



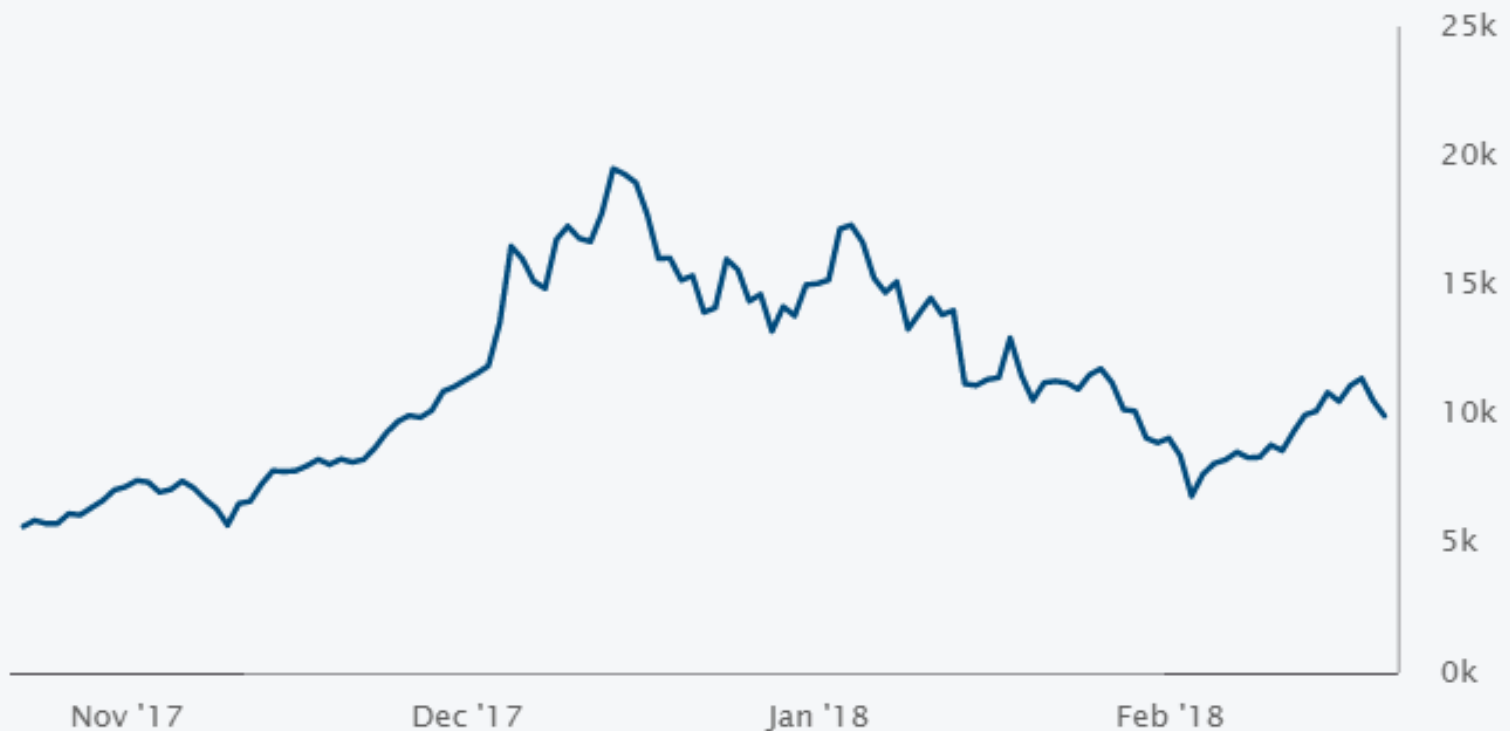
# 比特幣的原理 - 區塊鏈的應用

交大資訊工程系 陳榮傑

<https://blockchain.info>

1 BTC = \$10,098.52

[Interactive Chart](#) →



# Outline

1. Birth of Bitcoin 比特幣誕生
2. Mining 挖礦
3. Hash 雜湊函數
4. Blockchain 區塊鏈
5. Transaction 交易
6. Consensus 共識
7. PKC 公鑰密碼

# Birth of Bitcoin

# 比特幣誕生

# Birth of Bitcoin

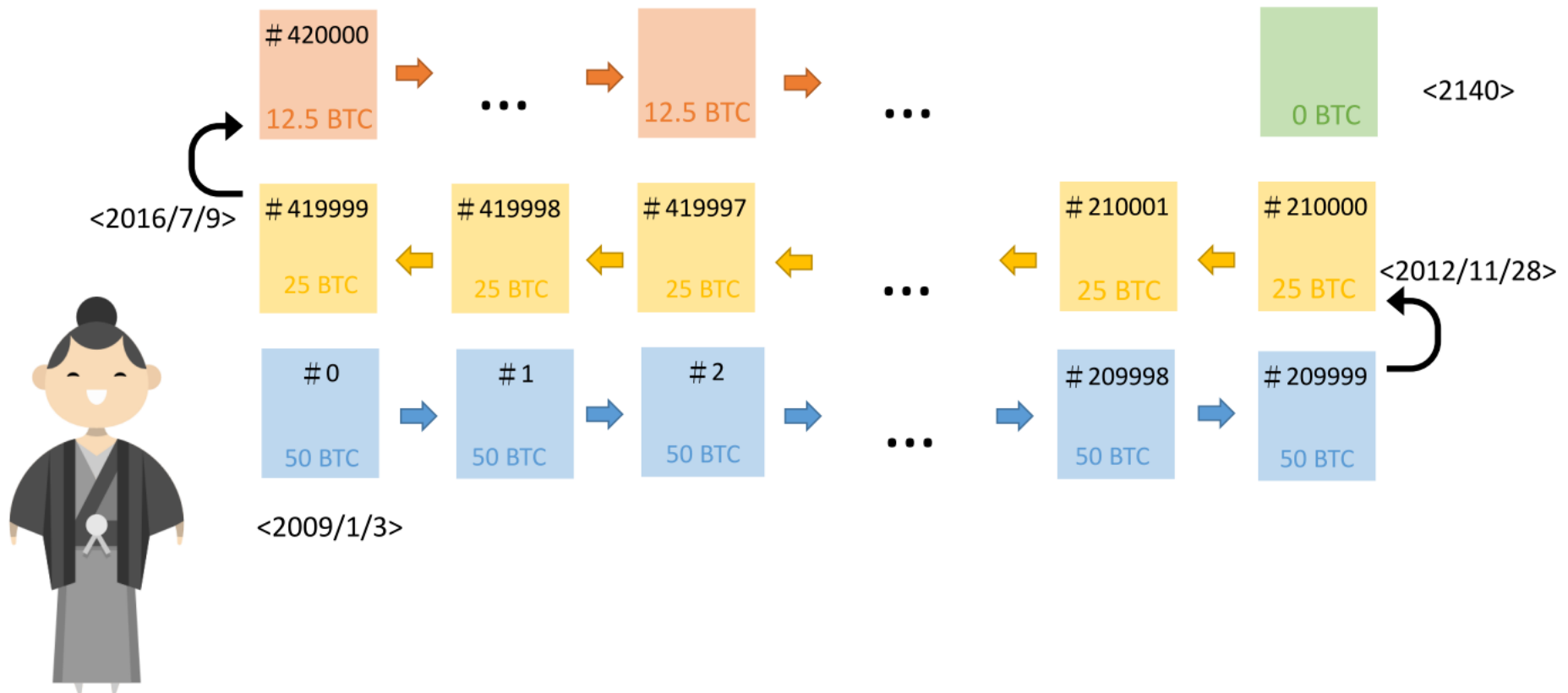
- Described by Satoshi Nakamoto (中本聰) in 2008
- Introduced as open-source software on the evening of January 3, 2009

## Bitcoin: A Peer-to-Peer Electronic Cash System

Satoshi Nakamoto  
satoshin@gmx.com  
www.bitcoin.org

**Abstract.** A purely peer-to-peer version of electronic cash would allow online payments to be sent directly from one party to another without going through a financial institution. Digital signatures provide part of the solution, but the main benefits are lost if a trusted third party is still required to prevent double-spending. We propose a solution to the double-spending problem using a peer-to-peer network.

# 比特幣的區塊鏈當作 公用帳本(public ledger)

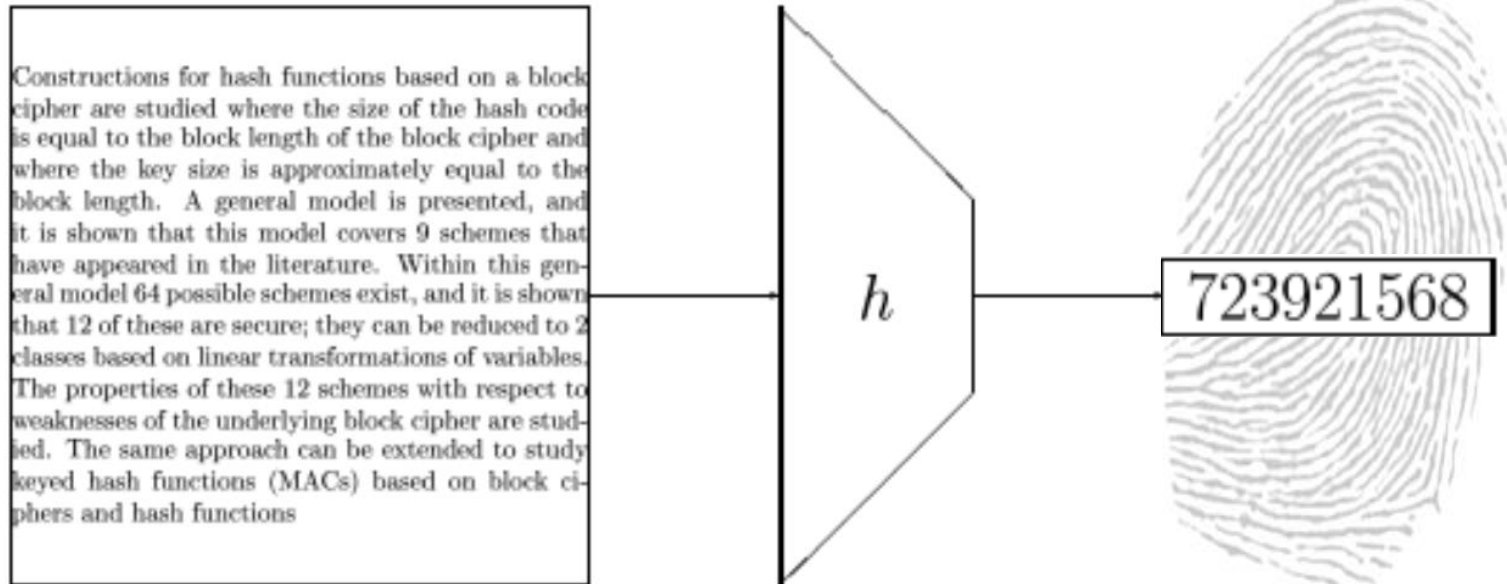


Block#0 的挖礦困難度(difficulty)為1  
即hash後前面32個bits為0, 以後每2016個  
blocks(約兩週)調整一次, 使約每十分鐘  
產生一block



# Hash Function 雜湊函數

- An efficient function mapping binary strings of **arbitrary length** to binary strings of **fixed length**, called the **hash-value** or **hash-code** (fingerprint, checksum)

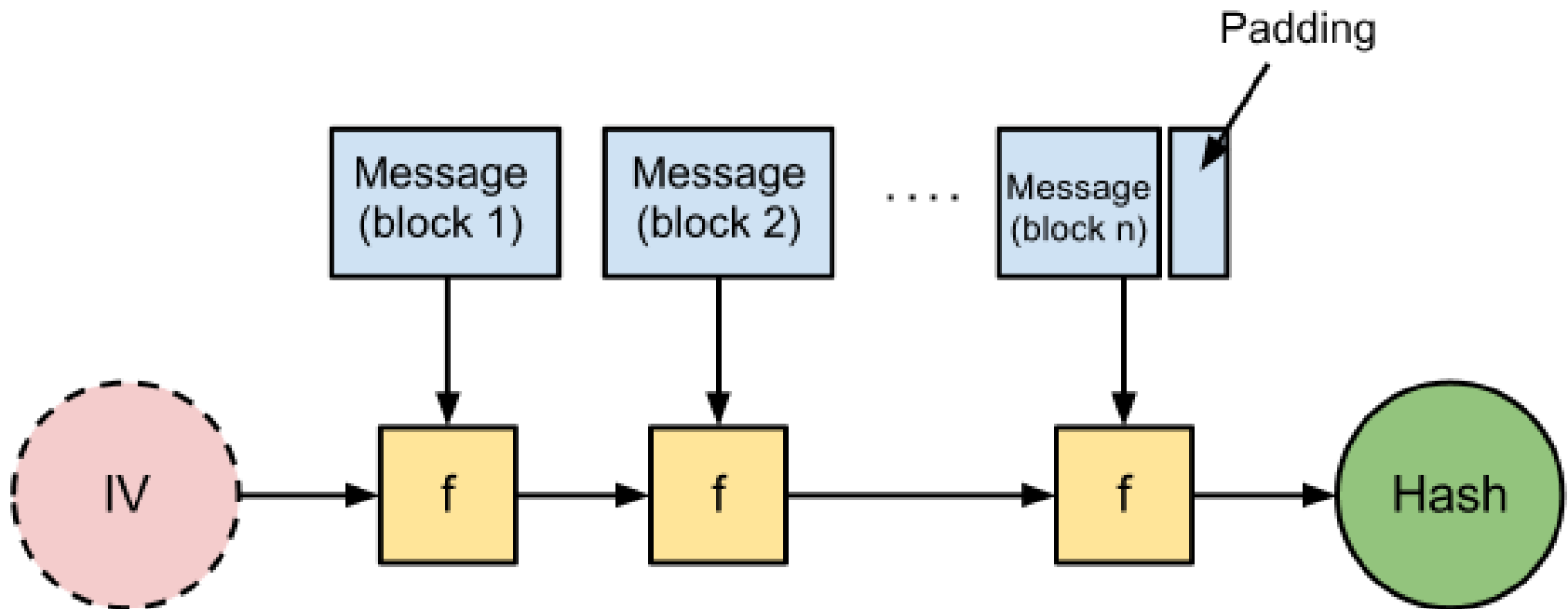


# SHA: Secure Hash Algorithm

- The Secure Hash Algorithm is a family of cryptographic hash functions published by the National Institute of Standards and Technology (NIST) as a U.S. Federal Information Processing Standard (FIPS)

| Algorithm and variant |                                       | Output size (bits) | Internal state size (bits)         | Block size (bits) | Rounds | Bitwise operations          | Security (bits)                 |
|-----------------------|---------------------------------------|--------------------|------------------------------------|-------------------|--------|-----------------------------|---------------------------------|
| <b>SHA-1</b>          | <b>FIPS 180</b>                       | 160                | 160                                | 512               | 80     | and, or, add, xor, rot      | Theoretical attack ( $2^{61}$ ) |
| <b>SHA-2</b>          | SHA-224                               | 224                | 256                                | 512               | 64     | and, or, xor, shr, rot, add | 112                             |
|                       | <b>SHA-256</b> Bitcoin                | 256                | $(8 \times 32)$                    |                   |        |                             | 128                             |
|                       | SHA-384                               | 384                | $512$<br>$(8 \times 64)$           | 1024              | 80     | and, or, xor, shr, rot, add | 192                             |
|                       | FIPS 180 SHA-512                      | 512                |                                    |                   |        |                             | 256                             |
|                       | SHA-512/224                           | 224                |                                    |                   |        |                             | 112                             |
|                       | SHA-512/256                           | 256                |                                    |                   |        |                             | 128                             |
| <b>SHA-3</b>          | SHA3-224                              | 224                | $1600$<br>$(5 \times 5 \times 64)$ | 1152              | 24     | and, xor, rot, not          | 112                             |
|                       | <b>SHA3-256</b> Ethereum (Keccak 256) | 256                |                                    | 1088              |        |                             | 128                             |
|                       | SHA3-384                              | 384                |                                    | 832               |        |                             | 192                             |
|                       | FIPS 202 SHA3-512                     | 512                |                                    | 576               |        |                             | 256                             |

# Merkle-Damgård Construction for SHA-1 / SHA-2



# SHA-256

- One iteration in a SHA-2 family compression function

- The blue components perform the following operations

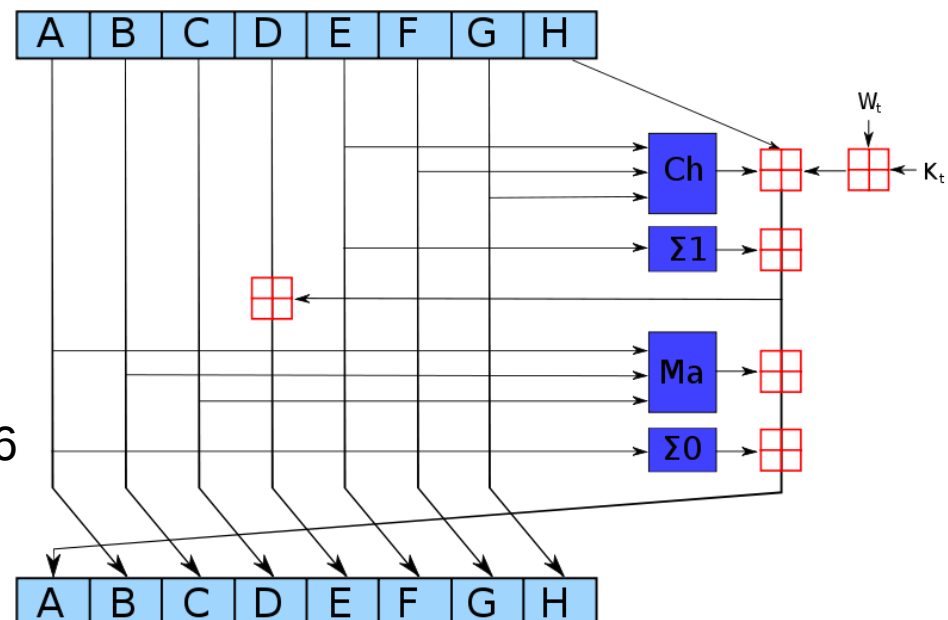
$$\text{Ma}(A, B, C) = (A \wedge B) \oplus (A \wedge C) \oplus (B \wedge C)$$

$$\text{Ch}(E, F, G) = (E \wedge F) \oplus (\neg E \wedge G)$$

$$\Sigma_0(A) = (A \ggg 2) \oplus (A \ggg 13) \oplus (A \ggg 22)$$

$$\Sigma_1(E) = (E \ggg 6) \oplus (E \ggg 11) \oplus (E \ggg 25)$$

- The bitwise rotation uses different constants for SHA-512
- The given numbers are for SHA-256
  -  is addition modulo  $2^{32}$



# Block #0

| Summary                      |                     |
|------------------------------|---------------------|
| Number Of Transactions       | 1                   |
| Output Total                 | 50 BTC              |
| Estimated Transaction Volume | 0 BTC               |
| Transaction Fees             | 0 BTC               |
| Height                       | 0 (Main Chain)      |
| Timestamp                    | 2009-01-03 18:15:05 |
| Received Time                | 2009-01-03 18:15:05 |
| Relayed By                   | Unknown             |
| Difficulty                   | 1                   |
| Bits                         | 486604799           |
| Size                         | 0.285 kB            |
| Weight                       | 0.896 kWU           |
| Version                      | 1                   |
| Nonce                        | 2083236893          |
| Block Reward                 | 50 BTC              |

| Hashes         |                                                                  |
|----------------|------------------------------------------------------------------|
| Hash           | 00000000019d6689c085ae165831e934ff763ae46a2a6c172b3f1b60a8ce26f  |
| Previous Block | 0000000000000000000000000000000000000000000000000000000000000000 |
| Next Block(s)  | 00000000839a8e6886ab5951d76f411475428afc90947ee320161bbf18eb6048 |
| Merkle Root    | 4a5e1e4baab89f3a32518a88c31bc87f618f76673e2cc77ab2127b7afdeda33b |



Be Your Own Bank.  
Use your Blockchain wallet  
to buy bitcoin now.

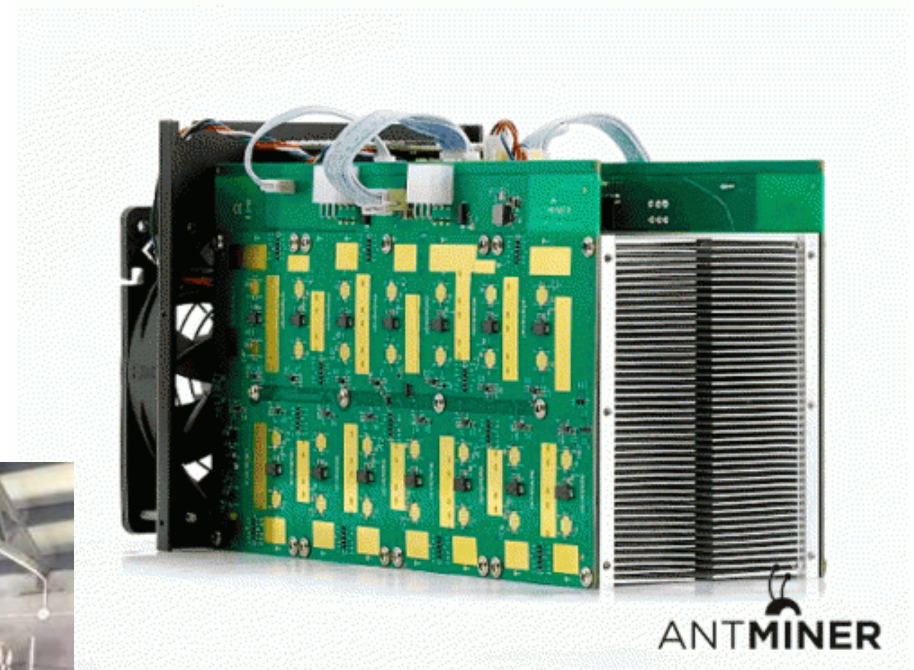
[GET STARTED →](#)

 **BLOCKCHAIN**

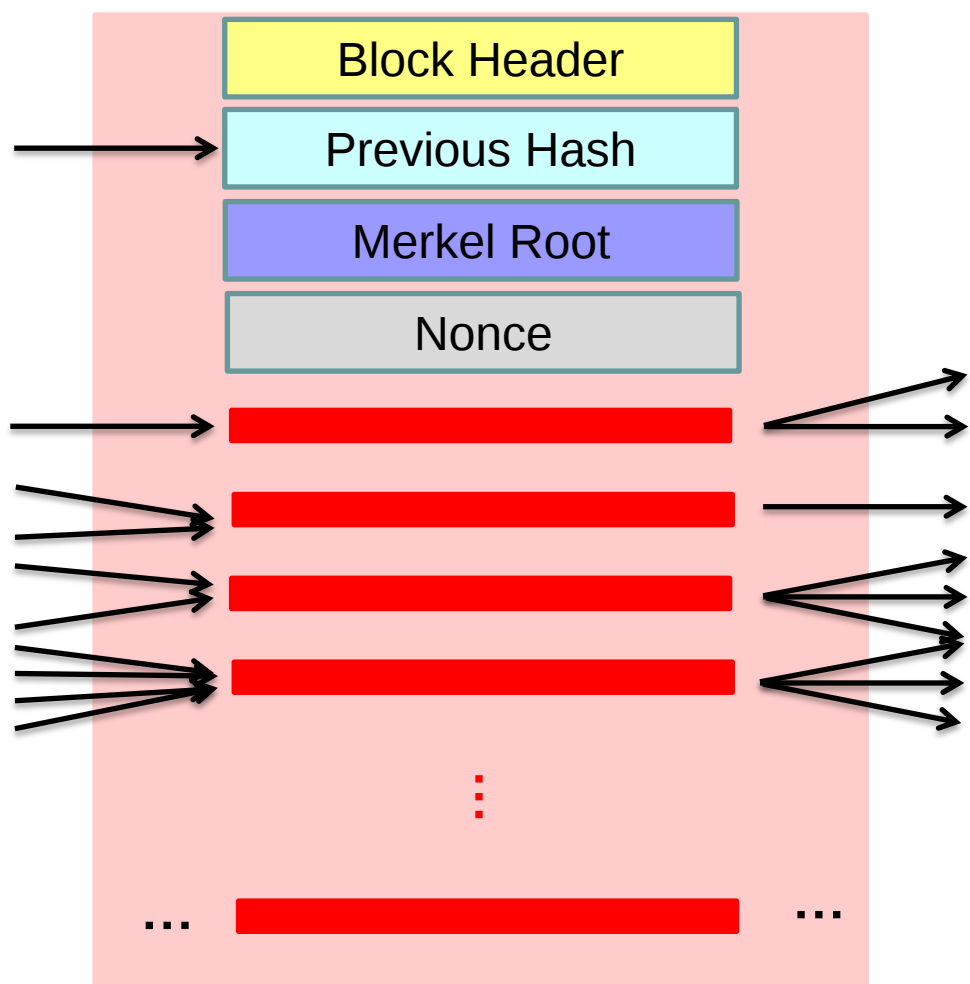
## Transactions

|                                                                  |                                                                                                                        |                     |
|------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------|---------------------|
| 4a5e1e4baab89f3a32518a88c31bc87f618f76673e2cc77ab2127b7afdeda33b |                                                                                                                        | 2009-01-03 18:15:05 |
| No Inputs (Newly Generated Coins)                                |  1A1zP1eP5QGefi2DMPTfTL5SLmv7DivfNa | 50 BTC              |
|                                                                  |                                                                                                                        | 50 BTC              |

# 比特幣礦機



# Block 區塊



# Block #534200

## Summary

|                              |                                     |
|------------------------------|-------------------------------------|
| Number Of Transactions       | 169                                 |
| Output Total                 | 624.18996341 BTC                    |
| Estimated Transaction Volume | 7.13403894 BTC                      |
| Transaction Fees             | 0.01235101 BTC                      |
| Height                       | <a href="#">534200 (Main Chain)</a> |
| Timestamp                    | 2018-07-29 03:59:19                 |
| Received Time                | 2018-07-29 03:59:19                 |
| Relayed By                   | <a href="#">SlushPool</a>           |
| Difficulty                   | 5,178,671,069,072.25                |
| Bits                         | 389437975                           |
| Size                         | 67.031 kB                           |
| Weight                       | 225.103 kWU                         |
| Version                      | 0x20000000                          |
| Nonce                        | 1962295696                          |
| Block Reward                 | 12.5 BTC                            |

## Hashes

|                |                                                                                      |
|----------------|--------------------------------------------------------------------------------------|
| Hash           | <a href="#">000000000000000000000035f53ec665dfcb65b6d946215d8dee6421d19f43373436</a> |
| Previous Block | <a href="#">000000000000000000000014cbbac6073eaaba44a63b6d78754a2bee4f9a065aa399</a> |
| Next Block(s)  |                                                                                      |
| Merkle Root    | 01296f783cf96ed0b71f5f5ca241ae6cc55b0842dad4e0a7d3cad43cb43a6af9                     |



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# Transactions

|                                                                                                                |                                                                                                                                                                                            |                                                      |
|----------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------|
| ff50337f00af30b0a6c5c572a91f15b03a2bbab03af6fde094d031e55d6a23e5                                               |                                                                                                                                                                                            | 2018-07-29 03:59:19                                  |
| No Inputs (Newly Generated Coins)                                                                              |  1CK6KHY6MHgYvmRQ4PAafKYDrg1ejbH1cE<br>Unable to decode output address<br>Unable to decode output address | 12.51235101 BTC<br>0 BTC<br>0 BTC<br>12.51235101 BTC |
| 18163d2c86d300881c4306c78436e8d9c53ba1ff40eb7ffd899a067621238354                                               |                                                                                                                                                                                            | 2018-07-29 03:57:17                                  |
| 1159SzdZyyqBwG1TG9KRJDwfaqZt4xdM3L<br>1Eb8buAUHvNtHA5XJu9CF6VAhrrGDnyXom<br>1N6EPcV47BRG8HmNTvPeyJ7sV3mUWqitsK |  1BuV7TDJ7ysr1BBFvTVf2AnD9rHYVDvuRv<br>36EQt6kESVT9X9oXMTj2PEyzxmm4n6owKy                                 | 0.00840876 BTC<br>0.02142695 BTC<br>0.02983571 BTC   |
| 795a3f5cc0ee990813c3dafde94404a06a5c9507bc80b650d09c238ffc22f7f0                                               |                                                                                                                                                                                            | 2018-07-29 03:58:23                                  |
| 18bY4zkJfErgmds7Wd1Q2d8uhPWqWyJDhE<br>1ABkcFYkXtZ3uLYJ4dxkG96P3T5ffAdpMa                                       |  1FzzXCEWm6sxxEDeebnFoTCTuSNGDMbfcU<br>34eFgSe3in1h4WsyftY4reDMkQaqaVrb14                                 | 0.24257527 BTC<br>0.60312843 BTC<br>0.8457037 BTC    |

# 比特幣發明人果然是他！澳洲企業家 Craig Steven Wright 終於坦言證實

中本聰一直是個謎樣的人物，2008年發表比特幣（Bitcoin）論文後，不僅創造出全新的金融模式，也發明了如今讓全球金融科技都瘋狂的區塊鏈技術



讚

2.7 萬

按讚加入iThome粉絲團



讚

分享

1,443

G+1

4

文/ 王宏仁 | 2016-05-02 發表

DR. CRAIG WRIGHT

*We wanted to create a forum about Bitcoin to dispel the myths out there and unleash its potential to change the world for the better.*



圖片來源: [www.drcaigwright.net/](http://www.drcaigwright.net/)

<http://www.ithome.com.tw/news/105677>

# 比特幣發明者是誰？Wright是中本聰還是騙子？

儘管部份人士相信澳洲企業家Craig Steven Wright就是比特幣發明者，但仍有資安專家、開發者質疑Wright是中本聰的真實性，認為Wright所提出的證據薄弱，要求提出的更有力的證據，例如展示第0區塊的相關私鑰才能證明他真的是中本聰。



讚

2.7 萬

按讚加入iThome粉絲團



讚

分享

55

G+1

3

文/ 陳曉莉 | 2016-05-03 發表

<http://www.ithome.com.tw/news/105687>

## 承認是中本聰後質疑聲四起，Wright不想再證明了

Wright向媒體承認自己是中本聰後謠言四起，Wright說，他的能力與性格都受到攻擊，當這些指控被駁回時，新的指控又出現了，他知道他承受不起...向相信他的人道歉。



讚

2.7 萬

按讚加入iThome粉絲團



讚

分享

112

G+1

0

文/ 陳曉莉 | 2016-05-06 發表

<http://www.ithome.com.tw/news/105769>



# Bitcoin Pizza Day: Celebrating the Pizzas Bought for 10,000 BTC

May 22, 2014 at 19:16 by Grace Caffyn

Today, bitcoiners the world over will celebrate the anniversary of the most expensive pizzas in history.

Bought on 22nd May 2010 by Laszlo Hanyecz, the programmer paid a fellow Bitcoin Talk forum user 10,000 BTC for two Papa John's pizzas. Back then – when the technology was just over a year old – that equated to roughly \$25, but is [\\$5.12m](#) by today's exchange rate.

<http://www.coindesk.com/bitcoin-pizza-day-celebrating-pizza-bought-10000-btc>

# 貨幣發行與交易確認

- 比特幣透過挖礦(mining)發行貨幣。每一區塊挖礦成功礦工得一筆酬金(reward)至2140共發行 21,000,000 BTCs
  - 2009.1.3 ~ 2012.11.28 (Block #0 ~ #209999): 50 bitcoins per block
  - 2012.11.28 ~ 2016.7.9 (#210000 ~ #419999): 25 bitcoins per block
  - ..... Done in 2140: All 21,000,000 bitcoins are issued
- 礦工確認新的交易(transactions)並把它們登錄在區塊鏈新產生的區塊內。
- 每筆交易要付手續費(transaction fee)給被確認的礦工。

# 現在已發行多少比特幣？

- 全部比特幣共 2100萬 BTCs
- 前21萬blocks共發行 1050萬 BTCs 佔 1/2
- 前42萬blocks共發行 3/4
- 前63萬blocks共發行 7/8
- 前51萬blocks(約現在)發行

$$3/4 + (9/21)*(1/8)=0.804$$

約八成

# 多少種密碼貨幣？

- 密碼貨幣(Cryptocurrency)

使用密碼學原理來確保交易安全及控制交易單位創造的交易媒介。密碼貨幣是數位貨幣(或稱虛擬貨幣)的一種。比特幣在2009成為第一個去中心化的密碼貨幣，這之後密碼貨幣一詞多指此類設計。自此之後數種類似的密碼貨幣被創造，它們通常被稱作altcoins。2018/5統計共有altcoins一千八百種，Ethereum，Ripple，Litecoin ...



在全家便利商店就可以將現金換成比特幣

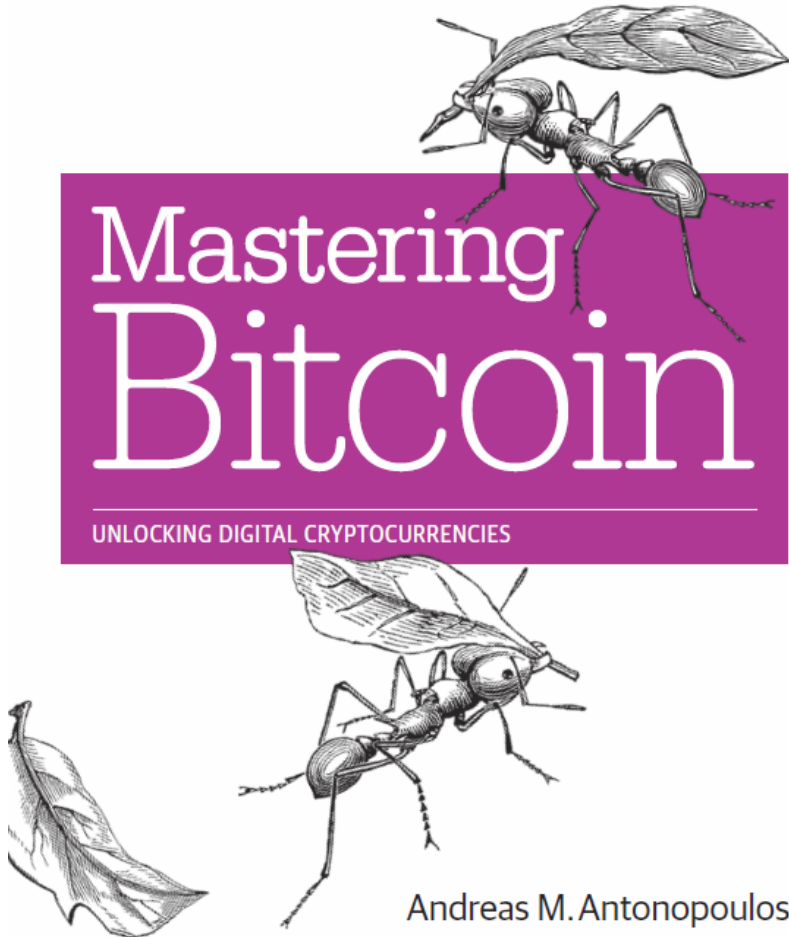


台灣第一家可以用加密貨幣買蛋糕的店 Coin Cake 開幕。

<https://www.inside.com.tw/2017/10/18/coin-cake>

# The Book “Mastering Bitcoin”

O'REILLY®



## Mastering Bitcoin

UNLOCKING DIGITAL CRYPTOCURRENCIES

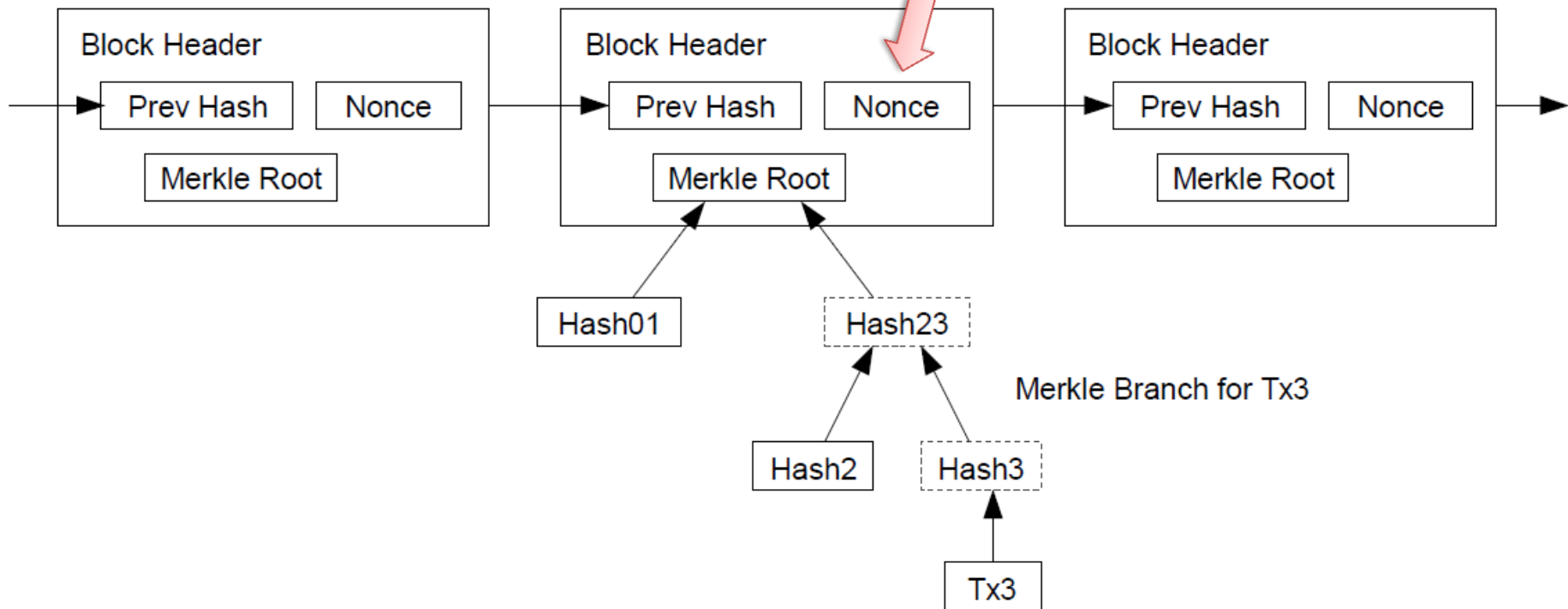
Andreas M. Antonopoulos

**MINING 挖礦**

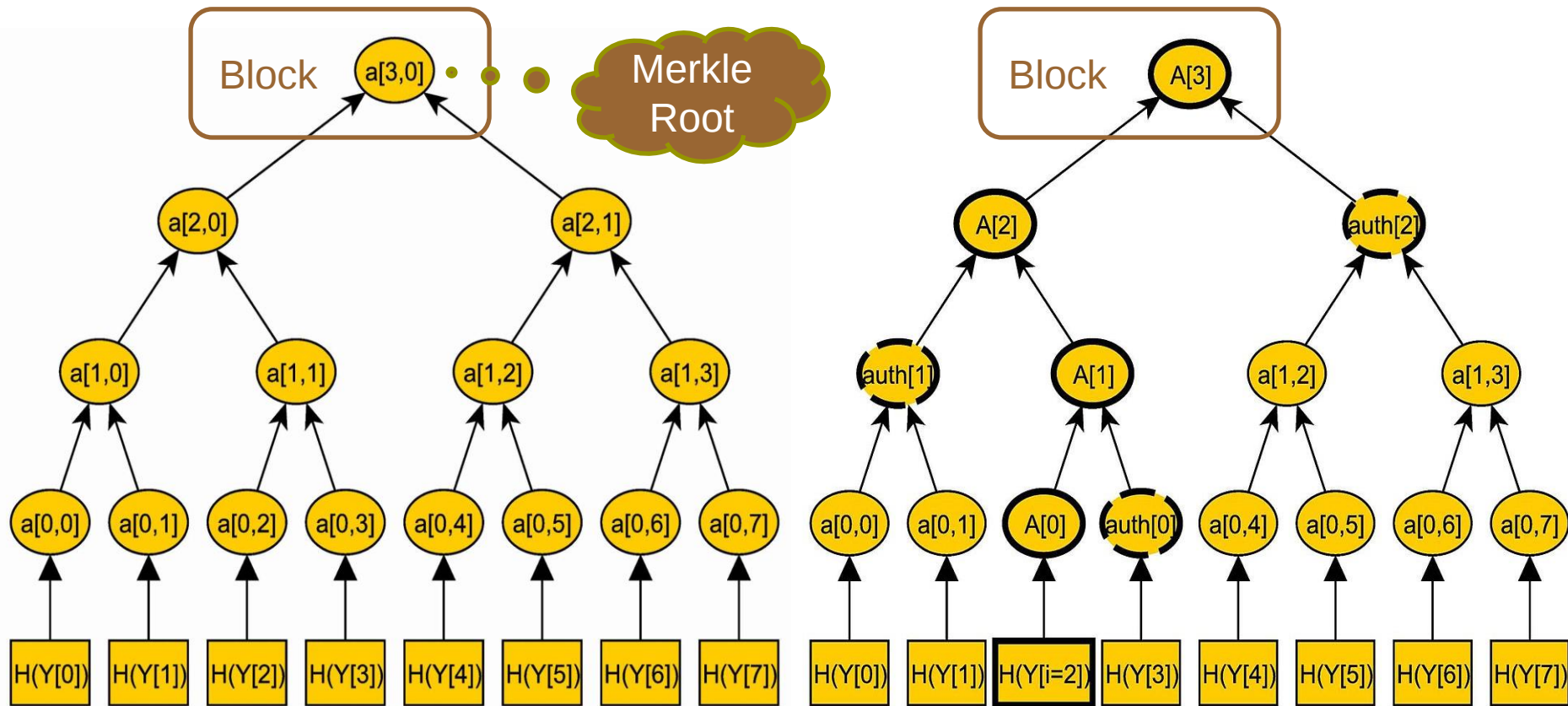
# Block Chain

Mining 挖礦

Longest Proof-of-Work Chain



# Merkle Tree / Hash Tree



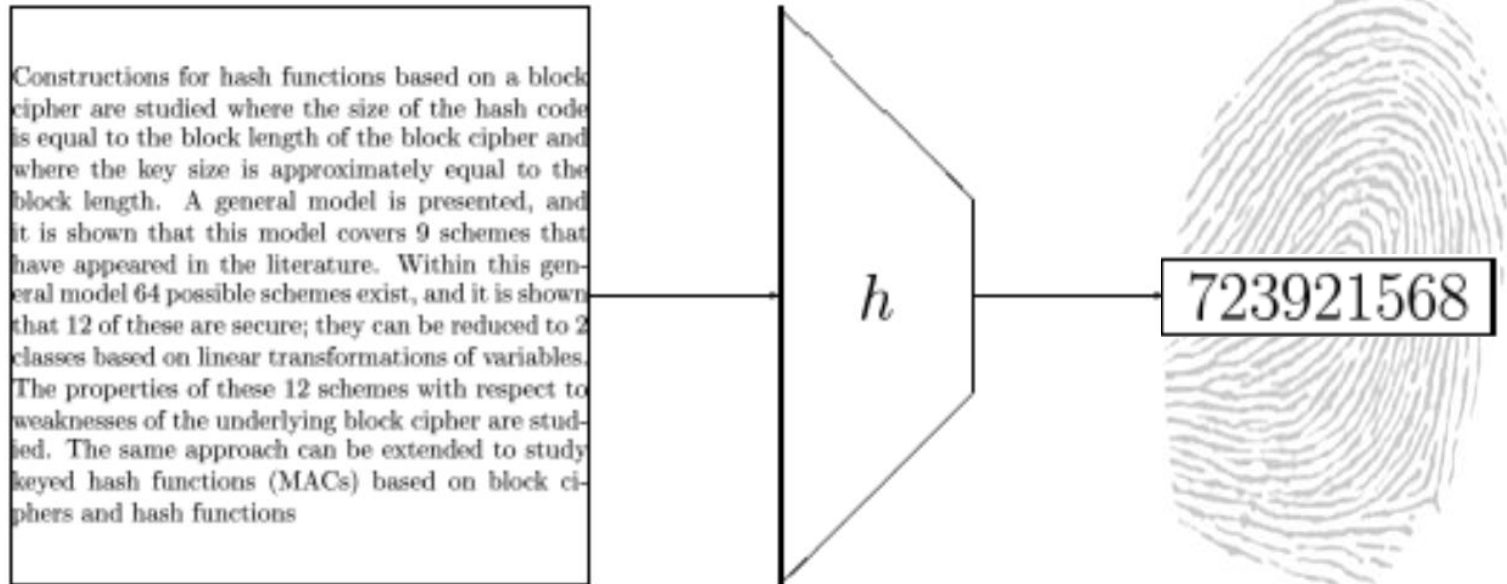
# Proof-of-Work 工作量證明

- “The proof-of-work involves scanning for a value that when hashed, such as with SHA-256, the hash begins with a number of zero bits.”

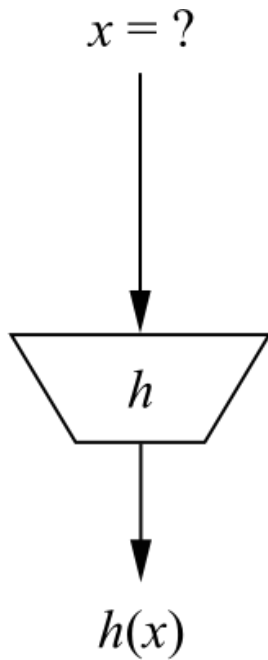
# **HASH 雜湊函數**

# Hash Function 雜湊函數

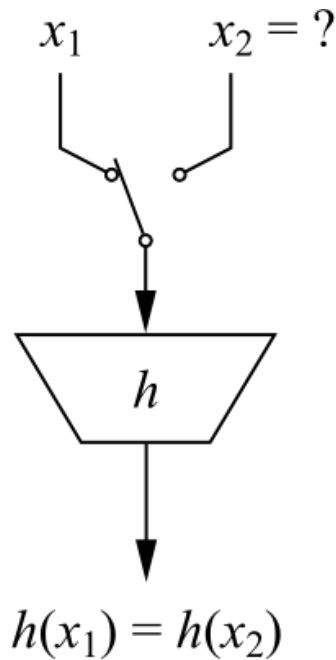
- An efficient function mapping binary strings of **arbitrary length** to binary strings of **fixed length**, called the **hash-value** or **hash-code** (fingerprint, checksum)



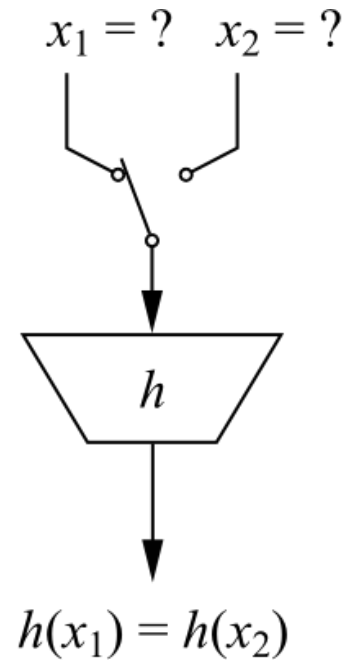
# Security Properties



preimage resistance



second preimage  
resistance



collision resistance

# SHA-3 Competition Winner:

## Keccak

- Designers:
  - Guido Bertoni (Italy) of STMicroelectronics
  - Joan Daemen (Belgium) of STMicroelectronics
  - Gilles Van Assche (Belgium) of STMicroelectronics
  - Michaël Peeters (Belgium) of NXP Semiconductors
- Not very fast in software implementation, but in hardware implementations it is notably faster than all other finalists
- In its largest instance, the state consists of a  $5 \times 5$  array of 64-bit words, 1600 bits total
  - Reduced versions are defined for smaller power-of-2 word sizes  $w$  down to 1 bit (25 bits total state)
  - Smaller state sizes can be used to test cryptanalytic attacks
  - Intermediate state sizes (e.g., from  $w = 4$ , 100 bits, to  $w = 32$ , 800 bits) also provide practical, lightweight, alternatives

# Applications

- Verifying the Integrity of Files or Messages
- Password Verification
- File or Data Identifier
- Pseudorandom Generation & Key Derivation
- Proof-of-Work (POW)

# Hash Function Usages

- Double SHA256, i.e., `SHA256(SHA256())`
  - Merkle Tree
  - Block Hash
  - Transaction ID
- `RIPEMD160(SHA256())`
  - Bitcoin Address

# Bitcoin Address

- Address = RIPEMD160(SHA256(public key representation))
- Example
  - ECDSA private key = 18E14A7B6A307F426A94F8114701E7C8E774E7F9A47E2C2035DB29A206321725
  - Public key  $P$  = 04 50863AD64A87AE8A2FE83C1AF1A8403CB53F53E486D8511DAD8A04887E5B235  
22CD470243453A299FA9E77237716103ABC11A1DF38855ED6F2EE187E9C582BA6
  - SHA256( $P$ ) = 600FFE422B4E00731A59557A5CCA46CC183944191006324A447BDB2D98D4B408
  - RIPEMD160(SHA256( $P$ )) = 010966776006953D5567439E5E39F86A0D273BEE
  - Address (Base58Check encoded): 16UwLL9Risc3QfPqBUvKofHmBQ7wMtjvM
  - [https://en.bitcoin.it/wiki/Technical\\_background\\_of\\_version\\_1\\_Bitcoin\\_addresses#How\\_to\\_create\\_Bitcoin\\_Address](https://en.bitcoin.it/wiki/Technical_background_of_version_1_Bitcoin_addresses#How_to_create_Bitcoin_Address)
- Base58 is a set of lower and capital letters and numbers without (0, O, l, I), i.e., 0 (number zero), O (capital o), l (lower L), I (capital i)

# BLOCKCHAIN 區塊鏈

# 比特幣交易登錄於區塊鏈的程序

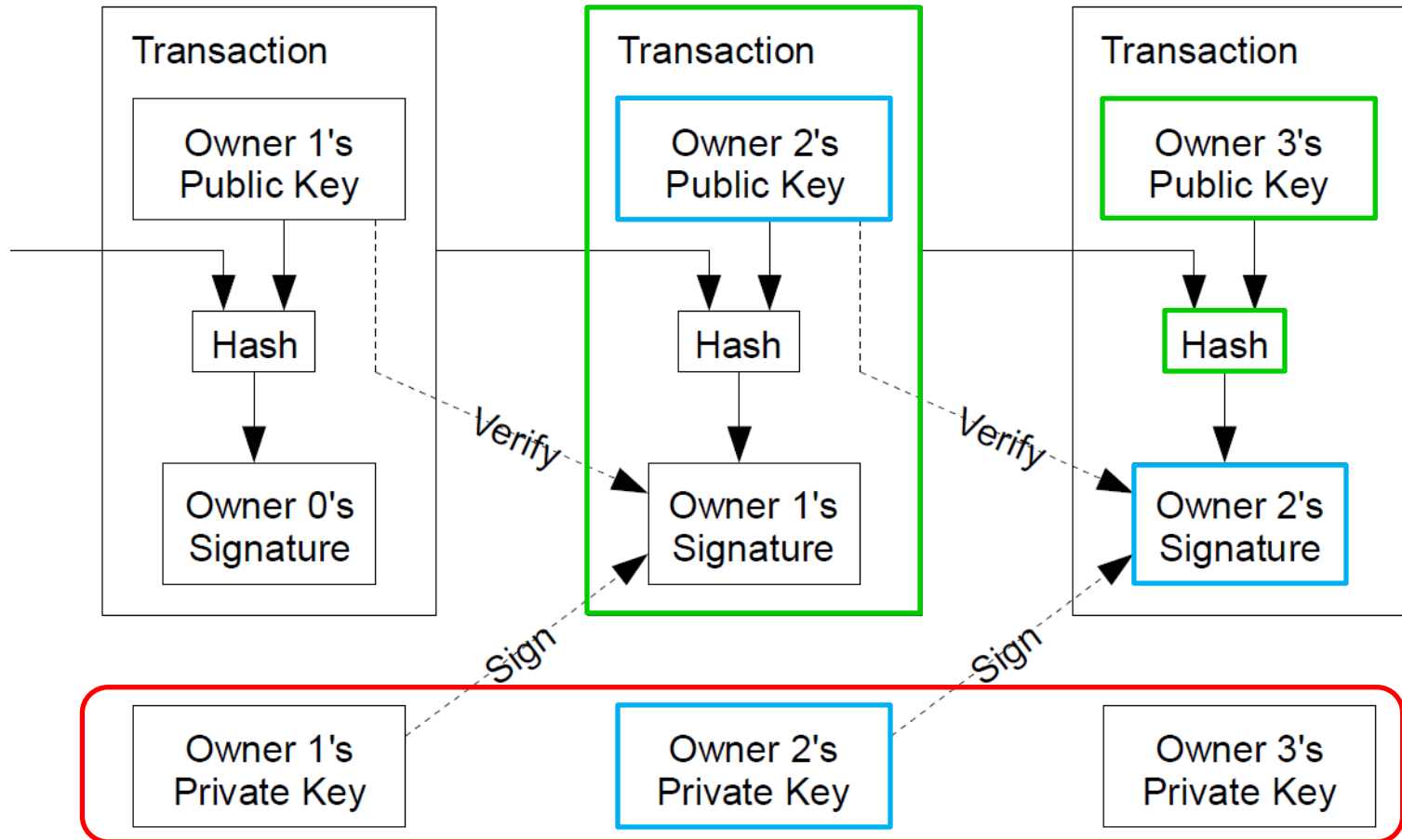
1. 當有一筆新的交易產生, 會通知所有的節點
2. 每個挖礦節點會將新的交易紀錄存到一個區塊中
3. 每個挖礦節點會執行工作量證明
4. 當節點執行完工作量證明後, 會將區塊資訊告知其他節點
5. 若區塊裡的所有交易皆為合法且尚未失效, 節點將接受此區塊
6. 若區塊被驗證為完成工作量證明, 則此區塊內容的 hash 將被當作下一個區塊的 previous hash

# 區塊鏈 Block Chain

- 區塊鏈技術是 Bitcoin 的基礎，受全世界重視的程度已經超越 Bitcoin 本身
- 一言以蔽之：「以 hash function 串接資料」
  - Hash function – 雜湊函數、赫序函數、哈希函數
- 探討區塊鏈的文獻極多，大部分著墨於應用，鮮少講清楚最根本的 hash function
  - Hash function 是基礎密碼演算法，實務應用非常普遍

# TRANSACTIONS 交易

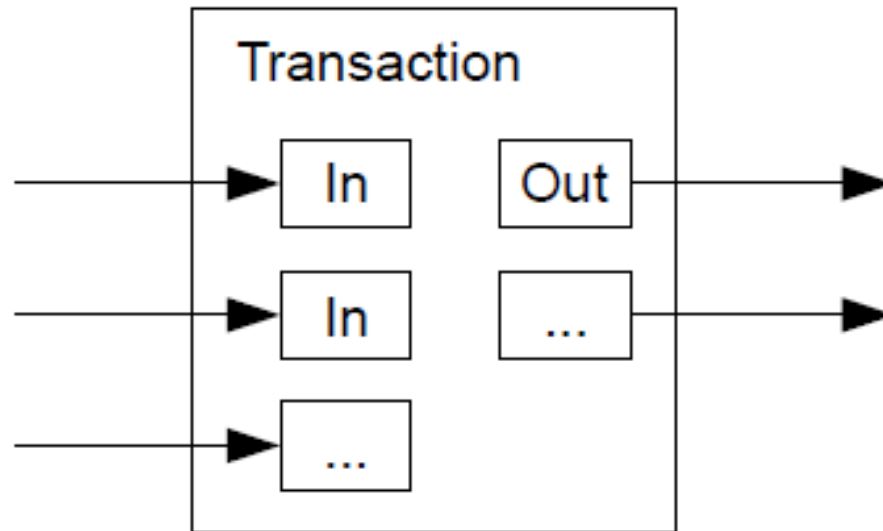
# Bitcoin Transactions 交易



Must be protected very well!!!

# Combining & Splitting Value

- “To allow value to be split and combined, transactions contain multiple inputs and outputs.”





### Transaction 7957a35fe64f80d234d76d83a2a8f1a0d8149a41d81de548f0a65a8a999f6f18

INPUTS From  
From (previous transactions Joe has received):  
Joe 0.1005 BTC

OUTPUTS To  
Output #0 Alice's Address 0.1000 BTC (spent)  
Transaction Fees: 0.0005 BTC

### Transaction 0627052b6f28912f2703066a912ea577f2ce4da4caa5a5fbd8a57286c345c2f2

INPUTS From  
7957a35fe64f80d234d76d83a2a8f1a0d8149a41d81de548f0a65a8a999f6f18 : 0  
Alice 0.1000 BTC

OUTPUTS To  
Output #0 Bob's Address 0.0150 BTC (spent)  
Output #1 Alice's Address (change) 0.0845 BTC (unspent)  
Transaction Fees: 0.0005 BTC

### Transaction 2bbac8bb3a57a2363407ac8c16a67015ed2e88a4388af58cf90299e0744d3de4

INPUTS From  
0627052b6f28912f2703066a912ea577f2ce4da4caa5a5fbd8a57286c345c2f2 : 0  
Bob 0.0150 BTC

OUTPUTS To  
Output #0 Gopesh's Address 0.0100 BTC (unspent)  
Output #1 Bob's Address (change) 0.0845 BTC (unspent)  
Transaction Fees: 0.0005 BTC

*Figure 2-4. A chain of transactions, where the output of one transaction is the input of the next transaction*

# Transaction View information about a bitcoin transaction

0627052b6f28912f2703066a912ea577f2ce4da4caa5a5fbd8a57286c345c2f2

1Cdid9KFAaatwczBwBttQcwXYCpvK8h7FK (0.1 BTC - Output)



1GdK9UzpHBzqzX2A9JFP3Di4weBwqgmoQA

- (Unspent) 0.015 BTC

1Cdid9KFAaatwczBwBttQcwXYCpvK8h7FK -

(Unspent) 0.0845 BTC

97 Confirmations

0.0995 BTC

## Summary

Size 258 (bytes)

Received Time 2013-12-27 23:03:05

Included In 277316 (2013-12-27 23:11:54 +9  
Blocks minutes)

## Inputs and Outputs

Total Input 0.1 BTC

Total Output 0.0995 BTC

Fees 0.0005 BTC

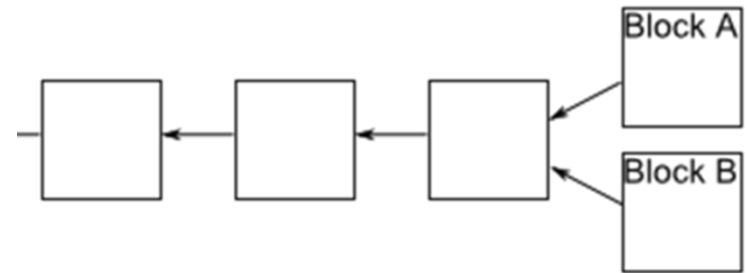
Estimated BTC Transacted 0.015 BTC

Figure 2-8. Alice's transaction to Bob's Cafe

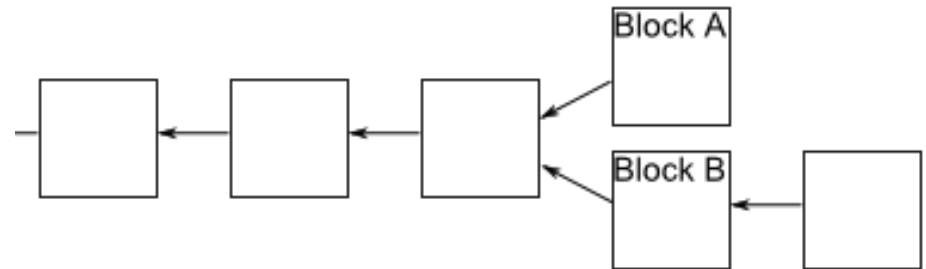
**CONSENSUS 共識**

# Block Forking 區塊分岔

- Occasionally, a fork appears in the block chain, i.e., two miners happen to validate a block of transactions near-simultaneously

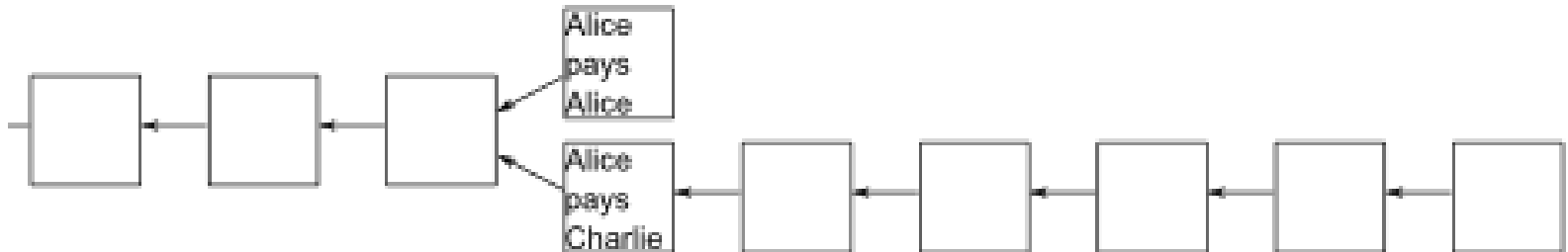


- If a fork occurs, people on the network keep track of both forks
- Miners only work to extend whichever fork is longest in their copy of the block chain



# Confirmations

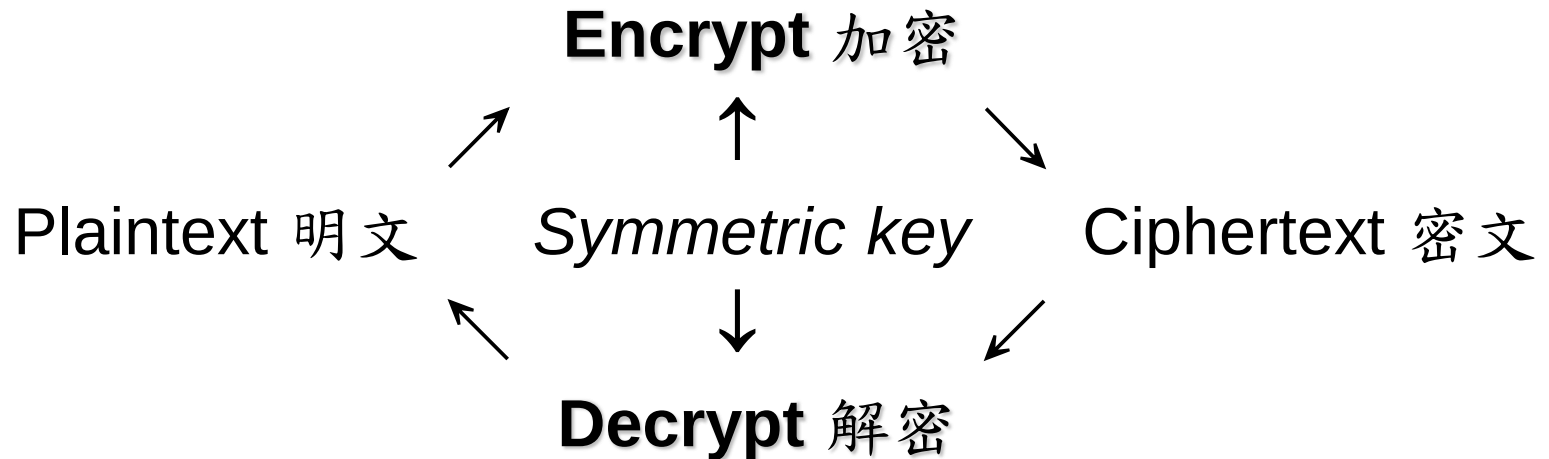
- A transaction is not considered confirmed until
  - It is part of a block in the longest fork
  - At least 5 blocks follow it in the longest fork
  - In this case, we say that the transaction has “6 confirmations”
- 10 minutes per block (in average)
- Payee must wait 60 minutes



# **PUBLIC KEY CRYPTOGRAPHY**

## **(PKC)公鑰密碼**

# Symmetric Cryptography



DES (Data Encryption Standard)

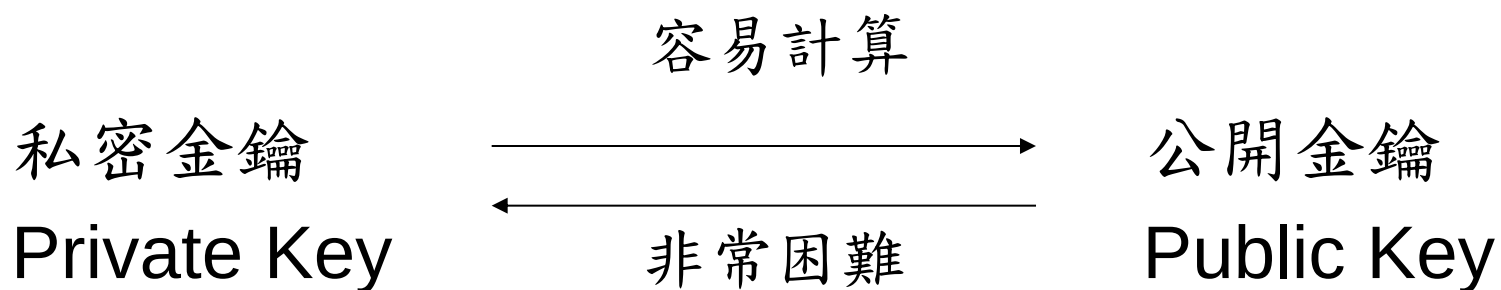
AES (Advanced Encryption Standard)

# Asymmetric Cryptography

- New Idea: Use the “mailbox” principle
  - Everyone can drop a letter
  - But only the owner has the correct key to open the box
- 1976: first publication of such an algorithm by Whitfield Diffie and Martin Hellman



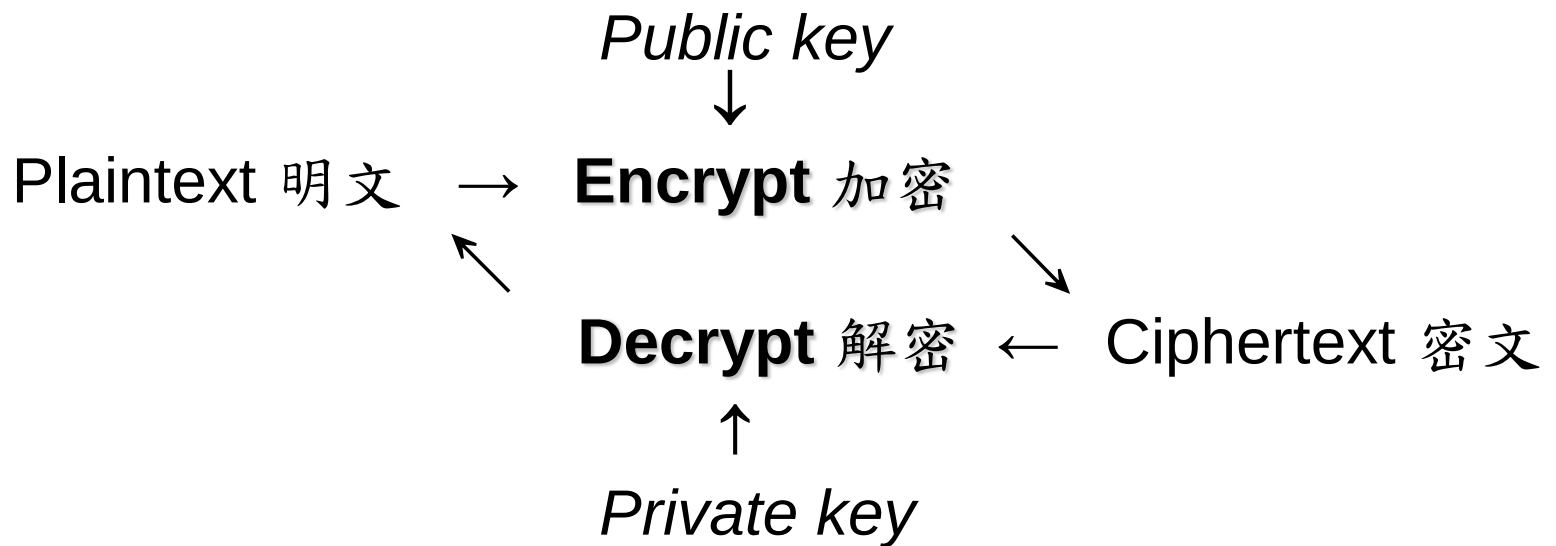
# 私密金鑰 與 公開金鑰



藉由數學工具達成此目的

Whit Diffie 和 Martin Hellman 於 1976 年提出此觀念

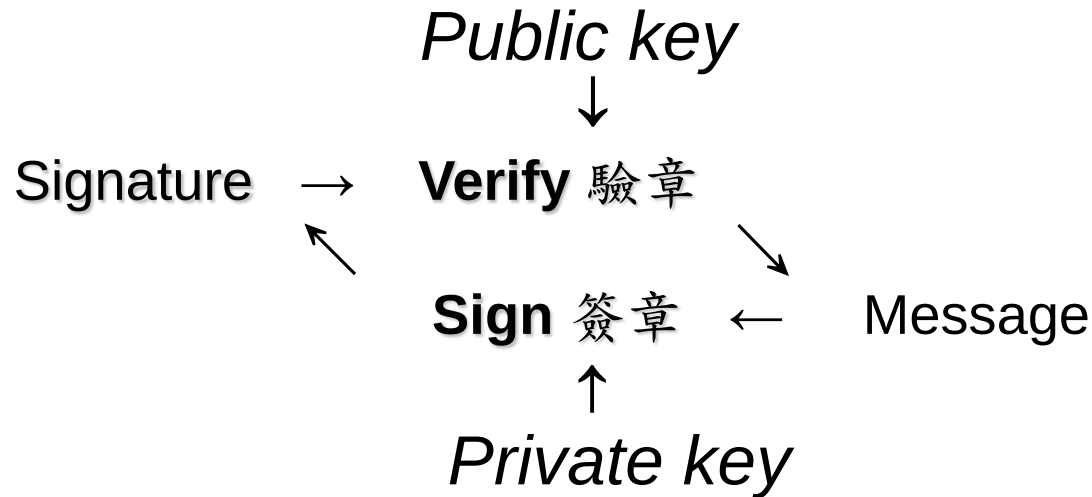
# Public Key Cryptosystem 公鑰密碼系統



RSA (Rivest – Shamir – Adleman 1977)

ECC (Elliptic Curve Cryptosystem 橢圓曲線密碼系統)

# Digital Signature 數位簽章

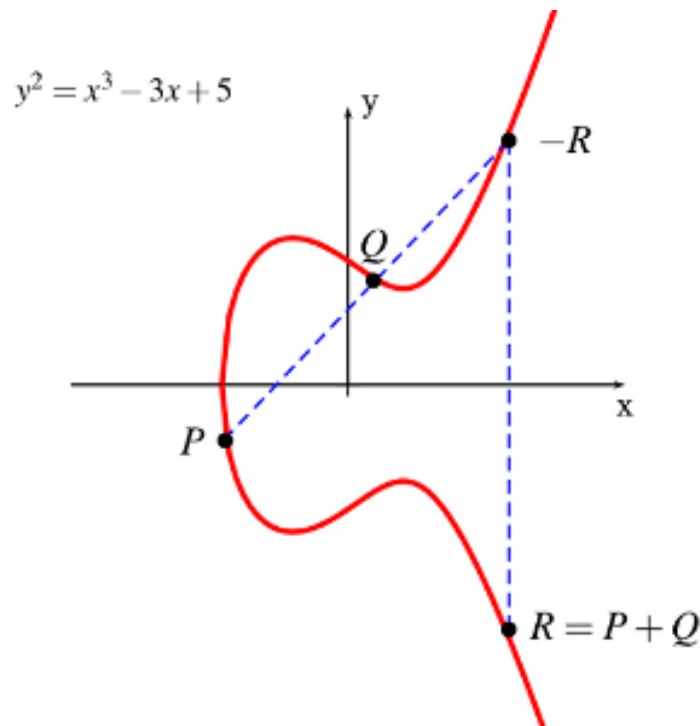


- \* 資料完整性 (Integrity)
- \* 身份鑑別性 (Authentication)
- \* 不可否認性 (Non-Repudiation)

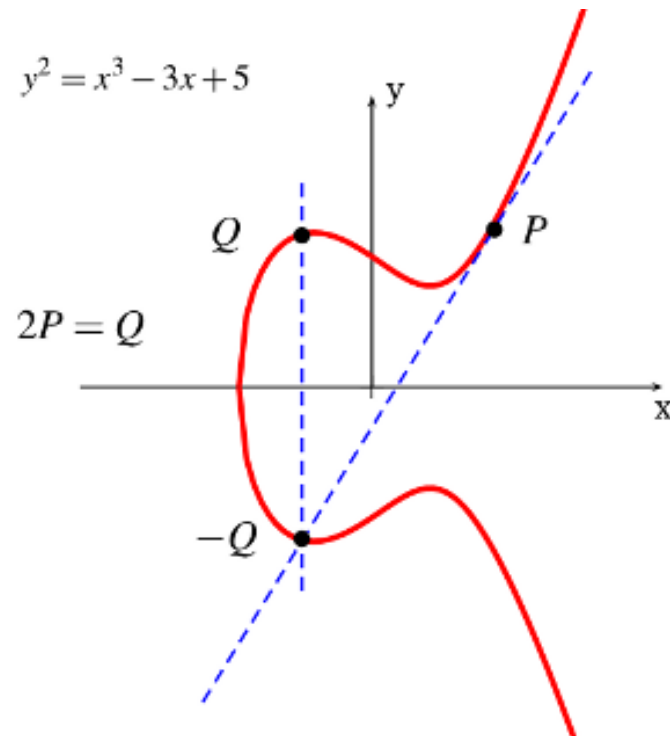
# Elliptic Curve

# Elliptic Curve 橢圓曲線

- The rich and deep theory of Elliptic Curves has been studied by mathematicians over 150 years
- Elliptic Curve over  $\mathbf{R} : y^2 = x^3 + ax + b$



Point Addition



Point Doubling

# 質數體 (Prime Field) 上的曲線

Addition:

$$(x_3, y_3) = (x_1, y_1) + (x_2, y_2)$$

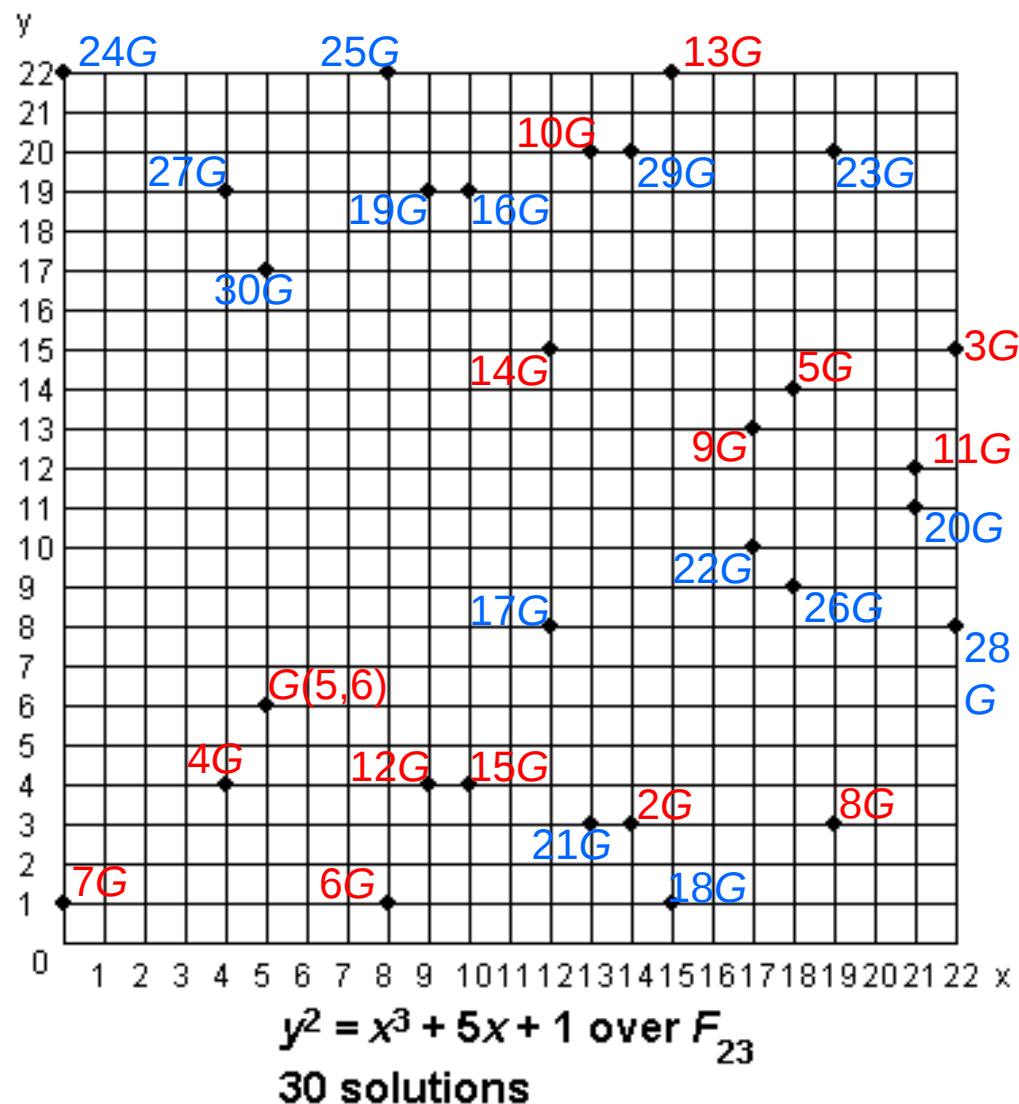
Doubling:

$$(x_3, y_3) = [2] (x_1, y_1)$$

$$s = \begin{cases} \frac{y_2 - y_1}{x_2 - x_1} \bmod p & \text{(addition)} \\ \frac{3x_1^2 + a}{2y_1} \bmod p & \text{(doubling)} \end{cases}$$

$$x_3 = s^2 - x_1 - x_2 \bmod p$$

$$y_3 = s(x_1 - x_3) - y_1 \bmod p$$



# Double and Add

**Example:**  $26P = (11010_2)P = (d_4d_3d_2d_1d_0)_2 P$ .

Step

#0  $P = 1_2P$

initial setting

#1a  $P + P = 2P = 10_2P$

DOUBLE (bit  $d_3$ )

#1b  $2P + P = 3P = 10^2P + 1_2P = 11_2P$

ADD (bit  $d_3 = 1$ )

#2a  $3P + 3P = 6P = 2(11_2P) = 110_2P$

DOUBLE (bit  $d_2$ )

#2b

no ADD ( $d_2 = 0$ )

#3a  $6P + 6P = 12P = 2(110_2P) = 1100_2P$

DOUBLE (bit  $d_1$ )

#3b  $12P + P = 13P = 1100_2P + 1_2P = 1101_2P$

ADD (bit  $d_1 = 1$ )

#4a  $13P + 13P = 26P = 2(1101_2P) = 11010_2P$

DOUBLE (bit  $d_0$ )

#4b

no ADD ( $d_0 = 0$ )

# Bitcoin 和 Ethereum 使用的曲線

The elliptic curve domain parameters over  $\mathbb{F}_p$  associated with a Koblitz curve secp256k1 are specified by the sextuple  $T = (p, a, b, G, n, h)$  where the finite field  $\mathbb{F}_p$  is defined by:

$$\begin{aligned} p &= \text{FFFFFFFF FFFFFFFF FFFFFFFF FFFFFFFF FFFFFFFF FFFFFFFF FFFFFFFE} \\ &\quad \text{FFFFFFFFC2F} \\ &= 2^{256} - 2^{32} - 2^9 - 2^8 - 2^7 - 2^6 - 2^4 - 1 \end{aligned}$$



256-bit prime

The curve  $E: y^2 = x^3 + ax + b$  over  $\mathbb{F}_p$  is defined by:

$$\begin{aligned} a &= \text{00000000 00000000 00000000 00000000 00000000 00000000 00000000} \\ &\quad \text{00000000} \\ b &= \text{00000000 00000000 00000000 00000000 00000000 00000000 00000000} \\ &\quad \text{00000007} \end{aligned}$$

橢圓曲線 secp256k1

<https://en.bitcoin.it/wiki/Secp256k1>

The base point  $G$  in compressed form is:

$$\begin{aligned} G &= \text{02 79BE667E F9DCBBAC 55A06295 CE870B07 029BFCDB 2DCE28D9} \\ &\quad \text{59F2815B 16F81798} \end{aligned}$$

and in uncompressed form is:

$$\begin{aligned} G &= \text{04 79BE667E F9DCBBAC 55A06295 CE870B07 029BFCDB 2DCE28D9} \\ &\quad \text{59F2815B 16F81798 483ADA77 26A3C465 5DA4FBFC 0E1108A8 FD17B448} \\ &\quad \text{A6855419 9C47D08F FB10D4B8} \end{aligned}$$

Finally the order  $n$  of  $G$  and the cofactor are:

$$\begin{aligned} n &= \text{FFFFFFFF FFFFFFFF FFFFFFFF FFFFFFFF BAAEDCE6 AF48A03B BFD25E8C} \\ &\quad \text{D0364141} \\ h &= \text{01} \end{aligned}$$



256-bit prime

# Key Pairs 金鑰對

- The base point  $G$  is fixed on the given Elliptic Curve
- $P = [m] G$ 
  - Given  $m$ , it is **easy and fast** to find the point  $P$ 
    - Using “double and add” for scalar multiplication
  - Given  $P$ , it is **extremely hard** to find the integer  $m$ 
    - Elliptic Curve Discrete Logarithm Problem (橢圓曲線離散對數問題)
  - A randomly generated integer  $m$  is a **private key**
    - A private key is used to sign Bitcoin transactions with ECDSA
  - The point  $P$  is the **public key** corresponding to  $m$ 
    - A public key is used by other nodes to verify Bitcoin transactions
    - A **Bitcoin address** is the hash value of a public key  $P$