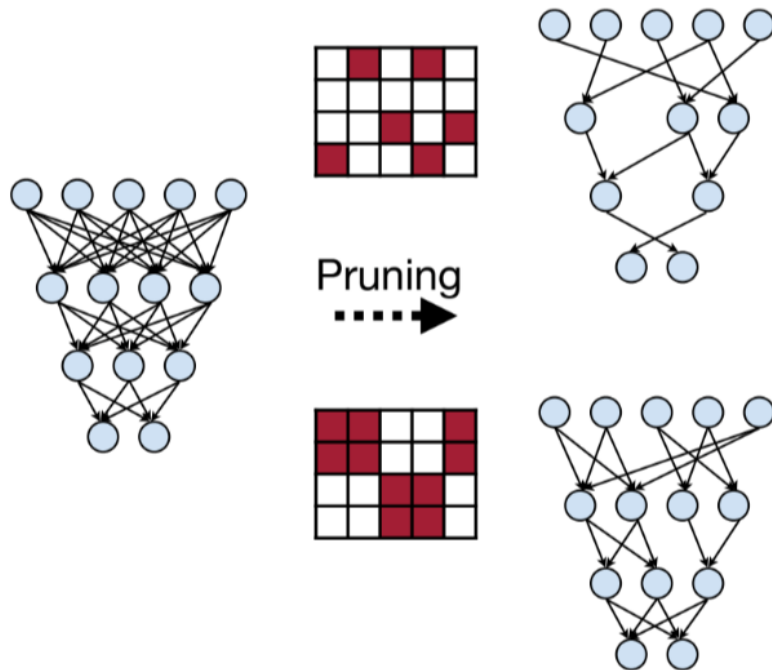
4 bit

8 x less  
memory  
footprint



# What is Neural Network Pruning ?

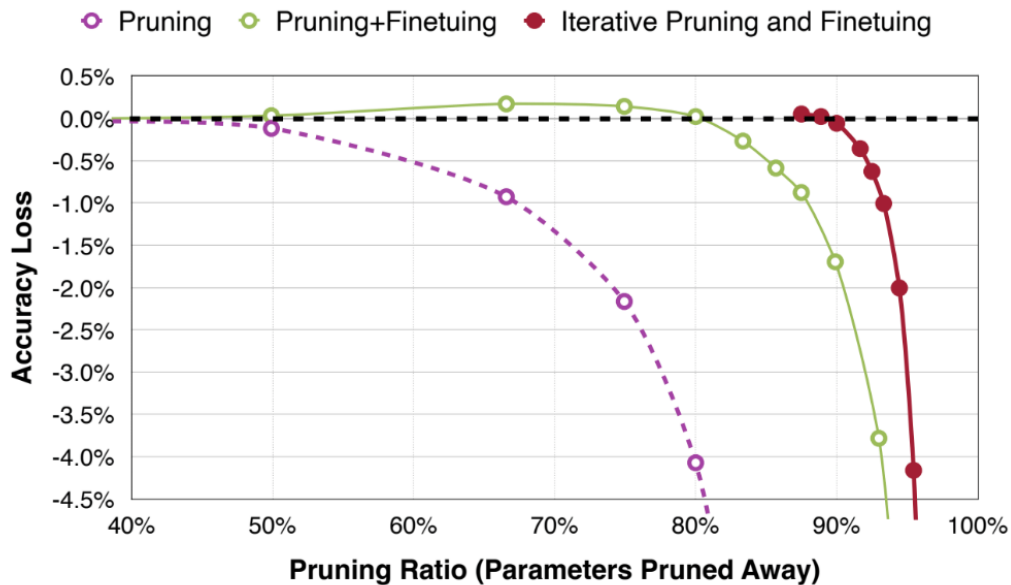
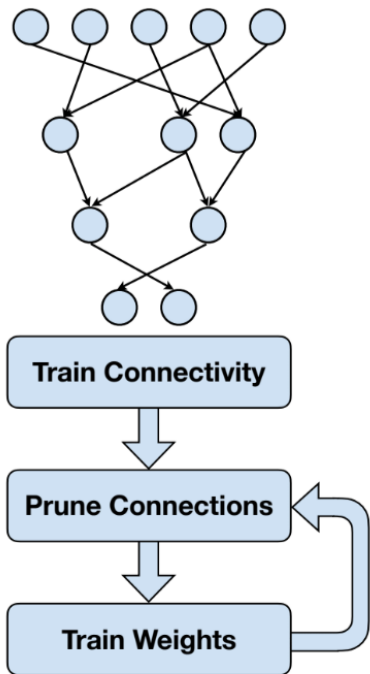
- **Neural Network Pruning**
  - Reducing the **parameter counts** of neural networks while maintaining model training accuracy
  - Create more zeros in weights
  - Reduce the size of weights through data compression





# Neural Network Pruning

- Challenge: Which weight values can become zero?





# Neural Network Pruning

- Make neural network smaller by removing synapses and neurons

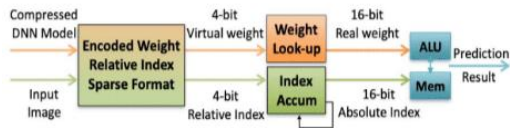
Neural Network	#Parameters			MACs
	Before Pruning	After Pruning	Reduction	Reduction
AlexNet	61 M	6.7 M	9 ×	3 ×
VGG-16	138 M	10.3 M	12 ×	5 ×
GoogleNet	7 M	2.0 M	3.5 ×	5 ×
ResNet50	26 M	7.47 M	3.4 ×	6.3 ×
SqueezeNet	1 M	0.38 M	3.2 ×	3.5 ×



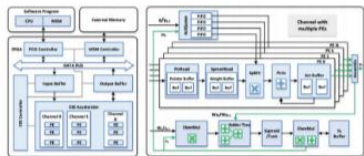


# Pruning in the Industry

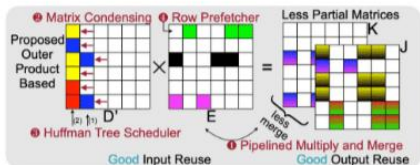
- Hardware support for sparsity



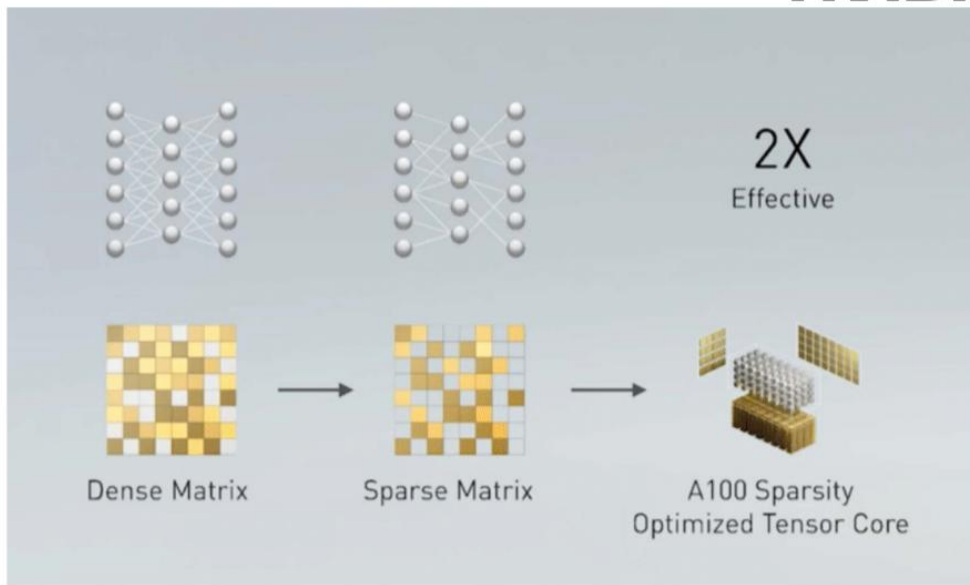
EIE [Han *et al.*, ISCA 2016]



ESE [Han *et al.*, FPGA 2017]



SpArch [Zhang *et al.*, HPCA 2020]  
SpAtten [Wang *et al.*, HPCA 2021]



**2:4 sparsity in A100 GPU**

**2X peak performance, 1.5X measured BERT speedup**



# Neural Network Pruning

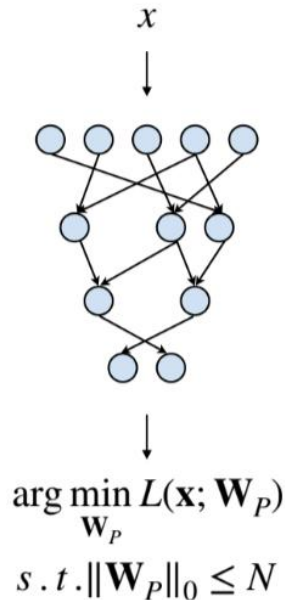
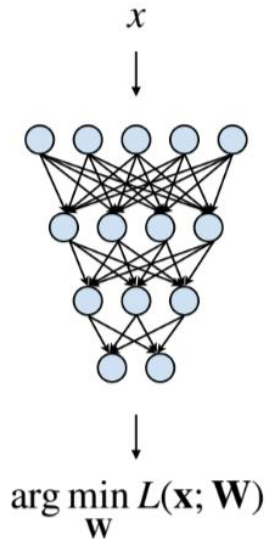
- In general, we could formulate the pruning as follows:

$$\arg \min_{\mathbf{W}_p} L(\mathbf{x}; \mathbf{W}_p)$$

subject to

$$\|\mathbf{W}_p\|_0 < N$$

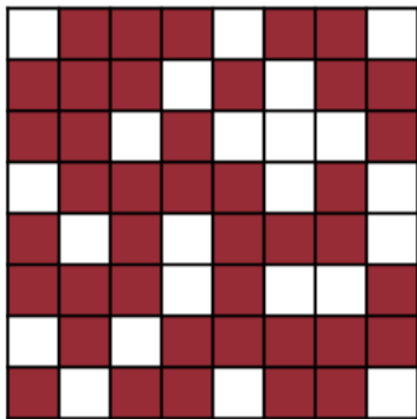
- $L$  represents the objective function for neural network training;
- $\mathbf{x}$  is input,  $\mathbf{W}$  is original weights,  $\mathbf{W}_p$  is pruned weights;
- $\|\mathbf{W}_p\|_0$  calculates the #nonzeros in  $\mathbf{W}_p$ , and  $N$  is the target #nonzeros.





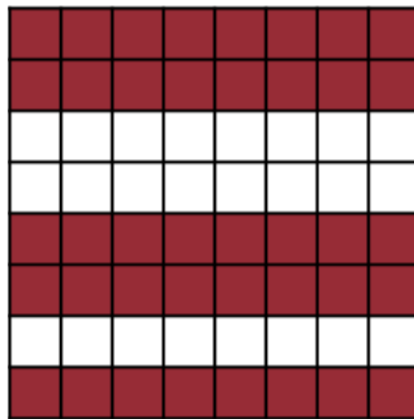
# Pruning at Different Granularities

- A simple example of 2D weight matrix



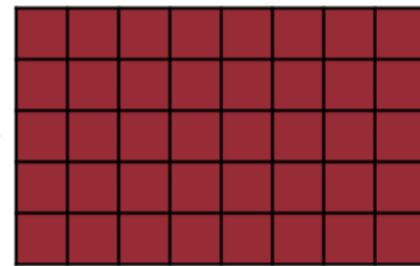
**Fine-grained/Unstructured**

- More flexible pruning index choice
- Hard to accelerate (irregular)



**Coarse-grained/Structured**

- Less flexible pruning index choice (a subset of the fine-grained case)
- Easy to accelerate (just a smaller matrix!)





# Pruning at Different Granularities

## The case of convolutional layers

- The weights of convolutional layers have 4 dimensions  $[c_o, c_i, k_h, k_w]$ :
  - $c_i$ : input channels (or channels)
  - $c_o$ : output channels (or filters)
  - $k_h$ : kernel size height
  - $k_w$ : kernel size width
- The 4 dimensions give us more choices to select pruning granularities

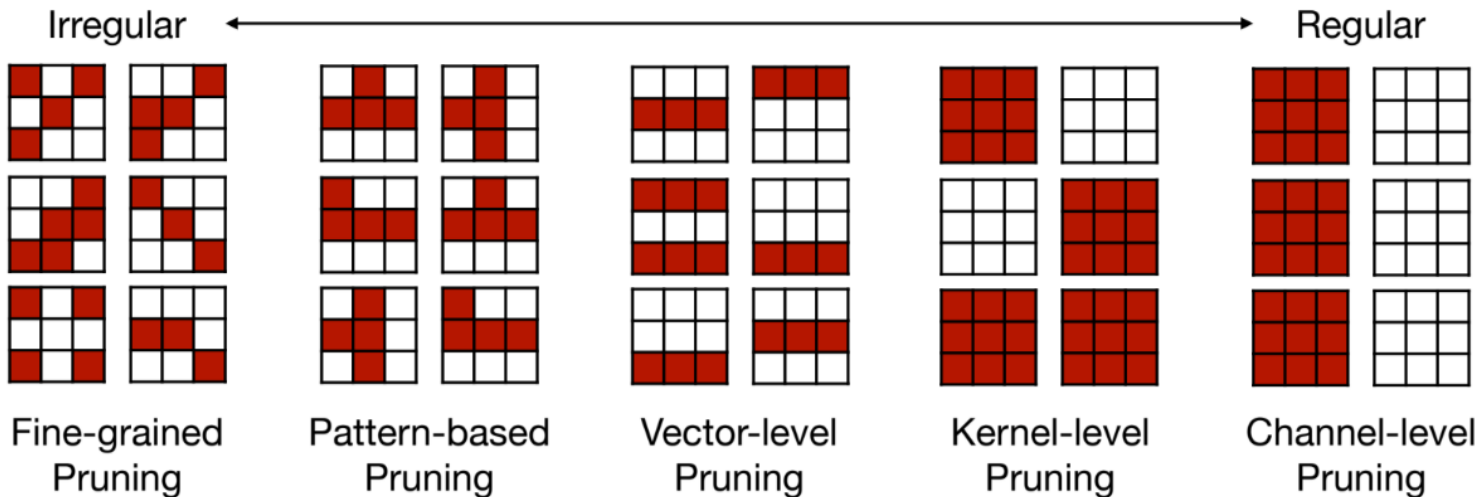
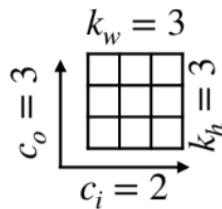


# Pruning at Different Granularities

## The case of convolutional layers

- Some of the commonly used pruning granularities

■ Preserved  
□ Pruned



like Tetris :)



# Pruning at Different Granularities

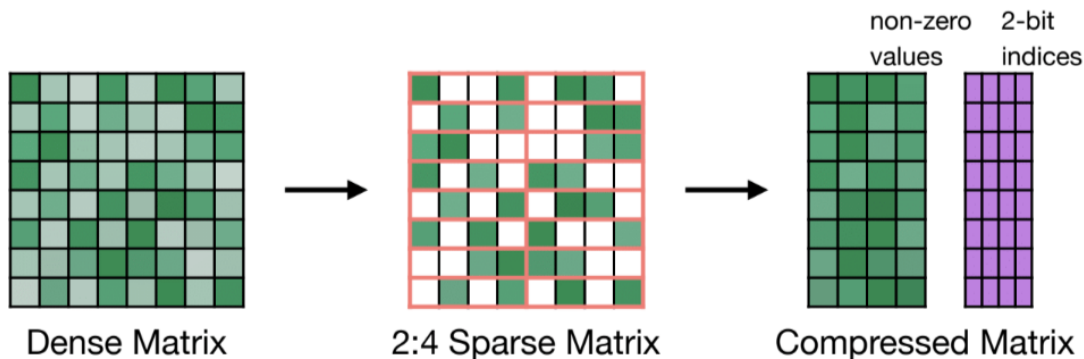
- **Fine-grained pruning**
  - Flexible pruning indices
  - Large compression ratio (flexibly find redundant weight)
  - Can deliver speedup on some customized hardware (EIE), but not GPU

Neural Network	#Parameters		
	Before Pruning	After Pruning	Reduction
AlexNet	61 M	6.7 M	9 ×
VGG-16	138 M	10.3 M	12 ×
GoogleNet	7 M	2.0 M	3.5 ×
ResNet50	26 M	7.47 M	3.4 ×



# Pruning at Different Granularities

- **Pattern-based pruning: N:M sparsity**
  - N:M sparsity means that in each contiguous M elements, N of them is pruned
  - A classic case is 2:4 sparsity (50% sparsity)
  - It is supported by NVIDIA's Ampere GPU, 2X speedup





# Pruning at Different Granularities

- **Pattern-based pruning: N:M sparsity**
  - Usually maintains accuracy

Network	Data Set	Metric	Dense FP16	Sparse FP16
ResNet-50	ImageNet	Top-1	76.1	76.2
ResNeXt-101_32x8d	ImageNet	Top-1	79.3	79.3
Xception	ImageNet	Top-1	79.2	79.2
SSD-RN50	COC02017	bbAP	24.8	24.8
MaskRCNN-RN50	COC02017	bbAP	37.9	37.9
FairSeq Transformer	EN-DE WMT'14	BLEU	28.2	28.5
BERT-Large	SQuAD v1.1	F1	91.9	91.9










# Pruning at Different Granularities

- **Channel pruning**



- Reduce channel numbers (leading to an neural network with smaller # of channels) -> speedup
- Con: smaller compression ratio

Sparsity=0.3   
Sparsity=0.3   
Sparsity=0.3   
Sparsity=0.3   
Sparsity=0.3 

...

Uniform Shrink

<

 Sparsity=0.5  
 Sparsity=0.3  
 Sparsity=0.7  
 Sparsity=0.2  
 Sparsity=0.3

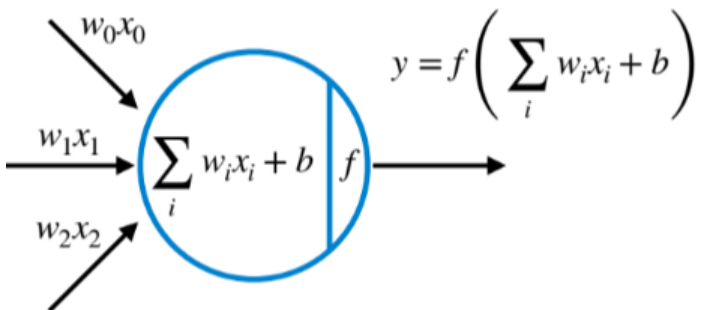
...

Channel Prune



# Pruning Criterion

- What synapses and neurons should we prune ?
  - The less important parameters should be removed
  - What is the less important parameter in a neural network?



## Example

$$f(\cdot) = \text{ReLU}(\cdot), \quad W = [10, -8, 0.1]$$

$$\Rightarrow y = \text{ReLU}(10x_0 - 8x_1 + 0.1x_2)$$

- If one weight will be removed, which one?



# Magnitude-based Pruning

- **Magnitude-based pruning**

- Considers weights with **large absolute values** are more important than other weights
- Remove weights with small magnitudes

$$\text{Importance} = |W|$$

fmap                  filter

1	1	1
1	1	1
1	1	1

 \* 

-8	3	2
1	-3	-2
1	1	1

 = -4

Without Pruning

fmap                  filter

1	1	1
1	1	1
1	1	1

 \* 

-8	3	2
0	-3	-2
0	0	0

 = -8

Error = -4

Magnitude-based Pruning



# Magnitude-based Pruning

- **Row-wise pruning**

- The L1-norm magnitude can be defined as

$$Importance = \sum_{i \in S} |w_i|, \text{ where } \mathbf{W}^{(S)} \text{ is the structural set } S \text{ of parameters } \mathbf{W}$$

## Example

3	-2
1	-5

**Weight**

L1-norm  
Row-wise

$ 3  +  -2 $
$ 1  +  -5 $

**Importance**

5
6



0	0
1	-5

**Pruned Weight**



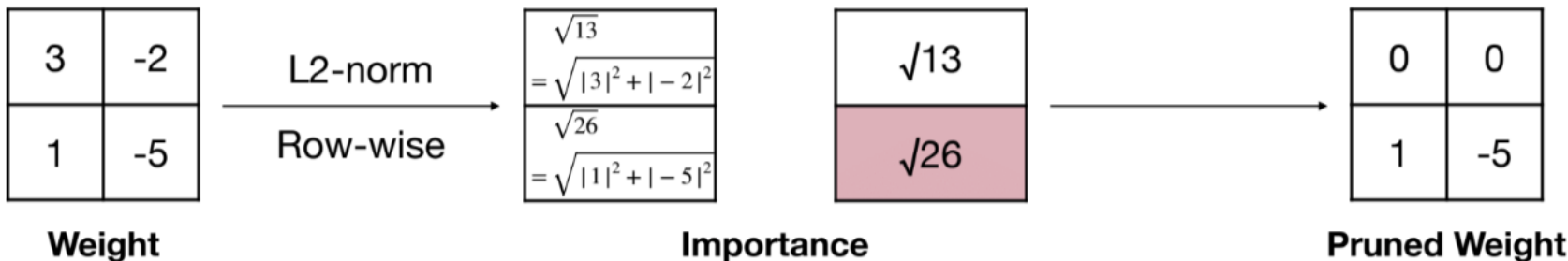
# Magnitude-based Pruning

- **A heuristic pruning criterion**

- The Lp-norm magnitude can be defined as

$$\|\mathbf{W}^{(S)}\|_p = \left( \sum_{i \in S} |w_i|^p \right)^{\frac{1}{p}}, \text{ where } \mathbf{W}^{(S)} \text{ is a structural set of parameters}$$

## Example





# Feature-Based Pruning

- **Feature-based pruning**

- Pruning based on the impact of the output feature map
- Achieve higher accuracy than magnitude-based pruning
- Complex evaluating the impact of the weights

fmap                  filter

1	1	1
1	1	1
1	1	1

 \* 

-8	3	2
1	-3	-2
1	1	1

 = -4

Without Pruning

fmap                  filter

1	1	1
1	1	1
1	1	1

 \* 

-8	0	0
1	0	0
1	1	1

 = -4  
Error = 0

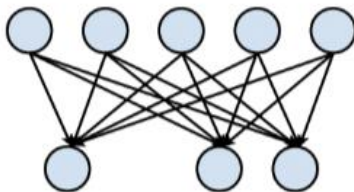
Feature-based Pruning



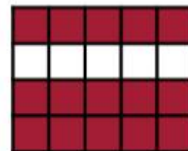
# Pruning Neurons

- When removing neurons from a neural network model
  - The less useful neurons are removed

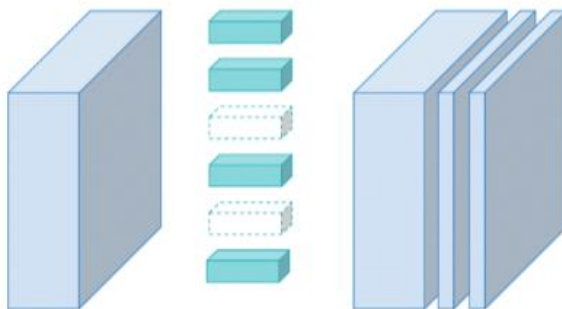
**Neuron Pruning  
in Linear Layer**



**Weight Matrix**



**Channel Pruning  
in Convolution Layer**





# Percentage-of-Zero-Based Pruning

- ReLU activation will generate zeros in the output activation
- The Average Percentage of Zero activations (APoZ) can be exploited to measure the importance of the neurons

Output Activations	Height = 4	Width = 4								Width = 4																							
		0	0.1	0.5	1	0.1	0.5	0	0	0	0	0.8	0	0.5	0	0.2	0.1	0.1	0.5	0	0	0	0.8	0.1	0								
		1.2	0.6	0.3	0.2	0.2	0.3	0	1	0.7	0	0.6	0.1	0	0.2	1.2	0	0	0.1	0	0.8	0	1	0.2	0	0.3							
		0	0.5	0	0.3	0.1	0	0	0.5	1.2	1	0	0.2	0	0.2	0.3	0.1	0	1.0	0	0.4	0	0.5	0	0.3	0.5							
		0.2	0	0	0.8	0.1	0.6	0.7	0.1	0.5	0	0.3	0.5	0.2	0.4	0	0	0.2	0	1.0	0	0.2	0	0.3	0								
Channel = 3														Batch = 2										Channel = 3									

**Average Percentage of Zeros (APoZ)**

$$= \frac{5 + 6}{2 \cdot 4 \cdot 4} = \frac{11}{32}$$

Channel 0

$$= \frac{5 + 7}{2 \cdot 4 \cdot 4} = \frac{12}{32}$$

Channel 1

$$= \frac{6 + 8}{2 \cdot 4 \cdot 4} = \frac{14}{32}$$

Channel 2





# Percentage-of-Zero-Based Pruning

- The Average Percentage of Zero activations (APoZ) can be exploited to measure the importance of the neurons
- The neuron with smaller APoZ is more important

Output Activations

Width = 4

Height = 4

0	0.1	0.5	1
1.2	0.6	0.3	0.2
0	0.5	0	0.3
0.2	0	0	0.8

0.1	0.5	0	0
0.2	0.3	0	1
0.1	0	0	0.5
0.1	0.6	0.7	0.1

0	0	0.8	0
0.7	0	0.6	0.1
1.2	1	0	0.2
0.5	0	0.3	0.5

Channel = 3

Width = 4

Height = 4

0.5	0	0.2	0.1
0	0.2	1.2	0
1.2	0	0.2	0.3
0.2	0.4	0	0

0.1	0.5	0	0
0	0.8	0	1
0.1	0	0.1	1.0
0.2	0	1.0	0

0	0.8	0.1	0
0.2	0	0	0.3
0	0.4	0	0.5
0.2	0	0.3	0

Channel = 3

Batch = 2

Average Percentage of Zeros (APoZ)

$\frac{5 + 6}{2 \cdot 4 \cdot 4} = \frac{11}{32}$ <p>Channel 0</p>	$\frac{5 + 7}{2 \cdot 4 \cdot 4} = \frac{12}{32}$ <p>Channel 1</p>	$\frac{6 + 8}{2 \cdot 4 \cdot 4} = \frac{14}{32}$ <p>Channel 2</p>
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# Takeaway Questions

- How does feature-based pruning work?
  - (A) Removing weights with small magnitudes
  - (B) Pruning through complex evaluation
  - (C) Removing inputs with small magnitudes
- What are goals of neural network pruning ?
  - Less number of weights
  - Less number of inputs
  - Less bits per weights



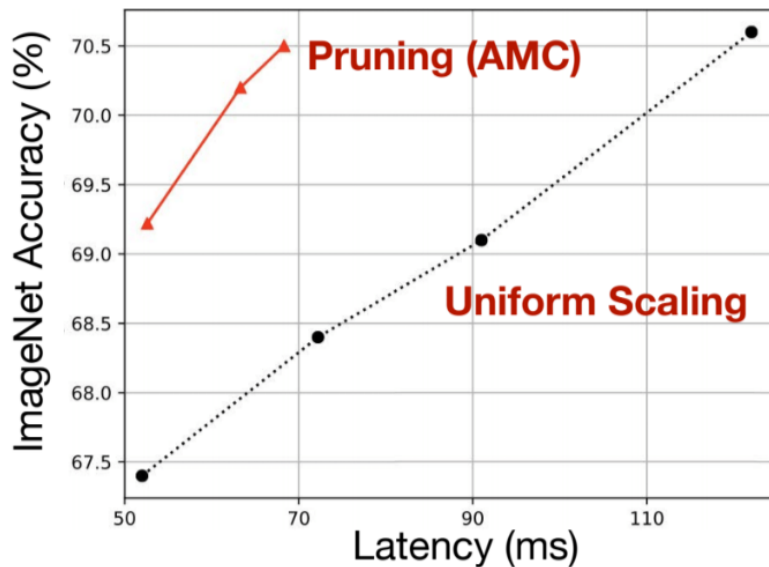
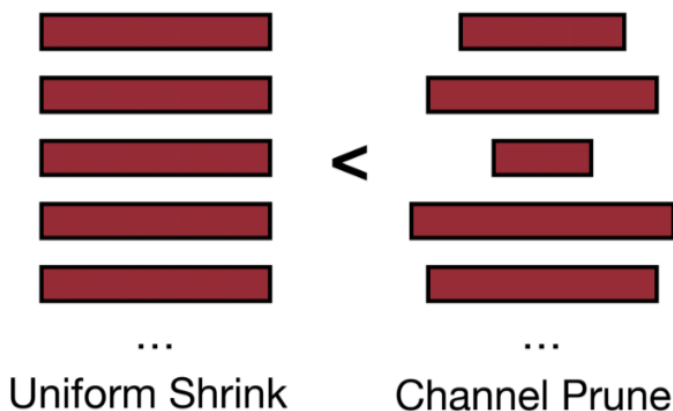
# Takeaway Questions

- What are benefits of network pruning ?
  - (A) Reduce the size of input data
  - (B) Small size of filter data
  - (C) Shorten the time to complete the DNN model inference



# Pruning Ratio

- How should we find per-layer pruning ratios ?
  - Non-uniform pruning is better than uniform shrinking





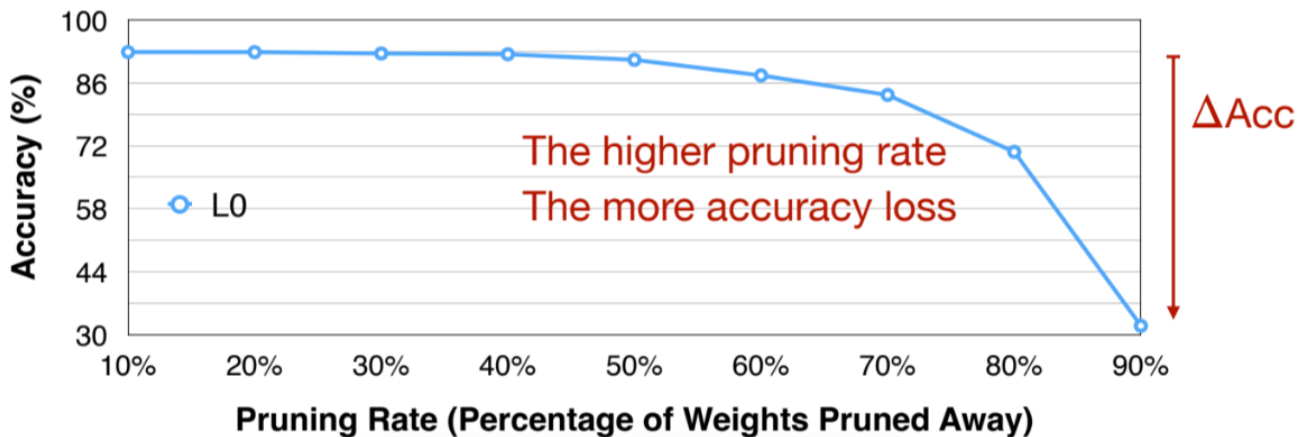
# Finding Pruning Ratios

- **Analyze the sensitivity of each layer**
  - Pruning ratios are varied across different layers
  - Some layers are more sensitive (e.g., first layer, why?)
  - Some layers are more redundant
  - Need to perform sensitivity analysis to determine the per-layer pruning ratio



# Finding Pruning Ratios

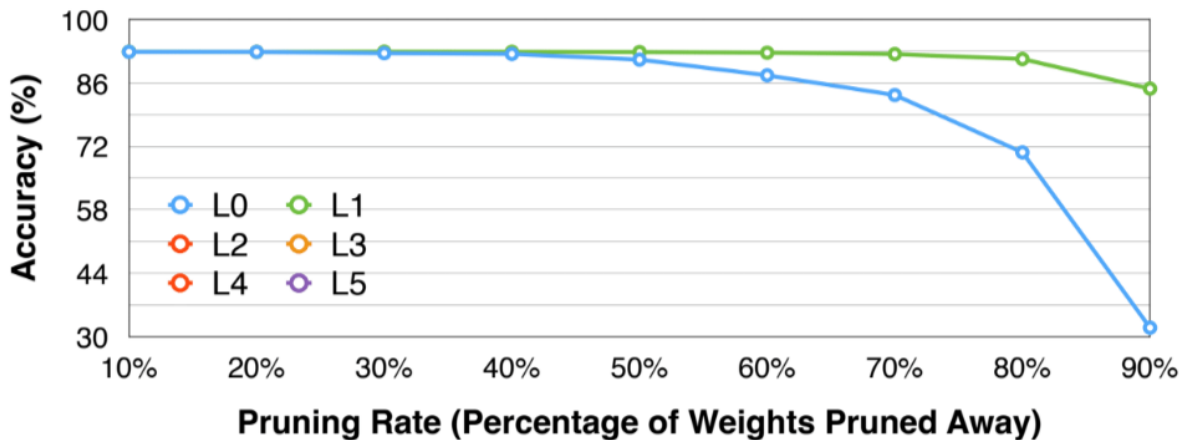
- The process of Sensitivity Analysis (\* VGG-11 on CIFAR-10 dataset)
  - Pick a layer  $L_i$  in the model
    - Prune the layer  $L_i$  with pruning ratio  $r \in \{0,0.1,0.2,...,0.9\}$  (or other strides)
    - Observe the accuracy degrade  $\Delta\text{Acc}_r^i$  for each pruning ratio





# Finding Pruning Ratios

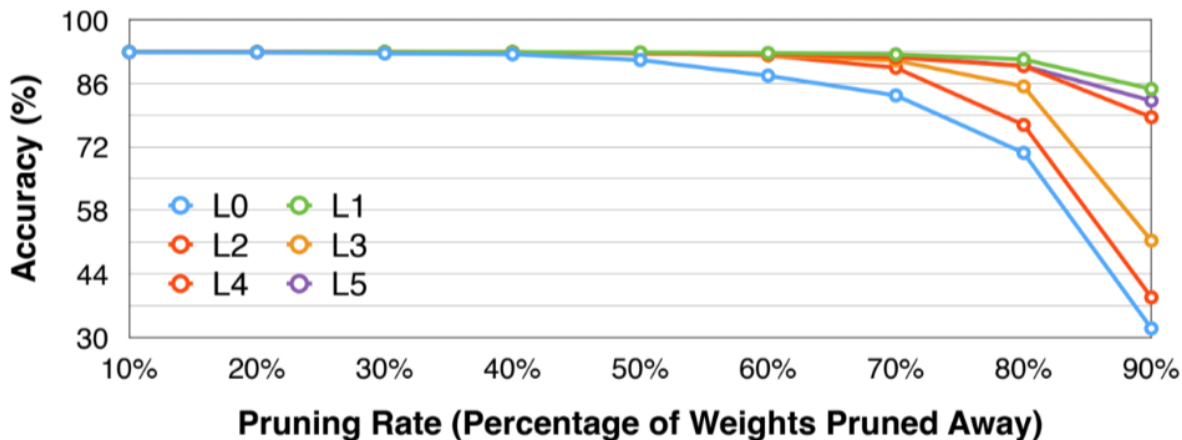
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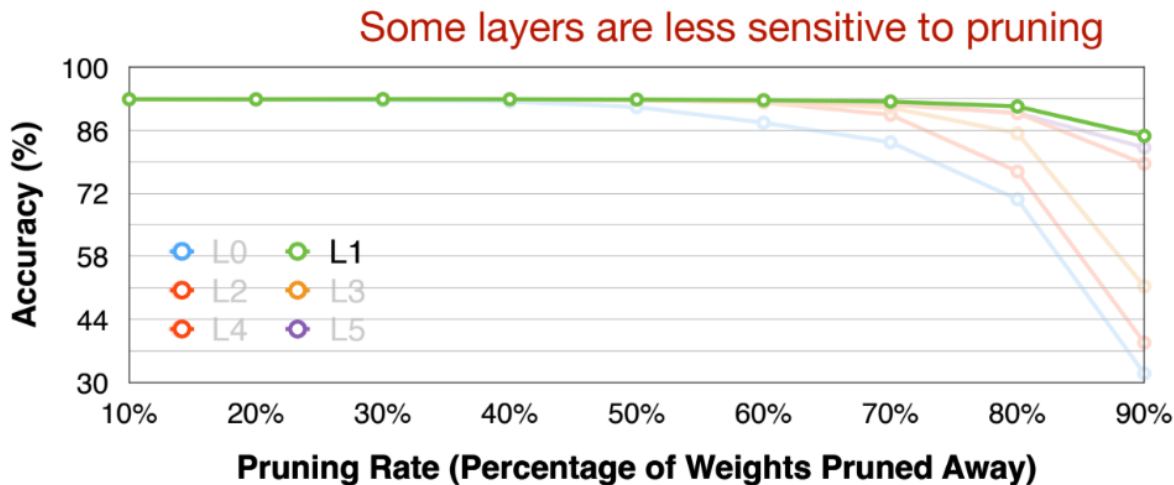






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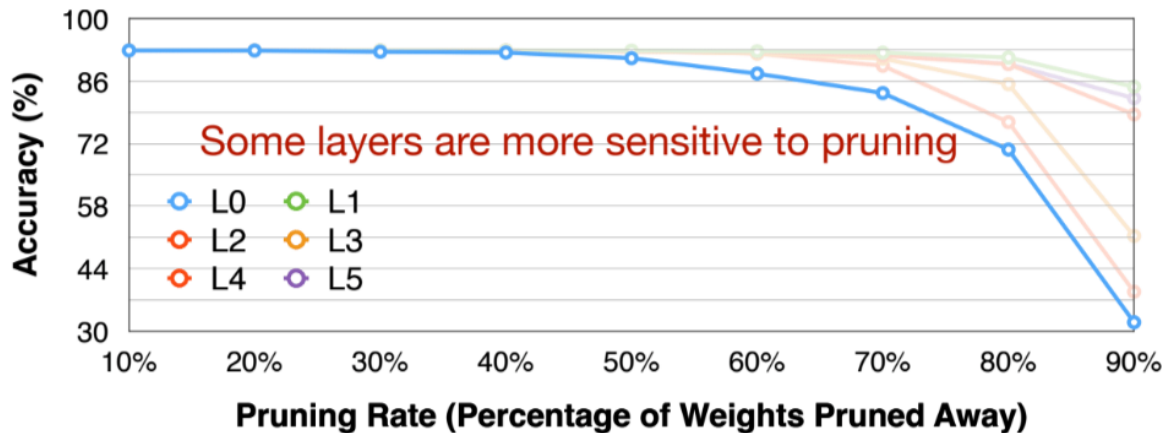
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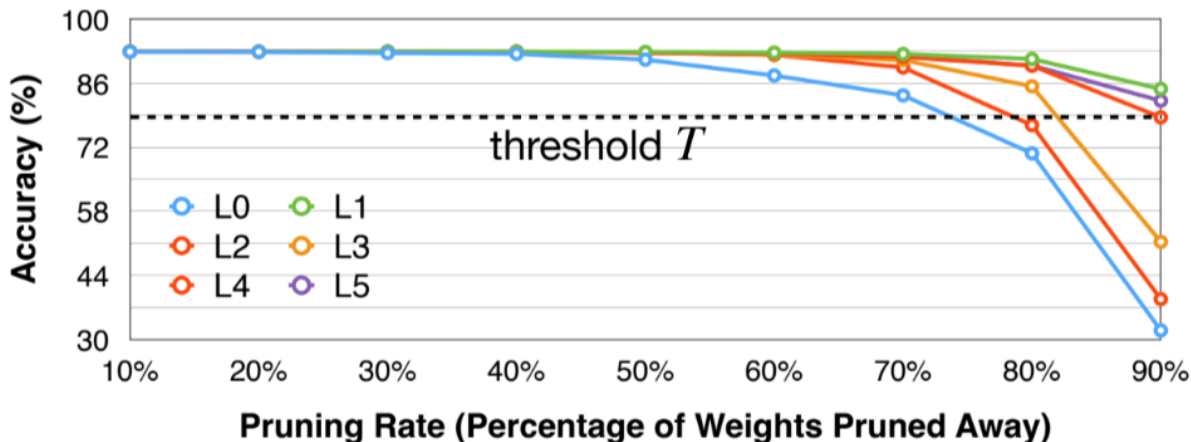
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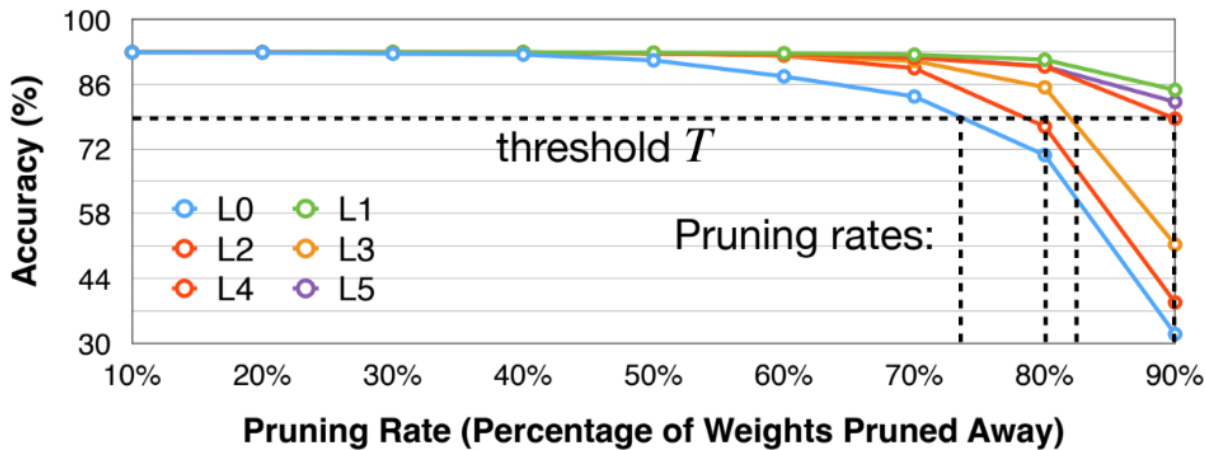
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    - Observe the accuracy degrade  $\Delta \text{Acc}_r^i$  for each pruning ratio
  - Repeat the process for all layers
  - Pick a degradation threshold  $T$  such that the overall pruning rate is desired





# Finding Pruning Ratios

- The process of Sensitivity Analysis (\* VGG-11 on CIFAR-10 dataset)
  - Pick a layer  $L_i$  in the model
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  - Repeat the process for all layers
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# Automatic Pruning

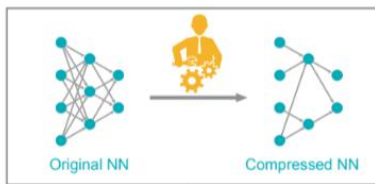
- Given an **overall** compression ratio, how do we **choose per-layer** pruning ratios ?
  - Sensitivity analysis ignores the interaction between layers
  - Conventionally, such process relies on human expertise and trials and errors



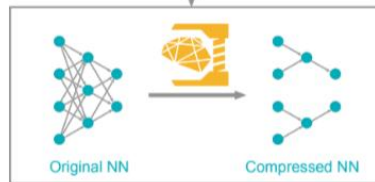
# AMC: AutoML for Model Compression

- Pruning as a reinforcement learning problem

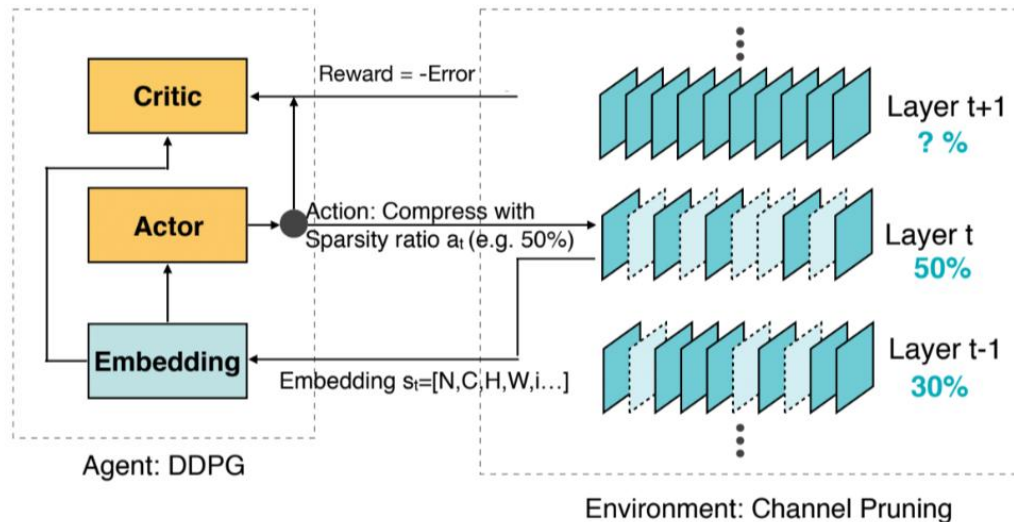
Model Compression by Human:  
Labor Consuming, Sub-optimal



AMC Engine



Model Compression by AI:  
Automated, Higher Compression Rate, Faster



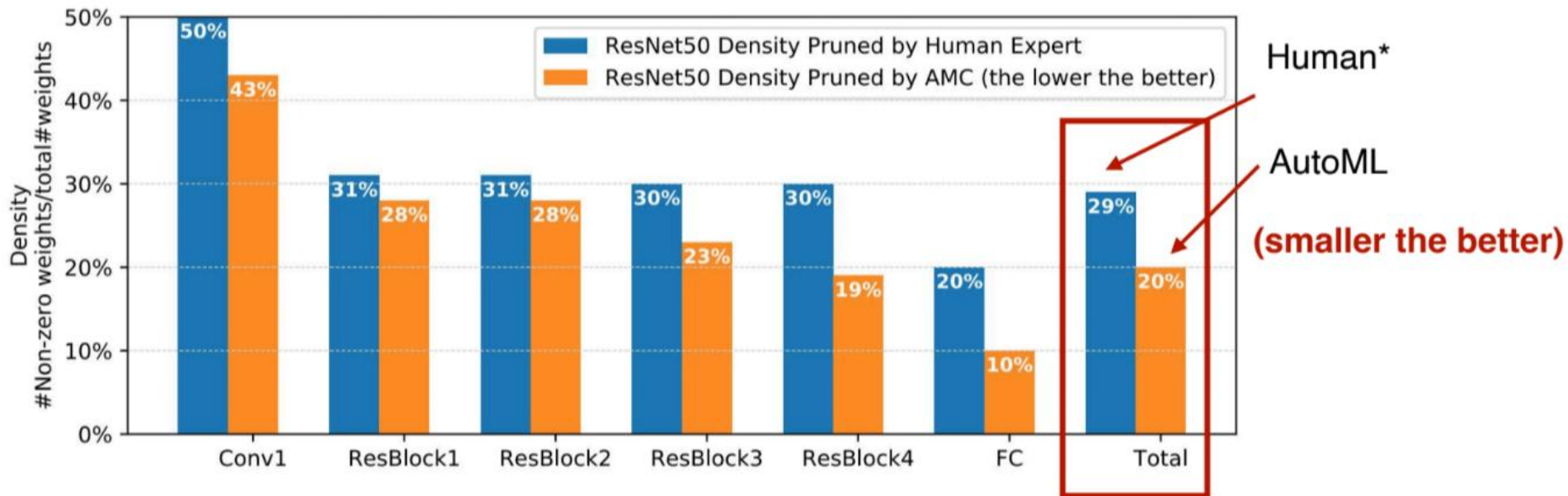


# AMC: AutoML for Model Compression

- AMC uses the following steps for the reinforcement learning problem
  - **State:** 11 features (including layer indices, channel numbers, kernel sizes, FLOPs, ...)
  - **Action:** A continuous number (pruning ratio)  $a \in [0,1)$
  - **Agent:** Deep Deterministic Policy Gradient (DDPG) agent, because it supports continuous action output
  - **Reward:** 
$$R = \begin{cases} -\text{Error}, & \text{if satisfies constrains} \\ -\infty, & \text{if not} \end{cases}$$



# AMC: AutoML for Model Compression







# AMC: AutoML for Model Compression



Model	MAC	Top-1	Latency*	Speedup	Memory
1.0 MobileNet	569M	70.6%	119.0ms	1x	20.1MB
AMC (50% <b>FLOPs</b> )	285M	<b>70.5%</b>	64.4ms	<b>1.8x</b>	14.3MB
AMC (50% <b>Time</b> )	272M	<b>70.2%</b>	59.7ms	<b>2.0x</b>	13.2MB
0.75 MobileNet	325M	68.4%	69.5ms	1.7x	14.8MB

\* Measured with TF-Lite on Samsung Galaxy S7 Edge, which has Qualcomm Snapdragon SoC  
Single core, Batch size = 1(mobile, latency oriented)



# Summary of Neural Network Pruning

- **Introduction to pruning**
  - What is the purpose of pruning ?
- **Determine the pruning granularity**
  - Fine-grain, channel-level pruning ....
- **Determine the pruning criterion**
  - What synapses/neurons should we prune ?
- **Determine the pruning ratio**
  - What should target sparsity be for each layer
- **Fine-tune/train pruned neural network**
  - How to improve performance of pruned models



# Takeaway Questions

- How to find prune ratios appropriately ?
  - (A) Randomly guess
  - (B) Sensitivity analysis
  - (C) Refer to the ratio in the batch normalization
- What are potential techniques used by automatic pruning ?
  - (A) Word embedding
  - (B) Iterative training
  - (C) Reinforcement learning