Problem 1 (6%). Consider the Turán's theorem and its proof, and let n be a multiple of k, where $k \geq 2$ is an integer.

Construct a graph $G = (V, E)$ with n vertices that contains no $(k+1)$-clique such that the number of edges attains the upper-bound given in the Turán's theorem, i.e.,

$$|E| \;=\; \left(1 - \frac{1}{k}\right) \cdot \frac{n^2}{2}.$$

Justify your answer.

Problem 2 (7%). Let $k > 0$ be an integer and let $p(n)$ be a function of n with $p(n) = \Omega\left((6k \ln n)/n\right)$ for large n. Prove that "almost surely" the random graph $G = G(n, p)$ has no independent set of size $n/2k$, i.e., show that

$$\Pr\left[\, \alpha(G) \geq \frac{n}{2k} \,\right] \;=\; o(1).$$

Problem 3 (7%). Let K_n denote the complete graph with n vertices. Show that it is possible to color K_n with at most $3\sqrt{n}$ colors so that there are no monochromatic triangles.