計概補充

Greedy Algorithm, Dynamic Programming Algorithm

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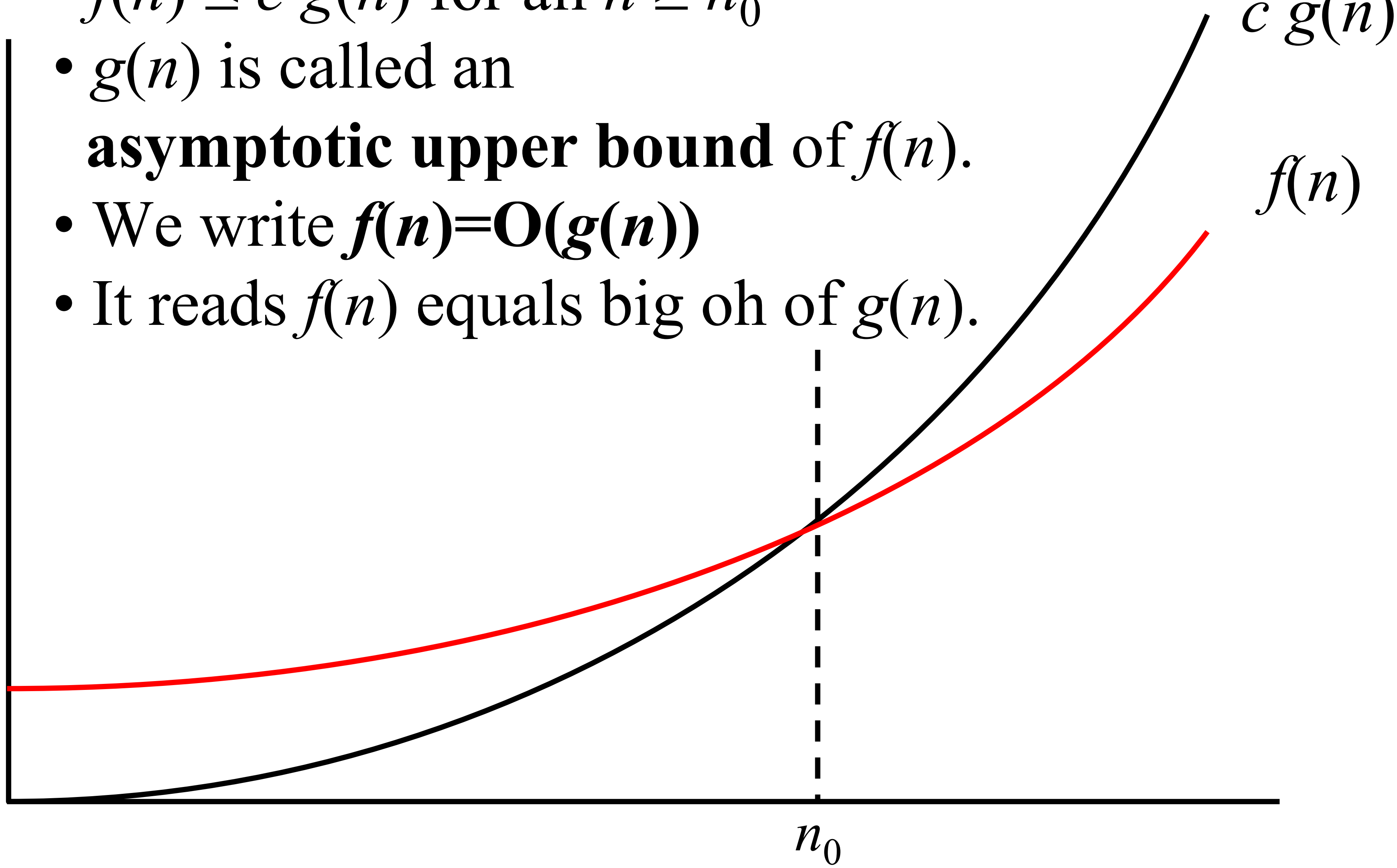
Agenda

- **Complexity: Big O, Big Omega, Big Theta**
- **Knapsack problem**
- **Greedy Algorithm**
- Making Change problem
- Knapsack problem
- **Dynamic Programming (DP)**
- DP vs. **Divide-and-Conquer**
- **Greedy** **vs.** **Dynamic Programming**
- **Quick Sort, merge Sort**
- **C++ STL sort( ), java.util.Arrays.sort( )**

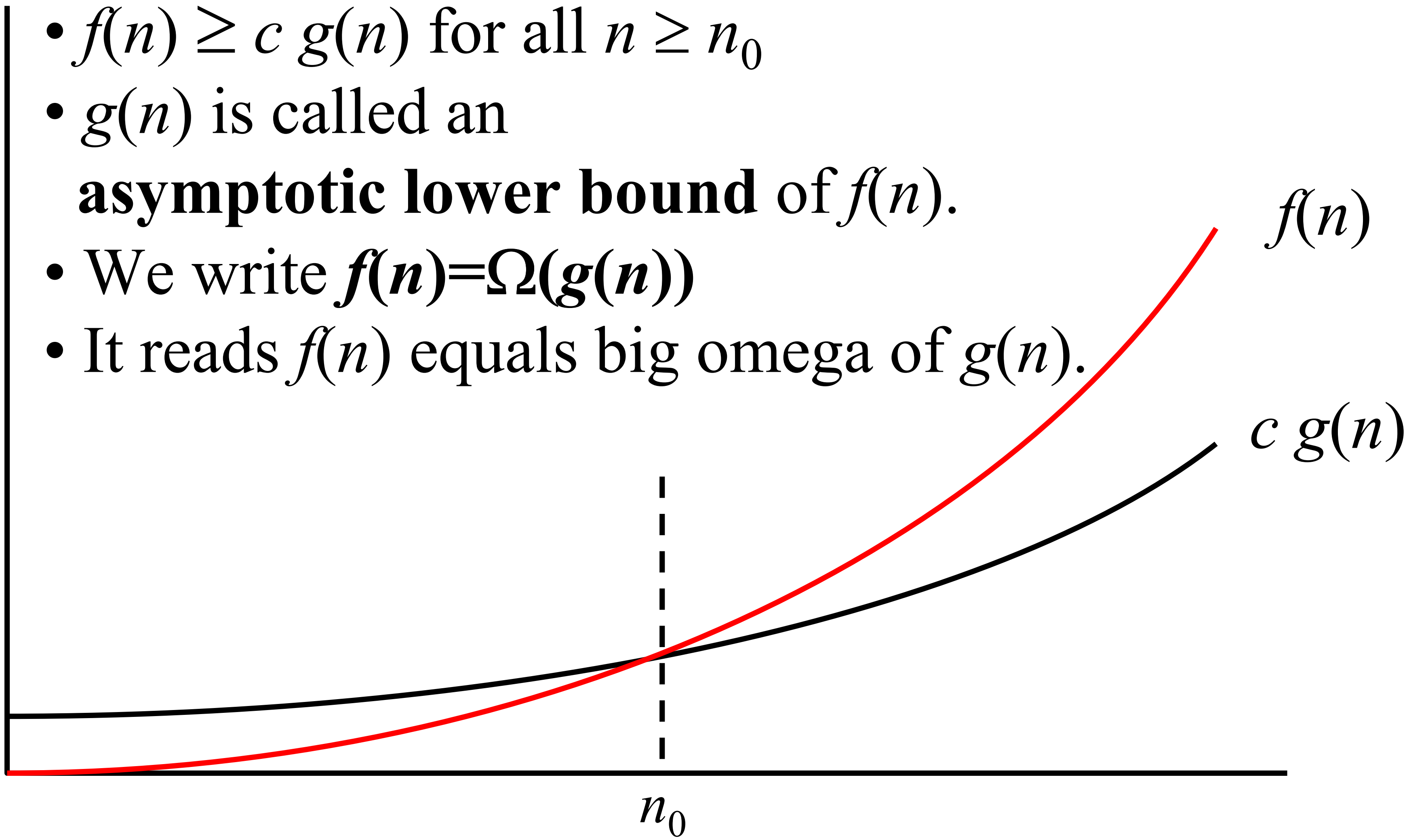
Asymptotic Upper Bound (Big O)

- $f(n) \le c\, g(n)$ for all $n \ge n_0$
- $g(n)$ is called an **asymptotic upper bound** of $f(n)$.
- We write $\boldsymbol{f(n)=O(g(n))}$
- It reads $f(n)$ equals big oh of $g(n)$.



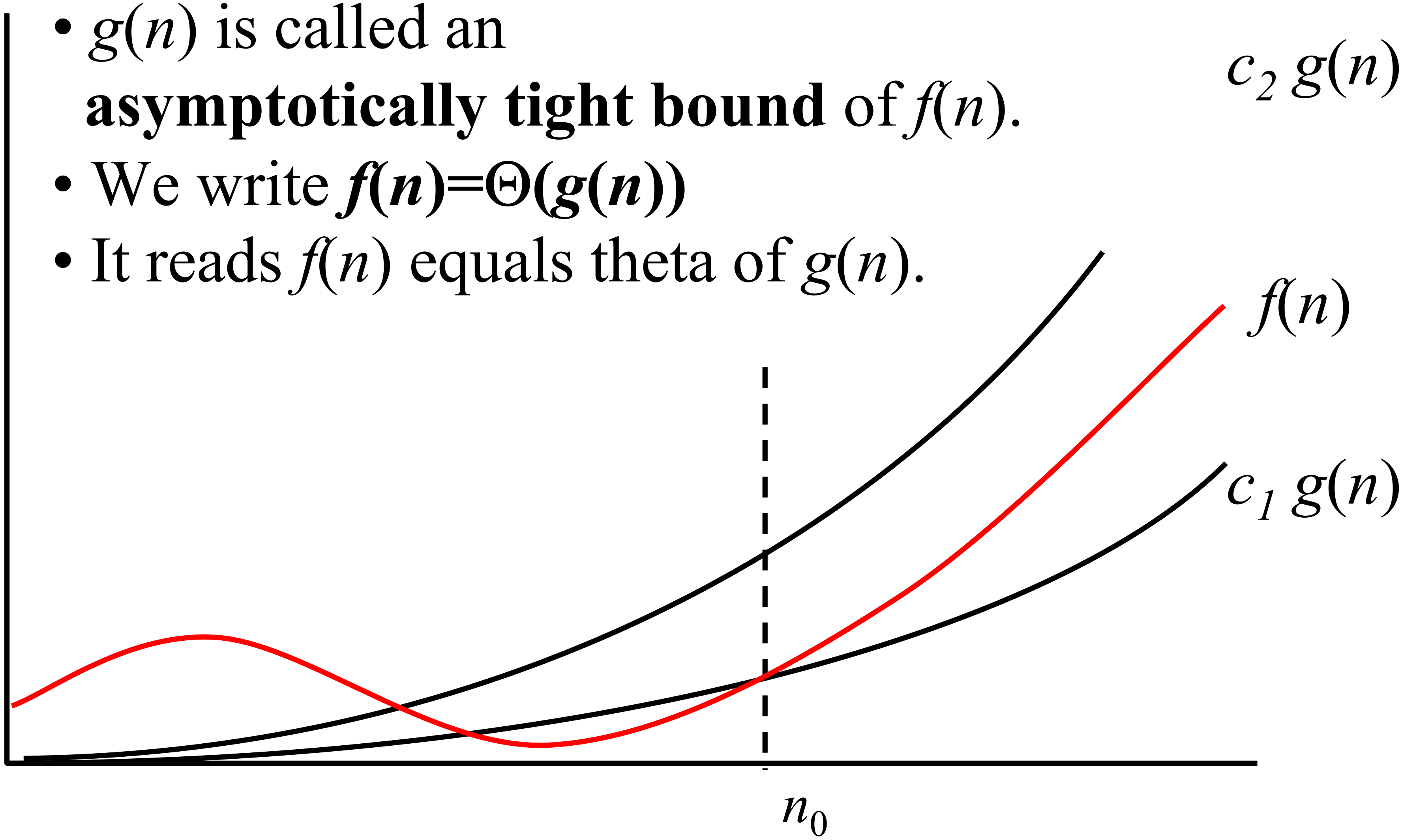
Asymptotic Lower Bound (Big Omega)

- $f(n) \geq c\, g(n)$ for all $n \geq n_0$
- $g(n)$ is called an **asymptotic lower bound** of $f(n)$.
- We write $\boldsymbol{f(n)=\Omega(g(n))}$
- It reads $f(n)$ equals big omega of $g(n)$.



Asymptotically Tight Bound (Big Theta)

- $f(n) = \mathrm{O}(g(n))$ and $f(n) = \Omega(g(n))$
- $g(n)$ is called an **asymptotically tight bound** of $f(n)$.
- We write $\boldsymbol{f(n)=\Theta(g(n))}$
- It reads $f(n)$ equals theta of $g(n)$.

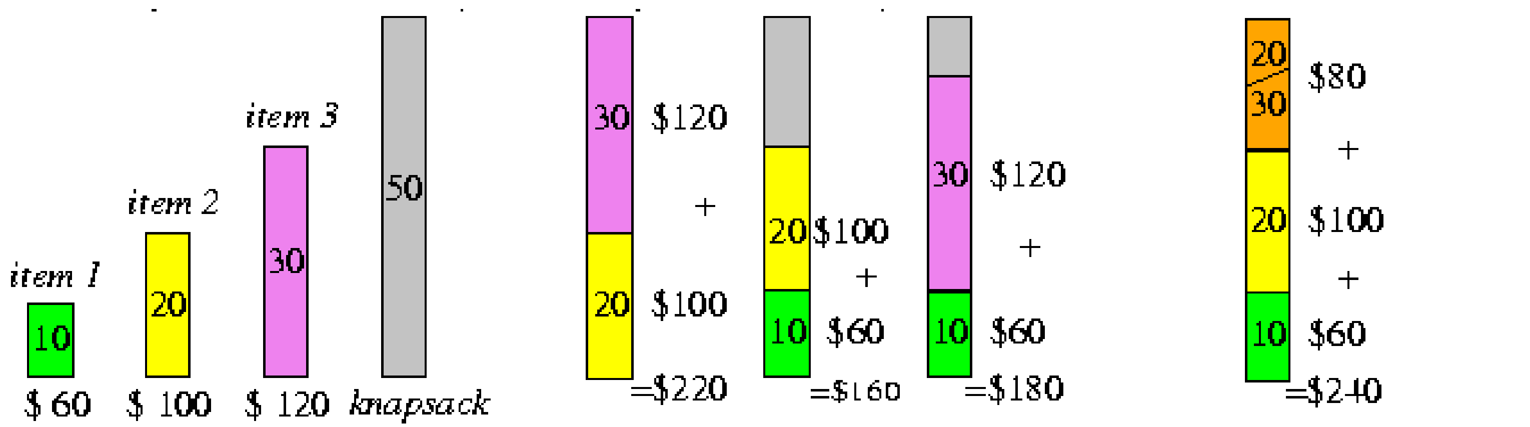


Algorithm types

- Algorithm types we will consider include:
  - Simple recursive algorithms
  - Backtracking algorithms
  - Greedy algorithms
  - Divide and conquer algorithms
  - Dynamic programming algorithms
  - Branch and bound algorithms
  - Brute force algorithms
  - Randomized algorithms

Knapsack Problem

- **Knapsack Problem:** Given n items, with ith item worth v_i dollars and weighing w_i pounds, a thief wants to take as valuable a load as possible, but can carry at most W pounds in his knapsack.
- **The 0-1 knapsack problem:** Each item is either taken or not taken (0-1 decision).
- **The fractional knapsack problem:** Allow to take fraction of items. (較簡單)
- **Exp:** $\vec{v} = (60, 100, 120)$, $\vec{w} = (10, 20, 30)$, $W = 50$



Item 1 has greatest value per pound

For the 0–1 version, any solution with item 1 is not optimal!

Greedy algorithm is optimal for the fractional version.

- Greedy solution by taking items in order of greatest value per pound is optimal for the fractional version, but not for the 0-1 version.
- The 0-1 knapsack problem is NP-complete, but can be solved in $O(nW)$ time by Dynamic Programming. **(A polynomial-time DP??)**

The Fractional Knapsack Problem

- Given: A set S of n items, with each item i having
  - b_i - a positive benefit
  - w_i - a positive weight
- Goal: Choose items with maximum total benefit but with weight at most W.
- If we are allowed to take fractional amounts, then this is the **fractional knapsack problem**.
  - In this case, we let x_i denote the amount we take of item i
  - Objective: maximize $$\sum_{i \in S} b_i (x_i / w_i)$$
  - Constraint: $$\sum_{i \in S} x_i \le W$$

The Fractional Knapsack Algorithm

- Greedy choice: Keep taking item with highest **value** (benefit to weight ratio)
  - Since $\sum_{i \in S} b_i (x_i / w_i) = \sum_{i \in S} (b_i / w_i) x_i$
  - Run time: O(n log n). Why?
- Correctness: Suppose there is a better solution
  - there is an item i with higher value than a chosen item j, but $x_i<w_i$, $x_j>0$ and $v_j<v_i$
  - If we substitute some j with i, we get a better solution
  - How much of i: min$\{w_i\text{-}x_i, x_j\}$
  - Thus, there is no better solution than the greedy one

Algorithm ***fractionalKnapsack***(***S***, ***W***)

Input: set ***S*** of items w/ benefit b_i and weight w_i; max. weight W

Output: amount x_i of each item i to maximize benefit w/ weight at most W

for ***each item i in S***

$x_i \leftarrow 0$

$v_i \leftarrow b_i / w_i$ {value}

$w \leftarrow 0$ {total weight}

while $w < W$

remove item i w/ highest v_i

$x_i \leftarrow \min\{w_i, W - w\}$

$w \leftarrow w + \min\{w_i, W - w\}$

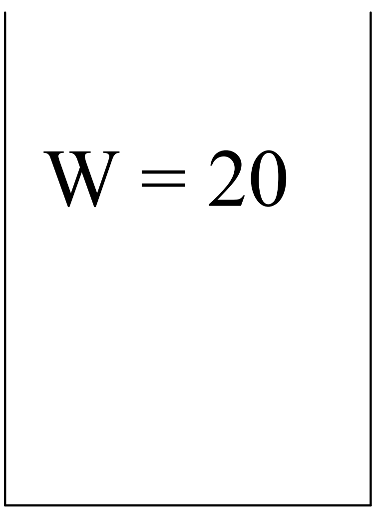
0-1 Knapsack problem (1/5)

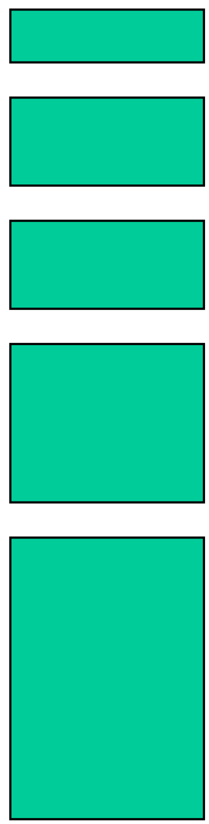
- Given a knapsack with maximum capacity W, and a set S consisting of n items
- Each item i has some **weight w_i** and **benefit value b_i** (all w_i , b_i and W are integer values)
- Problem:

  How to pack the knapsack to achieve maximum total value of packed items?

0-1 Knapsack problem (2/5)

This is a knapsack
Max weight: W = 20





Items	Weight w_i	Benefit value b_i
	2	3
	3	4
	4	5
	5	8
	9	10

0-1 Knapsack problem (3/5)

- Problem, in other words, is to find

$$\max \sum_{i \in T} b_i \text{ subject to } \sum_{i \in T} w_i \le W$$

- The problem is called a ***"0-1" Knapsack problem***, because each item must be entirely accepted or rejected.
- Another version of this problem is the "***Fractional Knapsack Problem***", where we can take fractions of items.

0-1 Knapsack problem (4/5)

Let's first solve this problem with a straightforward algorithm (Brute-force)

- ✓ Since there are n items, there are 2^n possible combinations of items.
- ✓ We go through all combinations and find the one with the most total value and with total weight less or equal to W.
- ✓ Running time will be $O(2^n)$

Brute force ?

- Brute = Beast 野獸
- Brute 是個英雄(**Marcus Junius Brutus**)，出現在**莎士比**
  取羅馬歷史上西元前44年凱撒遇刺事件所寫描述一個英雄
  墜落的悲劇 《凱撒大帝》。**貴族世家後** 代的**Brute 原爲凱**
  撒忠臣，因凱撒獨裁憂心羅馬前途決定刺殺凱撒。
- Brutus 刺殺凱撒大帝後自殺之前留下一句名言：「並非我愛凱撒較少，而是我愛羅馬更多。」

> In Shakespeare's "Julius Caesar," he is called Brute, and not Brutus, because "Brute" is in the Vocative Case in Latin.

- 1929年漫畫家西格(Elzie Segar)創作連載漫畫**大力水手卜派** (PopEye the Sailor)
- 1933 Fleischer Studios 改編爲戲劇, Brute Bluto是 Popeye的死對頭! 1961年更搬上電視卡通 (cartoon) !

0-1 Knapsack problem (5/5)

- Can we do better?
  - Yes, with an algorithm based on **Dynamic programming**
  - We need to carefully identify the subproblems

Key point:

If items are labeled *1..n*, then a subproblem would be to find an optimal solution for $S_k = \{items\ labeled\ 1,\ 2,\ ..\ k\}$

Algorithm types

- Algorithm types we will consider include:
  - Simple recursive algorithms
  - Backtracking algorithms
  - Greedy algorithms
  - **Divide and Conquer algorithms**
  - Dynamic programming algorithms
  - Branch and bound algorithms
  - Brute force algorithms
  - Randomized algorithms

The Greedy Method Technique

- **The greedy method** is a general algorithm design paradigm, built on the following elements:
  - **configurations**: different choices, collections, or values to find
  - **objective function**: a score assigned to configurations, which we want to either maximize or minimize
  - A **greedy algorithm** always makes the choice that looks best at the moment. (時到時擔當, 沒米再煮蕃薯塊湯)
  - Top-down algorithmic structure
    - With each step, reduce problem to a smaller problem
- It works best when applied to problems with the **greedy-choice** property:
  - a globally-optimal solution can always be found by **a series of local improvements** from a starting configuration.

The greedy method cannot always find an optimal solution!

Coin Changing problem

- Problem: A dollar amount to reach and a collection of coin amounts to use to get there.
  - Configuration: A dollar amount yet to return to a customer plus the coins already returned
  - Objective function: **Minimize number of coins returned.**

Coins in USA:	**1 ¢**	**5 ¢**	**10 ¢**	**25 ¢**	**50 ¢**

- Greedy solution: Always return the largest coin you can
  - Example 1: Coins are valued \$.32, \$.08, \$.01
    - Has the greedy-choice property, since no amount over \$.32 can be made with a minimum number of coins by omitting a \$.32 coin (similarly for amounts over \$.08, but under \$.32).

Greedy Algorithm for Coin Changing problem

This algorithm makes change for an amount *A* using coins of denominations

$$denom[1] > denom[2] > \cdots > denom[n] = 1.$$

```
Input Parameters: denom, A
Output Parameters: None
greedy_coin_change(int denom[], int A) {
   i = 1
   while (A > 0) {
      c = A/denom[ i ]
      println("use " + c +
            " coins of denomination " +
             denom[i])
      A = A - c * denom[i]
      i = i + 1
   }
}
```

Question ?

Suppose there are unlimited quantities of coins of each denomination.

What property should the denominations *c1, c2, …, ck* have so that the greedy algorithm always yields an optimal solution?

- ✓ Consider this example:
  - Example 2: Coins are valued \$.30, \$.20, \$.05, \$.01
    - Does **not** have greedy-choice property, since \$.40 is best made with two \$.20′s, but the greedy solution will pick three coins (which ones?)

The greedy method cannot always find an optimal solution!

Consider the Coin set for examples

- For the following examples, we will assume coins in the following denominations:

  1¢ 5¢ 10¢ 21¢ 25¢

- We'll use 63¢ as our goal

- This example is taken from:
  Data Structures & Problem Solving using Java *by* Mark Allen Weiss

The greedy method cannot always find an optimal solution!

(1) A simple solution

- **We always need a 1¢ coin, otherwise no solution exists for making one cent**
- To make K cents:
  - If there is a K-cent coin, then that one coin is the minimum
  - Otherwise, for each value i < K,
    - Find the minimum number of coins needed to make i cents
    - Find the minimum number of coins needed to make K - i cents
  - Choose the i that minimizes this sum
- This algorithm can be viewed as **divide-and-conquer**, or as **brute force**
  - This solution is very **recursive**
  - It requires exponential work
  - It is ***infeasible*** to solve for 63¢

(2) Another solution

- We can reduce the problem recursively by choosing the first coin, and solving for the amount that is left
- For 63¢:
  - One 1¢ coin plus the best solution for 62¢
  - One 5¢ coin plus the best solution for 58¢
  - One 10¢ coin plus the best solution for 53¢
  - One 21¢ coin plus the best solution for 42¢
  - One 25¢ coin plus the best solution for 38¢
- Choose the best solution from among the 5 given above
- Instead of solving 62 recursive problems, we solve 5
- This is still a very expensive algorithm

(3) A dynamic programming solution

- Idea: Solve first for one cent, then two cents, then three cents, etc., up to the desired amount
  - *Save each answer in an array !*
- For each new amount N, compute all the possible pairs of previous answers which sum to N
  - For example, to find the solution for 13¢,
    - First, solve for all of 1¢, 2¢, 3¢, ..., 12¢
    - Next, choose the best solution among:
      - Solution for 1¢ + solution for 12¢
      - Solution for 2¢ + solution for 11¢
      - Solution for 3¢ + solution for 10¢
      - Solution for 4¢ + solution for 9¢
      - Solution for 5¢ + solution for 8¢
      - Solution for 6¢ + solution for 7¢

Example using dynamic programming

- Suppose coins are 1¢, 3¢, and 4¢
  - There's only one way to make 1¢ (one coin)
  - To make 2¢, try 1¢+1¢ (one coin + one coin = 2 coins)
  - To make 3¢, just use the 3¢ coin (one coin)
  - To make 4¢, just use the 4¢ coin (one coin)
  - To make 5¢, try
    - 1¢ + 4¢ (1 coin + 1 coin = 2 coins)
    - 2¢ + 3¢ (2 coins + 1 coin = 3 coins)
    - The first solution is better, so best solution is 2 coins
  - To make 6¢, try
    - 1¢ + 5¢ (1 coin + 2 coins = 3 coins)
    - 2¢ + 4¢ (2 coins + 1 coin = 3 coins)
    - 3¢ + 3¢ (1 coin + 1 coin = 2 coins) – best solution
  - Etc.

How good is the algorithm?

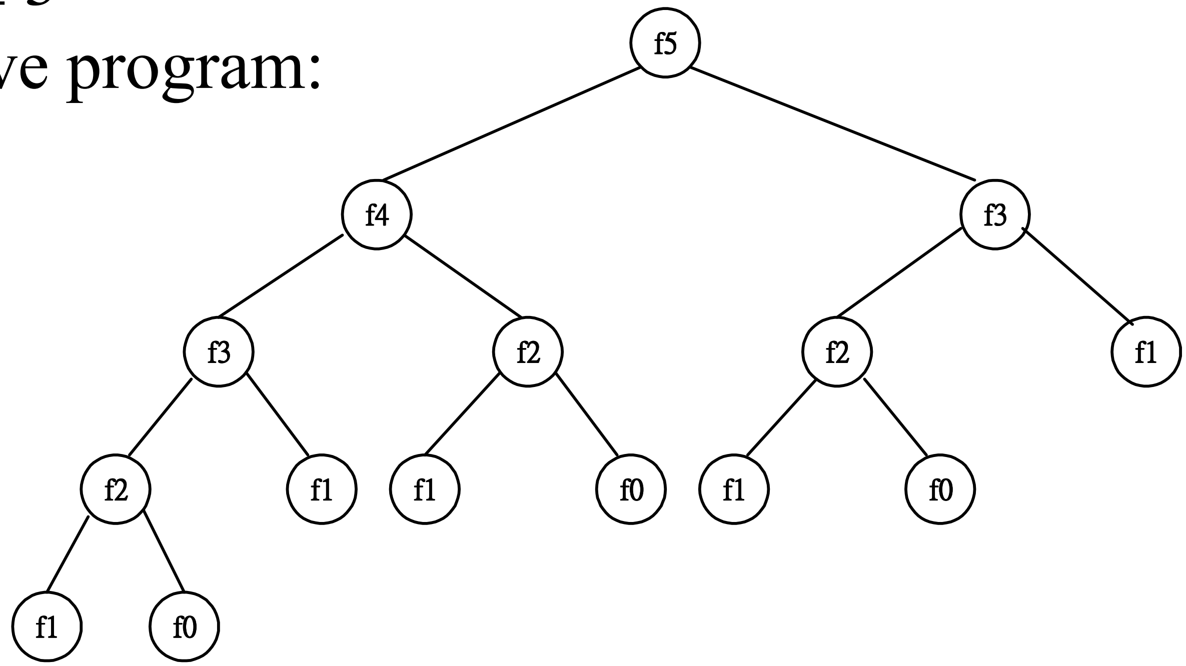
- The first algorithm is recursive, with a branching factor of up to 62
  - Possibly the average branching factor is somewhere around half of that (31)
  - The algorithm takes exponential time, with a large base
- The second algorithm is much better—it has a branching factor of 5
  - This is exponential time, with base 5
- The dynamic programming algorithm is O(N*K), where N is the desired amount and K is the number of different kinds of coins

Dynamic Programming (DP) ?

- Like divide-and-conquer, solve problem by combining the solutions to sub-problems.
- Differences between divide-and-conquer and DP:
  - Independent sub-problems, solve sub-problems independently and recursively, (so same sub(sub)problems solved repeatedly)
  - Sub-problems are dependent, i.e., sub-problems share sub-sub-problems, every sub(sub)problem solved just once, solutions to sub(sub)problems are stored in a table and used for solving higher level sub-problems.
- DP reduces computation by
  - Solving subproblems in a bottom-up fashion.
  - Storing solution to a subproblem the first time it is solved.
  - Looking up the solution when subproblem is encountered again.

Fibonacci sequence (1/4)

- Fibonacci sequence: 1 , 1 , 2 , 3 , 5 , 8 , 13 , 21 , …
  $F_i = 1$ if i ==1 or i == 2 (assume $F_0 = 0$)
  $F_i = F_{i-1} + F_{i-2}$ if $i \geq 3$
- Solved by a recursive program:



- Much replicated computation is done.
- It should be solved by a simple loop.

Fibonacci Sequence (2/4)

- Recursive logic:
  - $F_1 = F_2 = 1$
  - If $i > 2$ then $F_i = F_{i-1} + F_{i-2}$
- Directly translates into a recursive algorithm F:

```
F(i) {
    if (i = 1) or (i = 2)
      x ← 1
    else
      x ← F(i-1) + F(i-2)
    return x
    }
```

- F's call tree is **exponential**; F is recomputed many times for the same input value!

- We can speed things up by storing output values of F in an array A_F

```
F(i) {
    if (A_F != NULL) return A_F
    if (i = 1) or (i = 2)
      x ← 1
    else
      x ← F(i-1) + F(i-2)
    A_F ← x
    return x
    }
```

- Since there are n cells in A_F and each cell takes $O(1)$ time to compute, this is $O(n)$!

Fibonacci Sequence (3/4)

- We have sped things up by storing output values of F in an array A_F

```
F(i) {
    if (A_F != NULL) return A_F
    if (i = 1) or (i = 2)
        x ← 1
    else
        x ← F(i-1) + F(i-2)
    A_F ← x
    return x
    }
```

- But we don't need recursion at all, just a loop through A_F!

- The final algorithm is:

```
Fib(n) {
  A_F ← new array of n int's
  for (i = 1 to n) {
      if (i = 1) or (i = 2)
        x ← 1
      else
        x ← F(i-1) + F(i-2)
      A_F[i] ← x
      }
  return A_F[n]
}
```

- **This is a dynamic programming algorithm!**

Fibonacci Sequence (4/4) using Dynamic programming

- Dynamic programming calculates from **bottom to top**. (**bottom-up**)
- Values are stored for later use.
- This reduces repetitive calculation.

Pascal Triangle ?

Application domain of DP

- **Optimization problem**: find a solution with optimal (maximum or minimum) value.
- ***An* optimal** solution, not *the* optimal solution, since **may more than one optimal solution**, any one is OK.
- **Dynamic Programming** is an algorithm design method that can be used when the solution to a problem may be viewed as the result of a sequence of decisions

Typical steps of DP

- Characterize the structure of an optimal solution.
- Recursively define the value of an optimal solution.
- Compute the value of an optimal solution in a **bottom-up** fashion.
- Compute an optimal solution from computed/stored information.

Comparison with Divide-and-Conquer

- **Divide-and-conquer** algorithms split a problem into separate subproblems, solve the subproblems, and combine the results for a solution to the original problem
  - Example: **Quicksort**
  - Example: Mergesort
  - Example: Binary search

Divide-and-Conquer

分割征服 ;各個擊破;拆解

- **Divide-and-Conquer algorithms can be thought of as top-down algorithms**
- In contrast, a dynamic programming algorithm proceeds by solving small problems, then combining them to find the solution to larger problems
- **Dynamic programming can be thought of as bottom-up**

Divide and Conquer

- **Divide** the problem into a number of sub-problems (similar to the original problem but smaller);
- **Conquer** the sub-problems by solving them recursively (if a sub-problem is small enough, just solve it in a straightforward manner (base case).)
- **Combine** the solutions to the sub-problems into the solution for the original problem

Divide and Conquer example : Merge Sort

- **Divide** the n-element sequence to be sorted into two subsequences of n/2 element each
- **Conquer:** Sort the two subsequences recursively **using merge sort**
- **Combine:** **merge** the two sorted subsequences to produce the sorted answer
- **Note**: during the recursion, if the subsequence has only one element, then do nothing.

Comparison with Greedy

- Common: optimal substructure
  - **Optimal substructure:** An optimal solution to the problem contains within its optimal solutions to subproblems.
    - E.g., if A is an optimal solution to S, then $A' = A - \{1\}$ is an optimal solution to $S' = \{i \in S: s_i \geq f_1\}$.
- Difference: greedy-choice property
  - **Greedy:** A global optimal solution can be arrived at by making a locally optimal choice.
  - Dynamic programming needs to check the solutions to subproblems.
- DP can be used if greedy solutions are not optimal.

The greedy method cannot always find an optimal solution!

Greedy vs. Dynamic Programming

The knapsack problem is a good example of the difference.

0-1 knapsack problem: not solvable by greedy.

- n items.
- Item i is worth \$$v_i$, weighs w_i pounds.
- Find a most valuable subset of items with total weight $\leq W$.
- Have to either take an item or not take it—can't take part of it.

Fractional knapsack problem: solvable by **greedy**

- Like the 0-1 knapsack problem, but can take fraction of an item.
- Both have optimal substructure.
- But the fractional knapsack problem has the greedy-choice property, and the 0-1 knapsack problem does not.
- To solve the fractional problem, rank items by value/weight: $\boldsymbol{v_i / w_i}$.
- Let $v_i / w_i \geq v_{i+1} / w_{i+1}$ for all i .

The greedy method cannot always find an optimal solution!

Divide-and-Conquer in Sorting

- Mergesort
  - O(n log n) always, but O(n) storage
- Quick sort
  - O(n log n) average, O(n^2) worst in time
  - O(log n) storage
  - Good in practice (>12)

Quick Sort (1/6)

Algorithm *quick_sort*(array *A,* from, to)
Input: from - pointer to the starting position of array *A*
to - pointer to the end position of array *A*
Output: sorted array: *A*'

1. Choose any one element as the pivot;
2. Find the first element *a* = *A*[i] larger than or equal to pivot from *A*[from] to *A*[to];
3. Find the first element *b* = *A*[j] smaller than or equal to pivot from *A*[to] to *A*[from];
4. If i < j then exchange *a* and *b;*
5. Repeat step from 2 to 4 until j <= i;
6. If from < j then recursive call *quick_sort*(*A,* from, j);
7. If i < to then recursive call *quick_sort*(*A,* i, to);

Quick Sort (2/6)

- Quick sort

  main idea: from

  Choose 5 as pivot

  to

  1st step: 3 9 1 6 **5** 4 8 2 10 7

  i *j*

  2nd step: 3 2 1 6 **5** 4 8 9 10 7

  3rd step: 3 2 1 4 **5** 6 8 9 10 7

  Smaller than any integer right to 5

  greater than any integer left to 5

Quick Sort (3/6)

- Quick sort

from **pivot** **to**

from **pivot** **to**

4th step: 3 2 1 4 5 6 8 10 9 7

5th step: 1 2 3 4 5

6th step: 5 6 7 8 10 9

7th step: 7 8 10 9

8th step: 9 10

Quick Sort (4/6)

```
public class QuickSorter {    // Java function should be in a class
    public static void sort (int[ ] a, int from, int to) {
        if ((a == null) || (a.length < 2)) return;
        int i = from, j = to;
        int pivot = a[(from + to)/2];
        do {
            while ((i < to) && (a[i] < pivot))  i++;
            while ((j > from) && (a[j] >= pivot))  j--;
            if (i < j) { int tmp =a[i]; a [i] = a[j]; a[j] = tmp;}
            i++;  j--;
        }while (i <= j);   exchange(a, i, (from+to)/2 );  /***/
        if (from < j) sort(a, from, j);
        if (i < to) sort(a, i, to);
    }
}
```

Quick Sort (5/6)

3, 4, 6, 1, 10, 9, 5, 20, 19 14, 12, 2, 15, 21, 13, 18, 17, 8, 16, 1

3, 4, 6, 1, 10, 9, 5, 20, 19, 1, 12, 2, 15, 21, 13, 18, 17, 8, 16, 14

i *j*

3, 4, 6, 1, 10, 9, 5 8, 19, 1, 12, 2, 15, 21, 13, 18, 17, 20, 16, 14

i *j*

3, 4, 6, 1, 10, 9, 5, 8, 13 ,1, 12, 2, **15**, 21, 19, 18, 17, 20, 16, 14

i *j*

3, 4, 6, 1, 10, 9, 5, 8, 13 , 1, 12, 2, 14 21, 19, 18, 17, 20, 16, 15

i

j

3, 4, 6, 1, 10, 9, 5, 8, 13 , 1, 12, 2

Quick Sort (6/6)

```
void qsort (int a[ ], int from, int to) {
    int n = to – from + 1;
    if ( (n < 2) || (from >= to) ) return;
    int k = (from + to)/2; int tmp =a[to]; a [to] = a[k]; a[k] = tmp;
    int pivot = a[to];    // choose a[to]  as the pivot
    int i = from,  j = to-1;
    while(i < j )  {
        while ((i < j) && (a[i] < pivot))  i++;
        while ((i < j) && (a[j] >= pivot))  j--;
        if (i < j) { tmp =a[i]; a [i] = a[j]; a[j] = tmp;}
    };
    tmp =a[i]; a [i] = a[to]; a[to] = tmp;   // exchange
    if (from < i-1) qsort(a, from, i-1);
    if (i < to) qsort(a, i+1, to);
}
```

有大 bugs 的程式碼

```
//buga.c with Big BUG (quick sort)
//  這sort程式有一個大  bug :
//    ..  i 往右時可能會走過頭！ j 往左時也可能會走過頭！
// program 會在執行時當掉！ Why?
void sort( int left, int right, double x[ ]) {   // 注意參數順序要與呼叫者相同
   int i, j;  double tmp;
   i = left;   j = right+1;   // i points to LEFT, j points to 最右的下一個
   while( i<=j ) {
      do { i++; } while(   x[i] >= x[left]);  // Bug Bug Bug
      do { j--; } while ( x[j] <= x[left]);   //Bug  Bug Bug
      if(i<j) {tmp=x[i]; x[i]=x[j]; x[j]=tmp; }
   }
   tmp=x[j]; x[j]=x[left]; x[left]=tmp;
   if(left< j-1) sort(left, j-1, x);
   if(j+1 < right) sort(j+1, right, x);
} // sort( )
```

有小 bugs 的程式碼

```
//bugb.c with small BUG   (quick sort)
//  這sort程式有一個小  bug：  (已經試著要 fix Big Bug)
//     ..   i 往右時可能會走過頭！ j 往左時也可能會走過頭！
// Please try to fix it
void sort( int left, int right, double x[ ]) {
   int i, j;  double tmp;
   i = left;    j = right+1;    // i points to LEFT, j points to 最右的下一個
   while( i<=j ) {
       do { i++; } while(  i < right && x[i] >= x[left]);  // Bug 還在
       do { j--; } while ( j > left &&x[j] <= x[left]);
       if(i<j) {tmp=x[i]; x[i]=x[j]; x[j]=tmp; }
   }
   tmp=x[j]; x[j]=x[left]; x[left]=tmp;
   if(left< j-1) sort(left, j-1, x);
   if(j+1 < right) sort(j+1, right, x);
}
```


比了幾次 data 才完成 sorting ?(1/2)

```
//sortg.c  -- quick sort
#include<stdio.h>
long n=0;
void sort( int left, int right, double x[ ]) {
  int i, j;  double tmp, pivot;
  i = left;   j = right+1;
  pivot = x[left];
  while( i<=j ) {
      do { i++;  ++n;} while(  i< j && x[i] >= pivot);
      if(i>= j) --n;
      do { j--;  ++n;} while ( i<=j  &&x[j] <= pivot);
      if(i > j) --n;
      if(i<j) {tmp=x[i]; x[i]=x[j]; x[j]=tmp; }
  }
  printf("= n=%ld", n);
  tmp=x[j]; x[j]=x[left]; x[left]=tmp;
  if(left< j-1) sort(left, j-1, x);
  if(j+1 < right) sort(j+1, right, x);
}
```

比了幾次 data 才完成 sorting ?(2/2)

```
double y[ ] = {15, 38, 12, 75, 39,88,20, 66, 49, 58};
#include<stdio.h>
void pout(double*, int);
int main( ) {
    printf("Before sort:\n");  pout(y, sizeof(y)/sizeof( y[0]) );
    sort(0, sizeof(y)/sizeof( y[0]) -1 ,y );
    printf("==The number of data comparisms, n = %ld\n", n);
//
    printf(" After sort:\n");  pout(y, sizeof(y)/sizeof( y[0]) );
}
void pout(double*p, int n) {
    int i;
    for(i=0; i<=n-1; ++i) {
        printf("%7.2f  ", p[i]);
    } printf(" \n");
}
```

qsort( ) in C Library

- There is a library function for quick sort in C Language: qsort( ).
- #include <stdlib.h>

```
void qsort(void *base, size_t num, size_t size,
     int (*comp_func)(const void *, const void *) )
```

void * base --- a pointer to the array to be sorted

size_t num --- the number of elements

size_t size --- the element size

int (*cf) (…) --- is a pointer to a function used to compare

int comp_func( ) 必須傳回 -1, 0, 1 代表 <， ==， >

C++ STL <algorithm>

站在巨人肩膀上

```
#include <algorithm>
using namespace std;
int x[ ] = { 38, 49, 15, 158, 25, 58, 88,66 };    // array of primitive data
#define  n  (sizeof(x)/sizeof(x[0]))
//…
sort(x, x+n);    // ascending order
  // what if we want to sort into descending order
sort(x, x+n, sortfun);  // with compare function
sort(y, y+k, sortComparatorObject);  // with Comparator Object
```

Comparison function?   Default: bool operator<(first, second)
C++ Comparison function 爲**bool**
須傳回 **true** 代表 第一參數 **<** 第二參數 **:** **ascending**

Comparator 內要有 bool operator( ) (Obj a, Obj b) { /*…*/ }

http://www.cplusplus.com/reference/algorithm/sort/

Java 的 java.util.Arrays.sort( )

- 只能用來 sort 物件的 array (Not primitive data)
- 可透過該物件的 compareTo( ), 當然該 class 須 implements **java.lang.Comparable**
- 也可透過傳給 sort 一個 Comparator, 就是某個有 implement **java.util.Comparator** 之 class 的實體物件; 注意 Java 沒辦法把函數當作參數傳!!!

Java 不可以把函數當作參數!
java.util.Arrays.sort( ) 要傳 Comparator 物件?
不傳則會用要被 sort 之 array 的物件內的 compareTo( )

不論是 compareTo( ), 還是**Comparator 內的 compare( )**
都是類似 C 的 qsort 用的比較函數, int, 要傳回 -1, 0, 1

Merge Sort (1/5)

Merging means the combination of two or more ordered sequence into a single sequence. For example, can merge two sequences: 503, 703, 765 and 087, 512, 677 to obtain a sequence: 087, 503, 512, 677, 703, 765. A simple way to accomplish this is to compare the two smallest items, output the smallest, and then repeat the same process.

$$\left\{\begin{matrix}503 & 703 & 765\\ 87 & 512 & 677\end{matrix}\right. \Rightarrow 87 \left\{\begin{matrix}503 & 703 & 765\\ 512 & 677 & \end{matrix}\right.$$

$$\Rightarrow 87 \quad 503 \left\{\begin{matrix}703 & 765\\ 512 & 677\end{matrix}\right.$$

The smaller one goes first

Merge Sort (2/5)

Algorithm *Merge*(s1, s2)

Input: two sequences: s1 - $x1 \le x2 \ldots \le x_m$ and s2 - $y1 \le y2 \ldots \le y_n$

Output: a sorted sequence: $z1 \le z2 \ldots \le z_{m+n}$.

1.[initialize] $i := 1, j := 1, k := 1$;

2.[find smaller] if $x_i \le y_j$ goto step 3, otherwise goto step 5;

3.[output x_i] $z_k.:= x_i$, $k := k+1$, $i := i+1$. If $i \le m$, goto step 2;

4.[transmit $y_j \le \ldots \le y_n$] $z_k, \ldots, z_{\mathrm{m+n}} := y_j, \ldots, y_n$. Terminate the algorithm;

5.[output y_j] $z_k.:= y_j$, $k := k+1$, $j := j+1$. If $j \le n$, goto step 2;

6.[transmit $x_i \le \ldots \le x_m$] $z_k, \ldots, z_{\mathrm{m+n}} := x_i, \ldots, x_m$. Terminate the algorithm;

Merge Sort (3/5)

Algorithm ***Merge-sorting***(s)

Input: a sequences s = < x_1, ..., x_m>

Output: a sorted sequence.

1. If |s| = 1, then return s;
2. $k := \lceil m/2 \rceil$;
3. s1 := Merge-sorting(x_1, ..., x_k);
4. s2 := Merge-sorting(x_{k+1}, ..., x_m);
5. return(*Merge*(s1, s2));

Merge(s1, s2) will merge the two halves s1 and s2 together

Mergesort (4/5) (in C Language)

```
public void mergeSort ( double arr [ ],
                                        int  from,  int  to )
{
   if  (from <= to)
       return;

   int middle = (from + to ) / 2;
   mergeSort (arr, from, middle);
   mergeSort (arr, middle + 1, to);

   if (arr [middle] > arr [middle+ 1] )
   {
      copy (arr, from, to, temp) ;
      merge (temp, from, middle, to, arr);
   } // if
}// mergeSort
```

"if not yet sorted"...

optional shortcut

double temp[ ] is initialized outside the mergesort method

Mergesort (5/5) time complexity

- **Takes roughly $n \cdot \log_2 n$ comparisons.**
- **Without the shortcut**, there is no best or worst case.
- **With the optional shortcut, the best case is when the array is already sorted: takes only ($n-1$) comparisons.**

http://en.wikipedia.org/wiki/Merge_sort

Thank You!

CHAPTER 5

Algorithms
補充

謝謝捧場

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