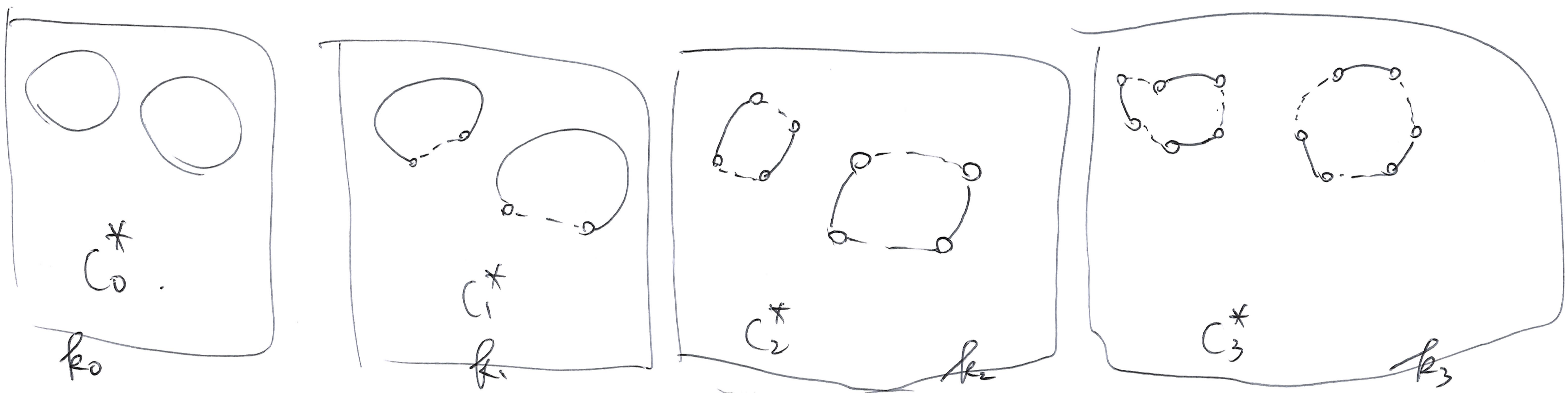
1-2

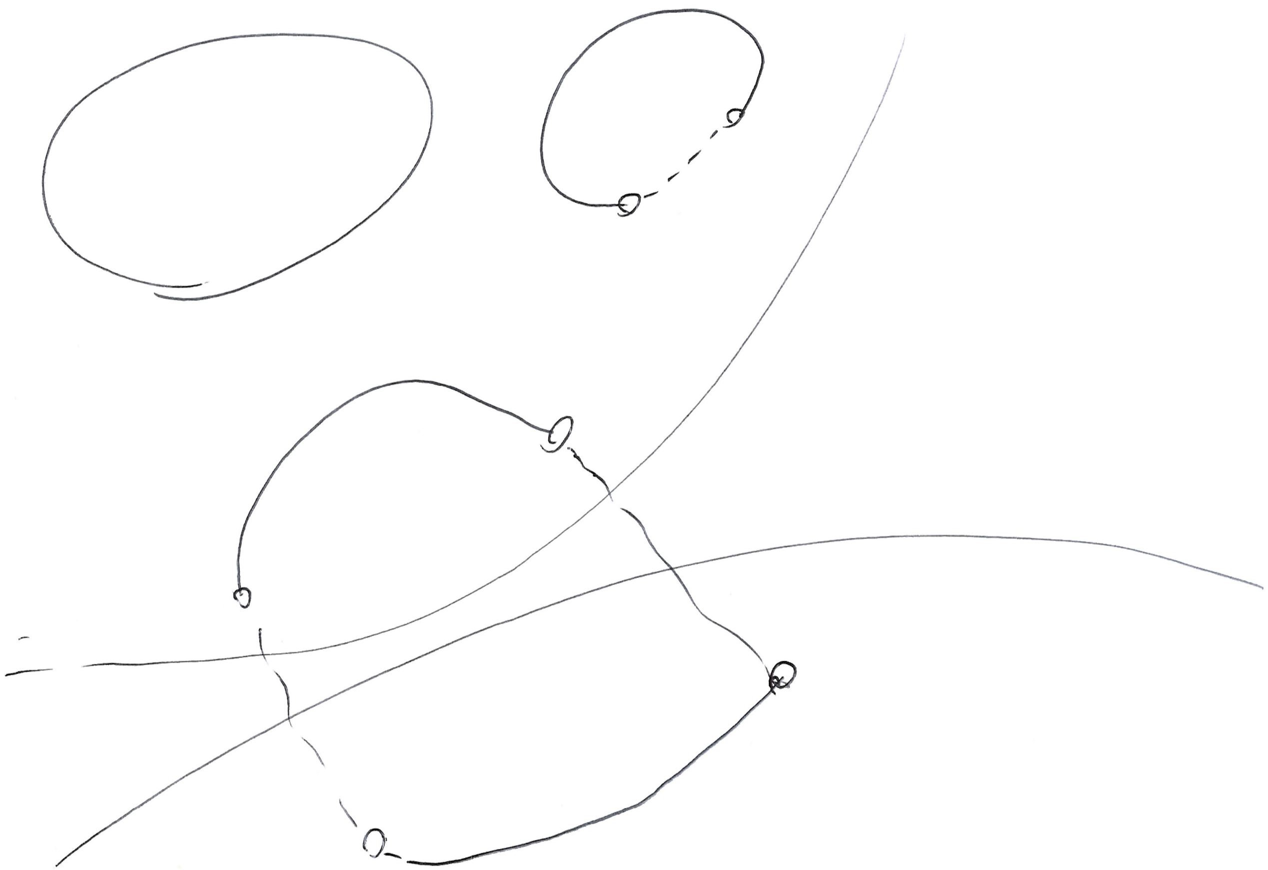
1. Remove edges with length $> \frac{1}{4}B$.
2. Compute MSTs for each component. $T_1, \dots, T_q$.
3. Compute Min-cost matching for odd-degree vertices. M^o.
4. Short-cut components in $\cup T_i \cup M^o$ to get Tours. $C_1 \cdots C_{q'}$.
5. Short-cut tours to get cycles of length $\le B$.

C_i^*: cycles in opt that contain i edges with length $> \frac{1}{4}B$.



Consider an opt solution on each connected component

$\boxed{g = \#\text{ of connected components.}}$



$$\boxed{A = \text{MSTs.}}$$

$$w(A) = \sum_{1\le j\le g} w(T_j)$$

$$\le k_0 \cdot B + \sum_{1\le j\le 3} k_j \cdot \frac{4-j}{4} \cdot B + \underbrace{\Big(k_0 + \sum_{1\le j\le 3} j \cdot k_j - g\Big)}_{\#\text{ of edges joinig segs into } g \text{ componemts.}} \cdot \frac{B}{4}.$$

$$= \frac{5}{4} k_0 \cdot B + \sum_{1\le j\le 3} k_j \cdot B - \frac{1}{4} g \cdot B.$$

To form a ~~lower~~ upper-bound for M°.

cost. $k_0 \cdot B$.

$k_1 \cdot B$.

for C_1^*. add the deleted edge back.

C_2^*.

C_3^*.

join the two end-points of the paths. $\sum_{3\le j\le 4} 2 \cdot k_j \cdot \frac{4-j}{4} \cdot B$

$$w(M^\circ) \le \frac{1}{2}\left(w(M_1^\circ) + w(M_2^\circ)\right)$$

$$\le \frac{1}{2}\cdot\left(k_0 B + k_1 B + 2\cdot\sum_{3\le j\le 4} k_j \cdot \frac{4-j}{4}\cdot B + 2\cdot\left(k_0 + \sum_{1\le j\le 3} j\cdot k_j - \xi\right)\cdot\frac{B}{4}\right)$$

$$\le \frac{3}{4} k_0 B + \frac{3}{4}\cdot k_1\cdot B + \sum_{3\le j\le 4} k_j\cdot B - \frac{1}{4}\cdot\xi\cdot B.$$

$$\Rightarrow w(\text{Tours}) \le 2\cdot k_0 B + \frac{7}{4} k_1\cdot B + 2\cdot\sum_{3\le j\le 4} k_j\cdot B - \frac{1}{2}\xi\cdot B.$$

$$\Rightarrow \frac{w(\text{Tours})}{\frac{B}{2}} + \xi \le 4\cdot\sum_j k_j \le 4K^*.$$