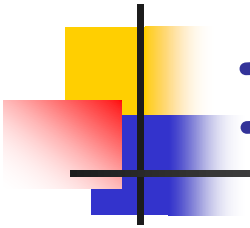
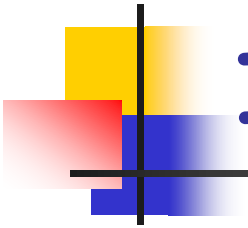
The Advanced Encryption Standard: Rijndael



Introduction

- In 1997, NIST put out a call for candidates to replace DES. Among the requirements were that the new algorithm should allow key size of 128, 192, and 256 bits, it should operate on blocks of 128 input bits, and it should work on different hardware, e.g., 8-bit (smard cards) and the 32-bit (PCs).

- In 1998, the cryptographic community was asked to comment on 15 candidate algorithms.



Introduction

- In August 1999, five finalists were chosen:

  MARS (from IBM)

  RC6 (from RSA Laboratories)

  Rijndael (from Joan Daemen and Vincent Rijmen)

  Serpent (from Anderson, Biham, and Knudsen)

  Twofish (from Schneier, Kelsey, Whiting, Wagner, Hall, and Ferguson)

- On Oct. 2, 2000, **Rijndael** was selected to be AES.
  - 3 main criteria: security, cost, algorithm and implementation characteristics

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- **Description of AES**
  - **Block length:**
    - 128 bits (Nb=4)
  - **Key length:**
    - 128 bits (Nk=4)
    - 192 bits (Nk=6)
    - 256 bits (Nk=8)
  - **Number of rounds N**

$S_{0,0}$	$S_{0,1}$	$S_{0,2}$	$S_{0,3}$
$S_{1,0}$	$S_{1,1}$	$S_{1,2}$	$S_{1,3}$
$S_{2,0}$	$S_{2,1}$	$S_{2,2}$	$S_{2,3}$
$S_{3,0}$	$S_{3,1}$	$S_{3,2}$	$S_{3,3}$

$k_{0,0}$	$k_{0,1}$	$k_{0,2}$	$k_{0,3}$	$k_{0,4}$	$k_{0,5}$	$k_{0,6}$	$k_{0,7}$
$k_{1,0}$	$k_{1,1}$	$k_{1,2}$	$k_{1,3}$	$k_{1,4}$	$k_{1,5}$	$k_{1,6}$	$k_{1,7}$
$k_{2,0}$	$k_{2,1}$	$k_{2,2}$	$k_{2,3}$	$k_{2,4}$	$k_{2,5}$	$k_{2,6}$	$k_{2,7}$
$k_{3,0}$	$k_{3,1}$	$k_{3,2}$	$k_{3,3}$	$k_{3,4}$	$k_{3,5}$	$k_{3,6}$	$k_{3,7}$

Nr	Nb = 4
Nk = 4	10
Nk = 6	12
Nk = 8	14

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- **Overview of AES:**
  - **AddRoundKey**, which xors the **RoundKey** with **State**.
  - For each of the first Nr-1 rounds: perform **SubBytes**(**State**), **ShiftRows**(**State**), **MixColumn**(**State**), **AddRoundKey**.
  - Final round: **SubBytes**, **ShiftRows**, **AddRoundKey**.
- All operations in **AES** are byte-oriented.
  - The plaintext x consists of 16 byte, $x_0, x_1, \ldots, x_{15}$.
  - Initially **State** is plaintext x (for 128-bit case):

$S_{0,0}$	$S_{0,1}$	$S_{0,2}$	$S_{0,3}$
$S_{1,0}$	$S_{1,1}$	$S_{1,2}$	$S_{1,3}$
$S_{2,0}$	$S_{2,1}$	$S_{2,2}$	$S_{2,3}$
$S_{3,0}$	$S_{3,1}$	$S_{3,2}$	$S_{3,3}$

x_0	x_4	x_8	x_{12}
x_1	x_5	x_9	x_{13}
x_2	x_6	x_{10}	x_{14}
x_3	x_7	x_{11}	x_{15}

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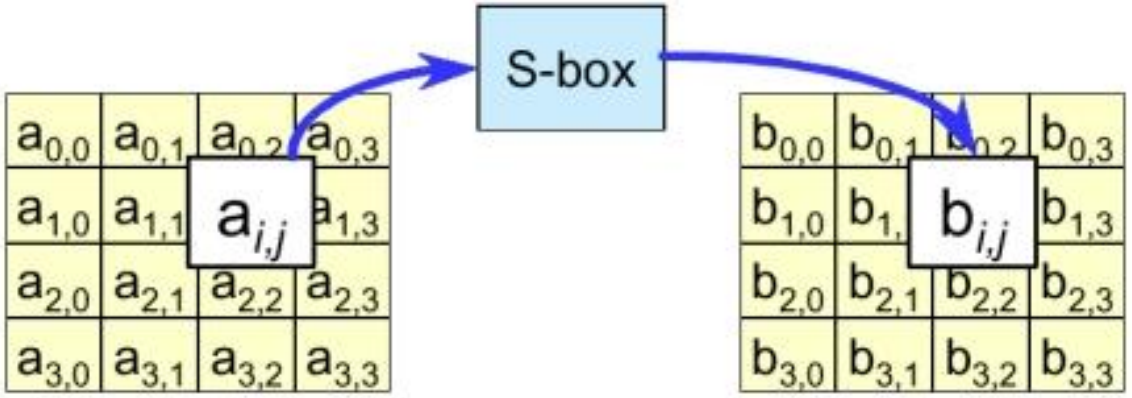
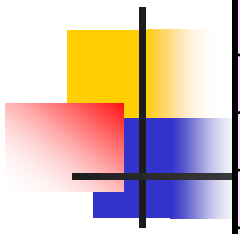
- **SubBytes:**
  - It performs a substitution on each byte of **State** using an S-box, say π_S.
  - π_S is a 16x16 array (**Figure A**). A byte is represented as two hexadecimal digits XY. So XY after substitution is $\pi_S(XY)$.





Example
C

Y X	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
0	63	7C	77	7B	F2	6B	6F	C5	30	01	67	2B	FE	D7	AB	76
1	CA	82	C9	7D	FA	59	47	F0	AD	D4	A2	AF	9C	A4	72	C0
2	B7	FD	93	26	36	3F	F7	CC	34	A5	E5	F1	71	D8	31	15
3	04	C7	23	C3	18	96	05	9A	07	12	80	E2	EB	27	B2	75
4	09	83	2C	1A	1B	6E	5A	A0	52	3B	D6	B3	29	E3	2F	84
5	53	D1	00	ED	20	FC	B1	5B	6A	CB	BE	39	4A	4C	58	CF
6	D0	EF	AA	FB	43	4D	33	85	45	F9	02	7F	50	3C	9F	A8
7	51	A3	40	8F	92	9D	38	F5	BC	B6	DA	21	10	FF	F3	D2
8	CD	0C	13	EC	5F	97	44	17	C4	A7	7E	3D	64	5D	19	73
9	60	81	4F	DC	22	2A	90	88	46	EE	B8	14	DE	5E	0B	DB
A	E0	32	3A	0A	49	06	24	5C	C2	D3	AC	62	91	95	E4	79
B	E7	C8	37	6D	8D	D5	4E	A9	6C	56	F4	EA	65	7A	AE	08
C	BA	78	25	2E	1C	A6	B4	C6	E8	DD	74	1F	4B	BD	8B	8A
D	70	3E	B5	66	48	03	F6	0E	61	35	57	B9	86	C1	1D	9E
E	E1	F8	98	11	69	D9	8E	94	9B	1E	87	E9	CE	55	28	DF
F	8C	A1	89	0D	BF	E6	42	68	41	99	2D	0F	B0	54	BB	16

Figure A
The AES S-box

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- The **AES** S-box can be defined algebraically. The permutation π_S incorporates operations in the finite field

$$\mathbf{F}_{2^8} = \mathbf{Z}_2[x]/(x^8 + x^4 + x^3 + x + 1).$$

- **FIELDINV**: the multiplicative inverse of a filed element
- **BINARYTOFIELD**: convert a byte to a field element
- **FIELDTOBINARY**: inverse operation
  - $\sum_{i=0}^{7} a_i x^i$

    corresponds to the byte

    $a_7 a_6 a_5 a_4 a_3 a_2 a_1 a_0$

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- **Algorithm B: SubBytes($a_7a_6a_5a_4a_3a_2a_1a_0$)**

external FieldInv, BinaryToField, FieldToBinary

$z \leftarrow$ BinaryToFiled($a_7a_6a_5a_4a_3a_2a_1a_0$)

if $z \neq 0$

then $z \leftarrow$ FieldInv(z)

$(a_7a_6a_5a_4a_3a_2a_1a_0) \leftarrow$ FieldToBinary(z)

$(c_7c_6c_5c_4c_3c_2c_1c_0) \leftarrow (01100011)$

comment: In the following loop, all subscripts are to be reduced modulo 8

for $i \leftarrow 0$ **to** 7

do $b_i \leftarrow (a_i + a_{i+4} + a_{i+5} + a_{i+6} + a_{i+7} + c_i) \bmod 2$

return ($b_7b_6b_5b_4b_3b_2b_1b_0$)

Operations in Algorithm B

- Take a byte $(a_7a_6a_5a_4a_3a_2a_1a_0)$ as $\sum_{i=0}^{7} a_i x^i$ in $\mathbf{F}_{2^8}$
- for $\sum_{i=0}^{7} a_i x^i \neq 0$, Find its multiplicative inverse $\sum_{i=0}^{7} d_i x^i$
- $(d_7d_6d_5d_4d_3d_2d_1d_0) \longrightarrow (a_7a_6a_5a_4a_3a_2a_1a_0)$
- $$\begin{bmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \\ b_6 \\ b_7 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \\ a_7 \end{bmatrix} + \begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \end{bmatrix}$$

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- **Example C: (illustrates Algorithm B)**
  - Suppose we begin with (hex) 53. In binary, it's 01010011, which represents the field element

$$x^6 + x^4 + x + 1$$

The multiplicative inverse (in $\mathbf{F}_{2^8}$) can be shown to be

$$x^7 + x^6 + x^3 + x$$

Thus we have

$$(a_7 a_6 a_5 a_4 a_3 a_2 a_1 a_0) = (11001010).$$

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$$
\begin{aligned}
b_0 &= a_0 + a_4 + a_5 + a_6 + a_7 + c_0 \bmod 2 \\
&= 0 + 0 + 0 + 1 + 1 + 1 \bmod 2 \\
&= 1 \\
b_1 &= a_1 + a_5 + a_6 + a_7 + a_0 + c_1 \bmod 2 \\
&= 1 + 0 + 1 + 1 + 0 + 1 \bmod 2 \\
&= 0,
\end{aligned}
$$

etc. The result is

$$(b_7 b_6 b_5 b_4 b_3 b_2 b_1 b_0) = (11101101).$$

which is **ED** in hex.

- This computation can be checked by verifying the entry in row 5 and column 3 of **Figure A**.

□

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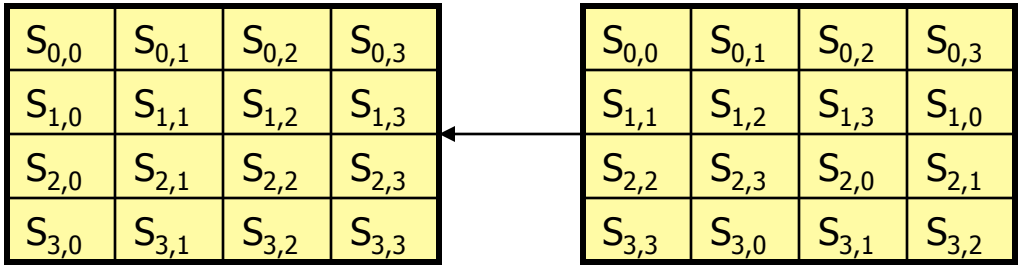
- ShiftRows:



  - Row 0: no shift
  - Row 1: shift 1
  - Row 2: shift 2
  - Row 3: shift 3

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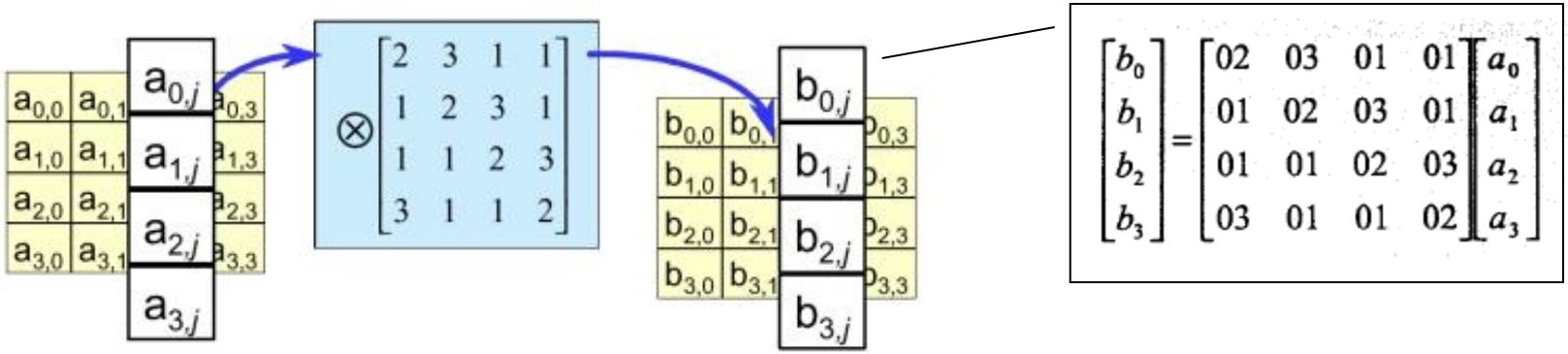
- **MixColumns: (Algorithm D)**
  - It is carried out on each of the four columns of **State**.
  - Each column of **State** is replaced by a new column which is formed by multiplying that column by a certain matrix of elements of the field $\mathbf{F}_{2^8}$.
  - **FieldMult** computes two inputs product in the field.



Note: 2 is x in $\mathbf{F}_{2^8}$ and 3 is x+1 in $\mathbf{F}_{2^8}$

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- **Algorithm D: MixColumn(c)**

external FieldMult, BinaryToField, FieldToBinary
for $i \leftarrow 0$ to 3
  do $t_i \leftarrow$ BinaryToField($s_{i,c}$)
$u_0 \leftarrow$ FieldMult(x,t_0) $\oplus$ FieldMult($x+1,t_1$) $\oplus$ t_2 $\oplus$ t_3
$u_1 \leftarrow$ FieldMult(x,t_1) $\oplus$ FieldMult($x+1,t_2$) $\oplus$ t_3 $\oplus$ t_0
$u_2 \leftarrow$ FieldMult(x,t_2) $\oplus$ FieldMult($x+1,t_3$) $\oplus$ t_0 $\oplus$ t_1
$u_3 \leftarrow$ FieldMult(x,t_3) $\oplus$ FieldMult($x+1,t_0$) $\oplus$ t_1 $\oplus$ t_2
for $i \leftarrow 0$ to 3
  do $s_{i,c} \leftarrow$ FieldToBinary(u_i)

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- KeyExpansion: (for 10-round **AES**)
  - 10-round, 128-bit key
  - We need 11 round keys, each of 16 bytes
  - Key scheduling algorithm is word-oriented (4 bytes), so a round key consists of 4 words
  - The concatenation of round keys is called the **expanded key**, which consists of 44 words, w[0], w[1],…, w[43].
  - See **Algorithm E**

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- **Notations of Algorithm E:**
  - Input: 128-bit key, **key**, key[0],…,key[15]
  - Output: words, **w**
  - **RotWord**: a cyclic shift of four bytes B_0,B_1,B_2,B_3

    $$\textbf{RotWord}\ (B_0,B_1,B_2,B_3)= (B_1,B_2,B_3,B_0)$$
  - **SubWord**: applies the S-box to each byte

    $$\textbf{SubWord}\ (B_0,B_1,B_2,B_3)=(B_0',B_1',B_2',B_3')$$

    where $B_i'=\textbf{SubBytes}(B_i)$
  - **RCon**: an array of 10 words, RCon[1],…,RCon[10], they are constants defined at the beginning

Algorithm E: KeyExpansion(key)

external RotWord, SubWord
RCon[1] ← 01000000
RCon[2] ← 02000000
RCon[3] ← 04000000
RCon[4] ← 08000000
RCon[5] ← 10000000
RCon[6] ← 20000000
RCon[7] ← 40000000
RCon[8] ← 80000000
RCon[9] ← 1B000000
RCon[10] ← 36000000
for $i \leftarrow 0$ **to** 3
  do w[i] ←(key[4i],key[4i+1],key[4i+2],key[4i+3])
for $i \leftarrow 4$ **to** 43
  do temp ← w[i-1]
    if $i \equiv 0 \pmod 4$
      then temp← SubWord(RotWord(temp)) ⊕RCon[i/4]
    w[i] ← w[i-4] ⊕ temp
return (w[0],…,w[43])

round key 0: w[0], w[1], w[2], w[3]
round key 1: w[4], w[5], w[6], w[7]
:
:
:
round key10: w[40], w[41], w[42],,w[43]

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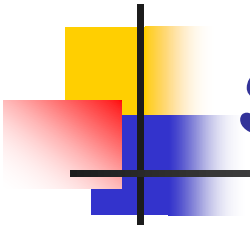

- Above are the operations need to encrypt in **AES**.
- To decrypt, we perform all operations and the key schedule in the reverse order.
- Each operation, **ShiftRows**, **SubBytes**, **MixColumns** must be replaced by their inverse operations.
  - **AddRoundKey** is its own reverse.

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- **Analysis of AES**
  - **AES** is secure against all known attacks.
  - Various aspects of design incorporate specific features to against specific attacks.
    - **Ex1**: Finite field inversion in S-box yields linear approximation and difference distribution tables close to uniform.
    - **Ex2**: MixColumns makes it impossible to find differential and linear attacks that involve "few" active S-boxes (**wide trail strategy**).