

# AN OVERVIEW OF CAD TECHNIQUES FOR SWITCHED CAPACITOR NETWORKS

L. Claesen, H. De Man, J. Rabaey and J. Vandewalle  
Katholieke Universiteit Leuven  
Kardinaal Mercierlaan 94  
3030 Heverlee

## Abstract

*In this paper, an overview of the different analysis techniques for switched capacitor filters, which recently have been presented, is given. The differences and the equivalences between those algorithms will be demonstrated, using the techniques, implemented in the DIANA program, as a guideline. Design in time and frequency domain and sensitivity and noise calculations are discussed.*

## 1. INTRODUCTION

Although the principle of using time variant networks containing switches, capacitors and operational amplifiers has already been proposed more than a decade ago, it has only recently been put into practice using MOS-LSI techniques. They are not only suited for the design of fully integrated filters but also for A/D and D/A-convertors, waveform generators etc...

The basic principles of these analog sampled-data systems are quite simple, but an accurate analysis of their behaviour in time and frequency domain is not so trivial: In fact, the signals which occur in these time-variant networks are often a mixture of discrete and continuous signals, which can be seen clearly when a continuous input-output path is present. Moreover, the sampling frequency  $F_s$ , defined as the frequency of the driving clock, is normally orders of magnitude higher than the signal frequency, making the use of traditional time domain simulators very costly.

Nevertheless time, frequency, sensitivity and noise analysis of such networks is very important, because the calculation of the influence of parasitic effects as stray capacitances, finite opamp-gain and bandwidth, aliasing, noise, component sensitivities, non linearities ... is almost impossible by hand calculations. This is the reason why during recent years a number of independent and parallel approaches to the mathematical description of s.c.-networks have taken place. Although almost all authors have adopted the same technique for simulation in the time domain, a large number of different approaches for the frequency domain exists. In this paper, it will be demonstrated that most of these techniques can be generalised, using the z-domain transfer matrix. It will also be shown how the adjoint s.c.-network, introduced by the authors, can be used to obtain more efficient frequency domain, sensitivity and noise calculations. Finally, an overview of the different methods to include the effects of resistances in the frequency domain response, will be given. It must be noted that, until now, only a small number of exact approaches to handle these effects exists. An approach, which combines the MNA-framework and the z-domain transfer matrix with classical ac-analysis techniques and which is implemented in DIANA will be presented. Bandwidth limitations due to operational amplifiers and RC-time constants are of prime interest when performing noise calculations. These effects limit the folding of noise from higher frequency bands to the base-band.

## 2. TIME DOMAIN ANALYSIS OF S.C.-NETWORKS.

In the time domain, different simulation modes are available: efficient simulations can be performed, assuming the equilibrium principle and using charge conservation. This can be implemented very easily in each simulation-program based on nodal or modified nodal analysis methods. Parasitic effects and resistive-capacitive transients can be checked using more detailed simulation-modes.

### 2.1. Ideal linear and non-linear s.c.-networks.

Different time-domain simulators have been set up for ideal s.c.-networks containing only capacitors, ideal switches, dependent and independent voltage and charge sources. All these methods are based on the fact that in the absence of resistive-capacitive time-constants, the Kirchhoff current equations in each node can be integrated yielding "Kirchhoff charge equations". E.g. referring to Fig. 1, one obtains for node ①:

$$q_{E,k}(t) + q_{S,k}(t) + q_{A,k}(t) + q_{C,k}(t) = 0$$

where

$$q_{j,k}(t) = \int_{t_k}^{t+\Delta k} i_j(t) dt \quad \text{is} \quad (1)$$

the charge, transferred in branch j in the time  $t-t_k$ .

The methods only differ in the way the equations are build up: using switching matrices and matrix multiplications [1, 6] or using topological partitioning [7]. The method, introduced by the authors [8-9], is based on the direct set up of the Modified Nodal Equations (MNA), using well defined stamps and Boolean controlled switches. In this way, only one matrix can serve for all clock-phases.

Charge-equation (1) can now be implemented easily in classical circuit-simulators, using the Backward Euler Integration Companion model, with a unit time-step  $\Delta = 1$  [9]. This has also been noted by Liou and Kuo [4], where a capacitor is replaced by an equivalent resistor (with value  $1/C$ ) in series with a dependent voltage source  $v_C$  or its Norton equivalent. (This allows them to derive a z-domain equivalent circuit).

For the MNA-approach, a final set of equations is obtained as described in Fig. 1B. This can also be written in matrix form:

$$\tilde{M}_k \cdot \tilde{X}_k(t) = \tilde{S}(\tilde{v}_{k-1}, \tilde{s}(t)) \quad (2)$$

where  $\tilde{s}(t)$  is the excitation vector (voltage and charge sources) and  $\tilde{v}_{k-1}$  the vector of the node voltages at the end of the previous clock-phase.

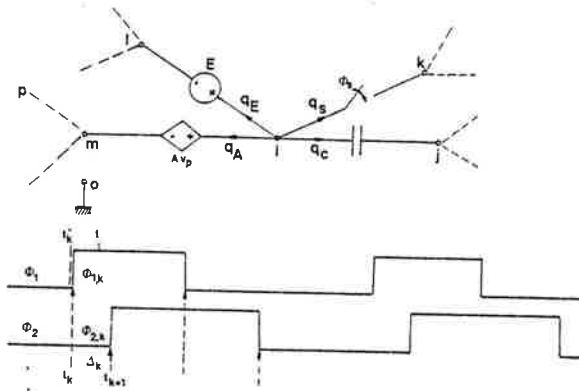


Fig. 1A.

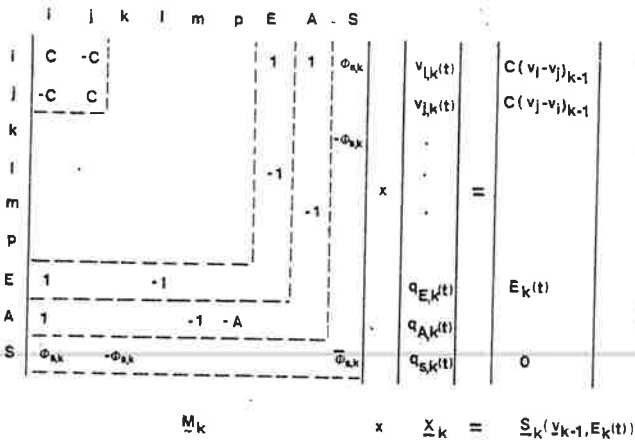


Fig. 1B.

Repetitive solving of (2), using classical sparse matrix techniques, leads to an efficient and accurate time domain simulator. Due to the introduction of the equilibrium principle, which assumes that the circuit goes immediately to the equilibrium state after a clock transition, only one evaluation of (2) has to be done for each time-slot, stepping from one slot to the next one.

Note that also non-linear capacitors and dependent sources can be implemented very easily in this framework, using the Newton-Raphson iteration technique. Interesting is also, that, due to the use of the Boolean controlled elements, no restrictions are imposed on the clocksequences and the number of clockphases.

Time domain simulation of ideal s.c.-networks has, amongst others, the following applications :

- computation of impulse responses of filters
- computation of sinusoidal, step or ramp responses
- verification of A/D and D/A response
- study of current and leakage current drift
- study of non-linear distortion in s.c.-networks.

## 2.2. Time domain simulation of second order effects in non-ideal s.c.-networks. Top-down design.

In analogy to the concept of mixed mode simulation in digital circuits, where one descends from the RTL (Register transfer language) level to the circuit level, the same concept can be used for the design of switched capacitor circuits in time (and also frequency) domain. Here the top level is above described simulator, while the down level means simulation including transistors, higher order models or transistor models for switches, one or more pole opamp-models and perhaps the detailed opamp simulation for one or more filter sections. This simulation strategy is called "Mixed Mode simulation for analog sampled MOSLSI circuits".

Note that this mode also allows for simultaneous simulation of analog and digital control.

Implementing this in existing simulators, as DIANA, is quite simple, using the above described MNA formulation for ideal s.c.-networks (top-level), which is completely compatible with traditional time domain simulation (down level).

## 3. ANALYSIS OF IDEAL S.C.-SYSTEMS IN THE FREQUENCY DOMAIN.

At the moment, a large number of approaches for the simulation of ideal s.c.-filters in the frequency domain are available. The equivalences and the deviations between those approaches will be demonstrated, starting from the theory of the z-domain transfermatrix, which can be seen as a generalisation of a large number of other theories [10].

### 3.1. z-domain equations.

The set-up of the z-domain equations, starting from the MNA-description will be explained.

We consider arbitrary linear networks containing ideal switches, capacitors, independent voltage and charge sources and dependent sources VCVS, QCQS, ... The switches are controlled by Boolean clock variables  $\phi_r(t) = 0$  or  $1$ .  $\phi_r(t) = 0$  (resp.,  $\phi_r(t) = 1$ ) corresponds to an open (resp., closed) switch at time  $t$  if this switch is driven by clock  $r$ . The time is partitioned into time slots  $\Delta_k = (t_k, t_{k+1}]$  such that the clock signals do not vary in  $\Delta_k$ , i.e.  $\phi_r(t) = \phi_{rk}$  for  $t \in \Delta_k$ . We assume that the clock signals are  $T$ -periodic, with  $N$  time slots (called  $N$  phases) in one period of duration  $T$ .

**Theorem 1 :** If any above defined SC network is excited by a piecewise-constant excitation  $\underline{w}(t) = \underline{w}_m$ ,  $\underline{u}(t) = \underline{u}_m$ ,  $t \in \Delta_m$ , then its response in the time domain is also piecewise-constant  $\underline{v}(t) = \underline{v}_m$ ,  $\underline{q}(t) = \underline{q}_m$  in  $\Delta_m$  and given by

$$\begin{bmatrix} \underline{A}_k & \underline{B}_k \\ \underline{C}_k & \underline{D}_k \end{bmatrix} \begin{bmatrix} \underline{v}_{k+\ell N} \\ \underline{q}_{k+\ell N} \end{bmatrix} = \begin{bmatrix} \underline{A}_k \cdot \underline{v}_{k+\ell N-1} \\ \underline{C}_k \cdot \underline{v}_{k+\ell N-1} \end{bmatrix} + \begin{bmatrix} \underline{B}_k \cdot \underline{w}_{k+\ell N} \\ \underline{D}_k \cdot \underline{u}_{k+\ell N} \end{bmatrix} \quad (3)$$

where  $\underline{v}(t)$  (resp.,  $\underline{u}(t)$ ) is the vector of the voltage responses at the nodes (resp., voltage sources in some selected branches) and where  $\underline{q}(t)$  (resp.,  $\underline{w}(t)$ ) is the vector of the charges transferred in some selected branches (resp., injected by charge sources in the nodes) between  $t_{k+\ell N}$  and  $t$  for  $t \in \Delta_{k+\ell N}$ , and where the contributions to  $\underline{A}_k$ ,  $\underline{B}_k$ ,  $\underline{C}_k$  and  $\underline{D}_k$  are given by the stamps of the classical modified nodal analysis and where the contribution of a switch  $S$  connecting node  $i$  to  $j$  and governed by clock  $\phi_r$ , is given by

$$\begin{matrix} i & j & S \\ \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} \phi_{rk} \\ -\phi_{rk} \end{bmatrix} & \begin{bmatrix} v_{ik} \\ v_{jk} \end{bmatrix} \\ \begin{bmatrix} \phi_{rk} & -\phi_{rk} \end{bmatrix} & \begin{bmatrix} -\phi_{rk} \\ \phi_{rk} \end{bmatrix} & \begin{bmatrix} q_{Sk} \\ 0 \end{bmatrix} \end{matrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (4)$$

where  $\phi$  denotes the complement of a Boolean variable. If the SC network is excited by an arbitrary excitation  $\underline{w}(t)$ ,  $\underline{u}(t)$ , then the piecewise-constant part of the input and output  $\underline{w}_m = \underline{w}(t_m)$ ,  $\underline{u}_m = \underline{u}(t_m)$ ,  $\underline{v}_m = \underline{v}(t_m)$  and  $\underline{q}_m = \underline{q}(t_m)$  are determined by (1) and the remainder waveforms  $\underline{v}^*(t) = \underline{v}(t) - \underline{v}_m$ ,  $\underline{u}^*(t) = \underline{u}(t) - \underline{u}_m$ ,  $\underline{v}^*(t) = \underline{v}(t) - \underline{v}_m$ , and  $\underline{q}^*(t) = \underline{q}(t) - \underline{q}_m$  for  $t \in \Delta_m$  are related by

$$\begin{bmatrix} \underline{A}_k & \underline{B}_k \\ \underline{C}_k & \underline{D}_k \end{bmatrix} \begin{bmatrix} \underline{v}^*(t) \\ \underline{q}^*(t) \end{bmatrix} = \begin{bmatrix} \underline{v}^*(t) \\ \underline{q}^*(t) \end{bmatrix} \text{ for } t \in \Delta_{k+\ell N} \quad (5)$$

Theorem 2 : given any above defined SC network with a piecewise-constant excitation. Define

$$\tilde{v}_m(z) \triangleq \mathcal{Z}\{v_{m+lN}\} = \sum_{l=0}^{\infty} v_{m+lN} z^{-l} \quad (6)$$

and analogous equations for  $\tilde{u}_m$ ,  $\tilde{q}_m$  and  $\tilde{w}_m$  where  $m=1, 2, \dots, N$ . Then these variables are related by:

$$\begin{array}{c|c|c|c} \begin{array}{c} \tilde{A}_1 \\ \tilde{A}_2 \\ \tilde{A}_3 \dots \tilde{A}_N \end{array} & \begin{array}{c} \tilde{B}_1 \\ \tilde{B}_2 \\ \tilde{B}_3 \dots \tilde{B}_N \end{array} & \begin{array}{c} \tilde{V}_1 \\ \tilde{V}_2 \\ \tilde{V}_3 \dots \tilde{V}_N \end{array} & \begin{array}{c} \tilde{W}_1 \\ \tilde{W}_2 \\ \tilde{W}_3 \dots \tilde{W}_N \end{array} \\ \hline \begin{array}{c} \tilde{C}_1 \\ \tilde{C}_2 \\ \tilde{C}_3 \dots \tilde{C}_N \end{array} & \begin{array}{c} \tilde{D}_1 \\ \tilde{D}_2 \\ \tilde{D}_3 \dots \tilde{D}_N \end{array} & \begin{array}{c} \tilde{Q}_1 \\ \tilde{Q}_2 \\ \tilde{Q}_3 \dots \tilde{Q}_N \end{array} & \begin{array}{c} \tilde{U}_1 \\ \tilde{U}_2 \\ \tilde{U}_3 \dots \tilde{U}_N \end{array} \end{array} \quad (7)$$

where the missing entries are zero, and the submatrices are defined in Theorem 1.

Corollary 1 : Any above defined SC-network is completely characterized by the z-domain transfer matrix  $\tilde{M} ::=$

$$\begin{array}{c|c|c|c} \begin{array}{c} \tilde{V}_1 \\ \tilde{V}_2 \\ \dots \\ \tilde{V}_N \end{array} & \begin{array}{c} \tilde{G}_{11} \tilde{G}_{12} \dots \tilde{G}_{1N} \\ \tilde{G}_{21} \tilde{G}_{22} \dots \tilde{G}_{2N} \\ \dots \\ \tilde{G}_{N1} \tilde{G}_{N2} \dots \tilde{G}_{NN} \end{array} & \begin{array}{c} \tilde{H}_{11} \tilde{H}_{12} \dots \tilde{H}_{1N} \\ \tilde{H}_{21} \tilde{H}_{22} \dots \tilde{H}_{2N} \\ \dots \\ \tilde{H}_{N1} \tilde{H}_{N2} \dots \tilde{H}_{NN} \end{array} & \begin{array}{c} \tilde{W}_1 \\ \tilde{W}_2 \\ \dots \\ \tilde{W}_N \end{array} \\ \hline \begin{array}{c} \tilde{Q}_1 \\ \tilde{Q}_2 \\ \dots \\ \tilde{Q}_N \end{array} & \begin{array}{c} \tilde{K}_{11} \tilde{K}_{12} \dots \tilde{K}_{1N} \\ \tilde{K}_{21} \tilde{K}_{22} \dots \tilde{K}_{2N} \\ \dots \\ \tilde{K}_{N1} \tilde{K}_{N2} \dots \tilde{K}_{NN} \end{array} & \begin{array}{c} \tilde{L}_{11} \tilde{L}_{12} \dots \tilde{L}_{1N} \\ \tilde{L}_{21} \tilde{L}_{22} \dots \tilde{L}_{2N} \\ \dots \\ \tilde{L}_{N1} \tilde{L}_{N2} \dots \tilde{L}_{NN} \end{array} & \begin{array}{c} \tilde{U}_1 \\ \tilde{U}_2 \\ \dots \\ \tilde{U}_N \end{array} \end{array} \quad (8)$$

The submatrices  $\tilde{G}_{kl}$ ,  $\tilde{H}_{kl}$ ,  $\tilde{L}_{kl}$  have a very simple interpretation. If only nonzero voltage sources are supplied in time slots  $l, l+N, l+2N, \dots$  and if the output node voltages are only observed at the end of time slots  $k, k+N, k+2N, \dots$  then  $\tilde{H}_{kl}(z)$  relates inputs at phase  $l$  to outputs at phase  $k$  i.e.

$$\tilde{v}_k(z) = \tilde{H}_{kl}(z) \tilde{u}_l(z) \quad (9)$$

Note that the z-domain transfer matrix  $\tilde{M}$  not only characterizes the behaviour of the system for piecewise constant inputs, but also for continuous inputs, which is interesting when continuous I/O-couplings are present : the remainder waveforms, as described in (5) can be recovered easily from  $\tilde{M}$  at  $z = \infty$ .

Solving equation (7) with selected excitations at the corresponding z-values, the different entries of the z-domain matrix can be obtained. This information is central for the calculation of the frequency domain transferfunctions, as is shown in next section.

### 3.2. Frequency domain transferfunctions.

Once the entries of the z-domain transfer matrix are calculated, using the methods described in 3.1. (or equivalent approaches), the exact frequency components of the output waveforms can be calculated, including the  $\sin(x)/x$  effects, the effects of the continuous I/O couplings and the different aliasing terms.

Since a SC network is a time-varying circuit, a sinusoidal excitation may generate many frequencies in the output. However often one is only interested in the following practical frequency domain

transfer function  $\tilde{H}(\omega)$  [1,3], which relates by definition the phasor  $\tilde{U}$  of the sinusoidal excitation  $y(t) = \tilde{U} e^{j\omega t}$  to the phasor at the same pulsation  $\omega$  in the output  $y(t)$ .

Standard Fourier transform techniques allow to prove that

$$\tilde{H}(\omega) = \sum_{k=1}^N e^{-j\omega t_{k+1}} \tilde{V}_k(\omega) \left( \sum_{l=1}^N e^{j\omega t_l} \tilde{H}_{kl}(e^{j\omega T}) \right) +$$

$$[(t_{k+1}-t_k)/T - \nu_k(\omega)] \tilde{H}_{kk}(\infty), \quad (10.a)$$

where

$$\tilde{V}_k(\omega) = 2 \left\{ \sin[p(t_{k+1}-t_k)/2] \exp[jp(t_{k+1}-t_k)/2] \right\} / T p \quad (10.b)$$

Similar expressions can be derived for the aliasing terms. [10]

The first term of the righthandside of (10a) handles the discrete signal part, including the effects of the  $\sin(x)/x$ , while the second part handles the continuous I/O-couplings. Equivalent equations have been deduced in [4,6,15].

### 3.3. Algorithmic aspects and overview.

As shown above, the frequency domain transferfunctions of ideal s.c.-systems can be obtained directly, solving (7) and combining the results using (10). The number of equations to be solved in this way is linear with the number of clock-phases and can easily grow to unacceptable dimensions. However, when inverting matrix  $\tilde{M}$  (7), the band-structured nature of this matrix can be taken into account in the L.U.-decomposition algorithm. Furthermore, compaction algorithms can be used to reduce the matrix-dimensions in a considerable way [12]. These algorithms are now implemented in the mixed mode simulator DIANA.

Equivalent approaches, deriving the frequency domain equations from a transformation of the time domain equations, were also presented by F. Brglez [1] and Y.P. Tividis [6], who used switching matrices and matrix multiplications to set up the equations in the time domain, by Liou & Kuo [4], using the state space, by Fang and Tividis [7], using topological methods for matrix compaction, and recently also by Hökenek and Moschytz [16] using the indefinite matrix.

Basic differences between these are the equation-set-up, the number of clockphases allowed and more important, the need for explicit equations. Moreover, most of them do not make use of the band-structured nature of  $\tilde{M}$  in the solution step, where they use closed form formula's incorporating a large number of inefficient matrix multiplications.

Other approaches use the principle of the z-domain equivalent circuit, converting the time variant s.c.-network into a time-invariant network in the z-domain, described by a set of equations, equivalent to (7). This idea was originated by Kurth [3] and Kuo [5] for 2 phase-networks and later generalised to N-phase s.c.-networks in [10] and [6]. Another equivalent circuit, using frequency dependent resistors, was presented by Knob and Dessoullavy [17], suffering however from the restrictions that all node voltages have to remain constant over one full period.

A third way of generating the z-domain matrix entries was described in [8,9], where the time domain simulator for ideal circuits is used to generate the impulse responses of the different clockphases.

These are transformed to the  $z$ - (or frequency) domain using Fast Fourier techniques. This method is very attractive, because it is the only one which can be implemented in any traditional simulator with an appropriate switch model with only minor adjustments. Disadvantages are the large number of time points which have to be handled for some filters (e.g. high  $Q$ -filters) and the constraints imposed by the FFT-algorithm.

Interesting methods for small s.c. networks were finally presented in [8] and [9], using respectively a pole-zero calculating method and a symbolic approach.

### 3.4. The adjoint switched network and its applications.

A large number of classical circuit simulators use the principle of the adjoint network in order to decrease drastically the number of calculations, needed for noise and sensitivity calculations. This concept has been generalised to the adjoint switched network in [1]. There it is demonstrated that the setup of adjoint s.c.-networks turns out to be equivalent to that of classical analog networks except for one additional step, namely the reversing of the time dependence of all time varying elements in one period, i.e. the new clock signal  $\phi_{rk}$  satisfies  $\phi_{rk} = \phi_{r, N-k+1}$ . The useful property of the adjoint network of a linear time-variant network is now that its impedance matrix and thus also the  $z$ -domain transfermatrix is the transpose of that of the original matrix. Note that it is also possible to set up an adjoint network, using  $z$ -domain equivalent

The principle of the adjoint switched network has recently been extended to derive both time and frequency domain s.c. adjoint networks, independent of the method used to formulate the original s.c.-circuit equations [20], using a modified form of Tellegen's Theorem.

The most interesting properties of this adjoint network are demonstrated here in short :  
- sensitivity analysis :

In sensitivity analysis one is basically interested in the variation in the input-output relations due to variations in one or several network parameters. This is especially important for filter design. The adjoint network can now be used to reduce considerably the computational efforts, brought about by sensitivity analysis. It can in fact be shown that

$$\frac{\partial \tilde{M}}{\partial \lambda} = -\tilde{M}^T \cdot \frac{\partial \tilde{M}^{-1}}{\partial \lambda} \cdot \tilde{M} \quad (11)$$

where  $\lambda$  is the considered variable,  $\tilde{M}$  and  $\tilde{M}^{-1}$  respectively the  $z$ -domain transfer matrices of the original and adjoint network and  $\tilde{M}^{-1}$  the admittance matrix as described in (7).  $T$  means transpose. Clearly, the righthandside of (11) is easier to calculate, since  $\frac{\partial \tilde{M}^{-1}}{\partial \lambda}$  is known from design, and  $\tilde{M}$  and  $\tilde{M}^{-1}$  can be found doing analyses on the original and the adjoint network, as described higher. It can easily be shown that, independent from the number of considered variables, one original and one adjoint network analysis are sufficient to calculate the sensitivity values. Expressions for sensitivities to capacitors, gains of voltage dependent voltage sources, clock switching time as well as group delays can be found in [1].

### - Noise calculations

The adjoint network is also most interesting when calculating the noise performance of an s.c.-network. This behaviour is most difficult to predict

due to folding of noise from higher bands to baseband. The noise spectrum can thus (for ideal s.c.-networks) be seen as an infinite sum, unbounded in magnitude [4], [1].

$$S_{vk}(i)(\omega) = \sum_{n=-\infty}^{+\infty} X_k^{(i.)}(\omega, \omega - n\omega_s) S_u(\omega - n\omega_s) X_k^{(i.)*}(\omega, \omega - n\omega_s) \quad (12)$$

where  $S_u$  is the spectral density matrix of the input noise sources,  $i$  is the output node,  $X_k$  is the noise transfer functions from frequency  $\omega - n\omega_s$  to  $\omega$ , with  $\omega$  the considered output frequency, and  $X_k^*$  is the complex conjugate and transpose of  $X_k$ . [11]. Essential for noise calculations is thus the evaluation of the transferfunctions from the noise sources to the output node, this considering the folding from higher frequency bands to the baseband. It can now be shown easily that all these transfers can be obtained in one analysis of the adjoint network, exciting the network at the output node and observing the transfers to the different noise sources.

Finally, it must be noted that, in reality, the infinite sums, as expressed in (12) is bounded due to bandlimitations, caused by resistive-capacitive timeconstants and opamp-poles. It is clear that the simulation mode for ideal s.c.-networks in the frequency domain, as described higher, is not suited for noise calculations. It is necessary to incorporate resistive effects in the transferfunctions as described in next section.

### 4. FREQUENCY DOMAIN ANALYSIS OF NON-IDEAL S.C.-NETWORKS INCLUDING RESISTIVE EFFECTS.

In contrast with the large number of approaches, existing for frequency domain analysis of ideal s.c.-networks, there are only a few techniques to handle the effects of switch resistances and opamp poles on the frequency response. Most of them are limited to small networks e.g. [21] or to pure opamp-poles [22]. An exact analysis in the state space has been presented by Ström and Signell [13]. State space techniques however are not compatible with today's currently used CAD-techniques. This method has been expanded by the authors to the MNA-frame work [4]. Instead of closed form formula's, involving matrix multiplications, the  $z$ -domain transfer matrix is introduced, combined with a number of classical ac-analyses : these ac-analyses, which are evaluations of the complex MNA-matrices, characterize the behaviour of different circuits, which are created in each clockphase, at input and output frequency. The results of these simulations are then combined using the  $z$ -domain transfer matrix, linking the different clock-phases together and incorporating the switching effects. This method is completely general without restrictions on the number of clock-phases, input and output signals.... The algorithm is in fact not only suited for analysis of s.c.-circuits, but even for all periodically switched, linear networks.

The study of the effects of these resistive-capacitive time-constants and opamp-poles on the frequency response is especially important in noise analysis (as already mentioned higher), when designing high  $Q$ -filters, filters with small ratio between sampling and signal frequencies, filters with high sampling frequencies... Simultaneous simulation of analog filters as anti-aliasing and smoothing filters and the s.c.-networks is possible. Normally, this mode is only used for detailed simulation of

smaller filter parts or can be considered as the down level of frequency domain simulation.

## Conclusions

An overview of the different existing CAD-techniques for the analysis of s.c.-networks in time and frequency domain has been given. We have described the principles of the Mixed Mode simulation for analog sampled MOSLSI circuits, as implemented in DIANA. Efficient time domain techniques for top down analysis were presented. It has been shown that the z-domain transfer matrix approach can be seen as a generalized frequency domain simulation tool, which can be extended easily to the analysis of non-ideal s.c.-networks including resistive effects. The adjoint network has proven to be a very efficient method for noise and sensitivity calculations, which can be easily extended to all existing approaches. Finally the MNA-framework can be seen as a natural tool to handle s.c. networks since it allows to set up and solve easily the equations and does not impose any restrictions.

Not much work has been done yet on optimization of switched capacitor circuits [5] nor distortion.

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