Formal Language Selected Homework Chapter 3.1

2. Does the expression $((0+1)(0+1)^*)^* 00 (0+1)^*$ denote the language in Example 3.5?

4. Find a regular expression for the set $\{a^n b^m : n \geq 3, m \text{ is even}\}$.

5. Find a regular expression for the set $\{a^n b^m : (n+m) \text{ is even}\}$.

6. Give regular expressions for the following languages.

(a) $L_1 = \{a^n b^m, n \geq 4, m \leq 3\}$.

(c) The complement of L_1.

10. Give a regular expression for $L = \{a^n b^m : n \geq 1, m \geq 1, nm \geq 3\}$.

13. Find a regular expression for $L = \{vwv : v, w \in \{a, b\}^*, |v| = 2\}$.

Sol. **2.** Yes, because $\left((0+1)(0+1)^*\right)^*$ denotes any string of 0's and 1's.

Sol. **6.** (a) Separate into cases $m = 0, 1, 2, 3$. Generate 4 or more a's, followed by the requisite number of b's. Solution: $aaaaa^* (\lambda + b + bb + bbb)$.

(c) The complement of the language in 6(a) is harder to find. A string is not in L if it is of the form $a^n b^m$, with either $n < 4$ or $m > 3$, but this does not completely describe $\overline{L}$. We must also consider the case where a b is followed by an a.

$$(\lambda + a + aa + aaa)\, b^* + a^* bbbbb^* + (a+b)^*\, ba\, (a+b)^*.$$

Sol. **10.** Split into three cases: (i) $m = 1$, $n \geq 3$, (ii) $n \geq 2$, $m \geq 2$, and (iii) $n = 1$, $m \geq 3$. Each case has a straightforward solution.

Sol. **13.** Enumerate all cases with $|v| = 2$ to get

$$aa\,(a+b)^*\, aa + ab\,(a+b)^*\, ab + ba\,(a+b)^*\, ba + bb\,(a+b)^*\, bb.$$

Sol 4. $\{a^n b^m : n \geq 3, b \text{ is even}\}$

$aaaa^*(bb)^*$

Sol. 5. $\{a^n b^m : (n+m) \text{ is even}\}$

$(aa)^*(bb)^* + a(aa)^* b(bb)^*$

16. Give regular expressions for the following languages on $\Sigma = \{a, b, c\}$.

(a) all strings containing exactly one a,

(b) all strings containing no more than three a's,

(c) all strings that contain at least one occurrence of each symbol in Σ,

17. Write regular expressions for the following languages on $\{0, 1\}$.

(a) all strings ending in 01,

(b) all strings not ending in 01,

(c) all strings containing an even number of 0's, Sol: **16.** (c) You just have to get each symbol once. The term

$$(a+b+c)^*\, a\, (a+b+c)^*\, b\, (a+b+c)^*\, c\, (a+b+c)^*$$

will do this, but is not enough since the a will precede the b, etc. For the complete solution you must generate all permutations of the three symbols, giving six terms that can be added. The answer, although quite long, is conceptually not hard.

Sol: **17.** (c) Create two 0's, interspersed with 1's, then repeat. But don't forget the case when there are no 0's at all. Solution: $(1^*01^*01^*)^* + 1^*$.

Sol: 16. (a) $(b+c)^* a (b+c)^*$

Sol: 17. (a) $(0+1)^* 01$

18. Find regular expressions for the following languages on $\{a, b\}$.

(a) $L = \{w : |w| \bmod 3 = 0\}$.

(b) $L = \{w : n_a(w) \bmod 3 = 0\}$.

Sol. **18.** (a) Create all strings of length three and repeat. A short solution is $((a + b \cancel{+c})(a + b \cancel{+c})(a + b \cancel{+c}))^*$.

(b) $(b^*ab^*ab^*ab^*)^* + b^*$

(Skip Problem 23, 25, 26, 27)