There are 6 problems, accounting for 100% in total.

Problem 1 (15%)**.** Prove the following statements.

- $n^2 + n \notin O(n \log n)$.
- $n^2 + n \notin o(n^2)$.

Problem 2 (15%)**.** We proved that the solution of $T(n) = 2T(\lfloor n/2 \rfloor) + n$ is $O(n \log n)$. Show that the solution of this recurrence is also $\Omega(n \log n)$ and conclude that the solution is $\Theta(n \log n)$.

Problem 3 (20%)**.** Show that the solution to $T(n) = 2T(\lfloor n/2 \rfloor + 17) + n$ is $O(n \log n)$.

Problem 4 (20%)**.** Argue that the solution to the recurrence $T(n) = T(n/3) + T(2n/3) + cn$, where c is a constant, is $\Omega(n \log n)$ by appealing to a recursion tree.

Problem 5 (10%)**.** Use the master method to give tight asymptotic bounds for the following recurrences.

1. $T(n) = 4T(n/2) + n$.
2. $T(n) = 4T(n/2) + n^2$.
3. $T(n) = 4T(n/2) + n^3$.

Problem 6 (20%)**.** Show how to sort n integers in the range of 0 to $n^2 - 1$ in $O(n)$ time.