

Wireless Communication Systems @CS.NCTU

Lecture 8: Successive Interference Cancellation

Instructor: Kate Ching-Ju Lin (林靖茹)

Agenda

- Successive Interference Cancellation
- ZigZag decoding

SRN and SNR_{dB}

- $\text{SNR} = \frac{P_{\text{signal}}}{P_{\text{noise}}}$
- Unit of power: watt

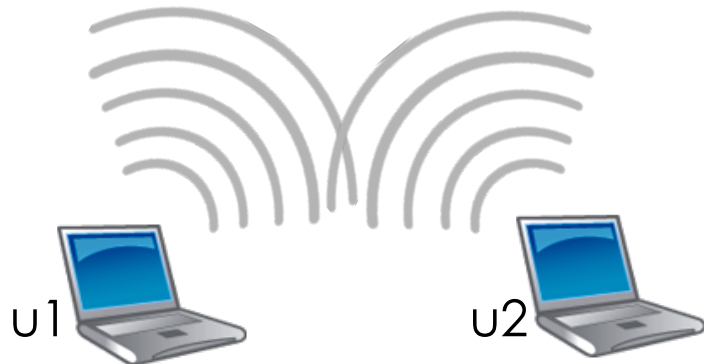
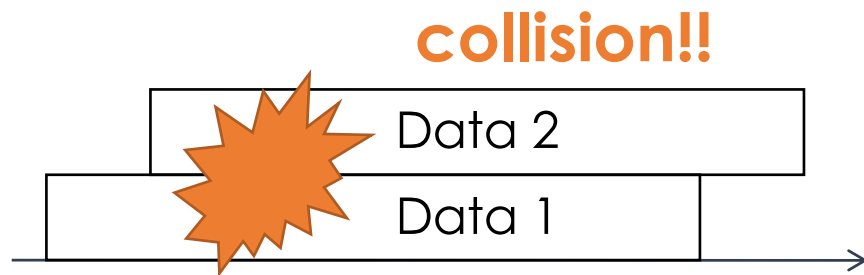
Logarithmic unit of power: decibel (**dBm**)

$$P_{\text{dB}} = 10 \log_{10} P$$

- $\text{SNR}_{\text{dB}} = 10 \log_{10} \left(\frac{P_{\text{signal}}}{P_{\text{noise}}} \right)$
 $= 10 \log_{10}(P_{\text{signal}}) - 10 \log_{10}(P_{\text{noise}})$
 $= P_{\text{signal,dBm}} - P_{\text{noise,dBm}}$

Example:
signal = -70 dBm
noise = -90 dBm
SNR = -70 - (-90) = 20 dB

Scenario



$$y = h_1x_1 + \boxed{h_2x_2} + n$$

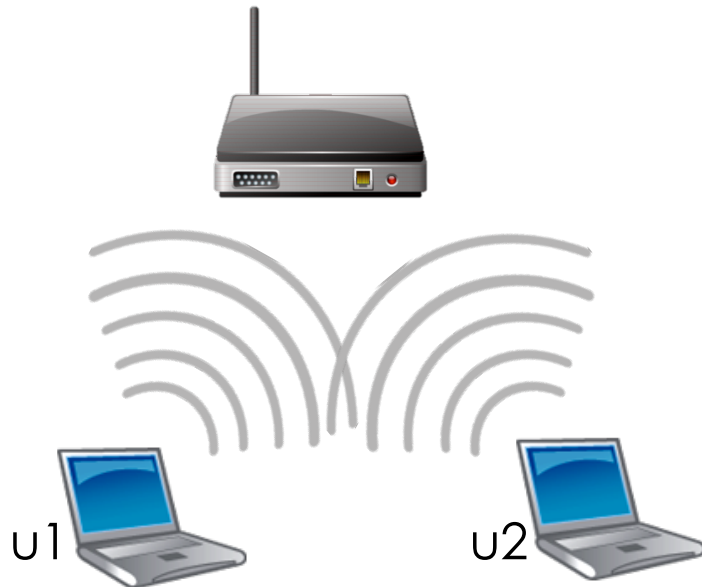
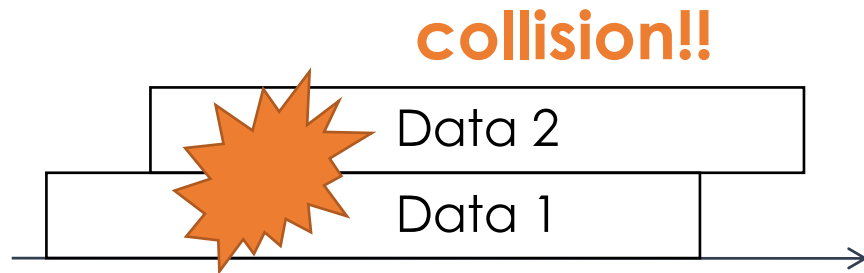
interference noise

$$\text{SINR} = \frac{P_1}{P_2 + N_0}$$

Reliably decode when the rate is no larger than capacity

$$R \leq C = \log\left(1 + \frac{P_1}{P_2 + N_0}\right)$$

Scenario



$$y = h_1x_1 + \boxed{h_2x_2} + n$$

interference

noise

$$\text{SINR} = \frac{P_1}{P_2 + N_0}$$

Example:

signal = -70 dBm

Interference = -75 dBm

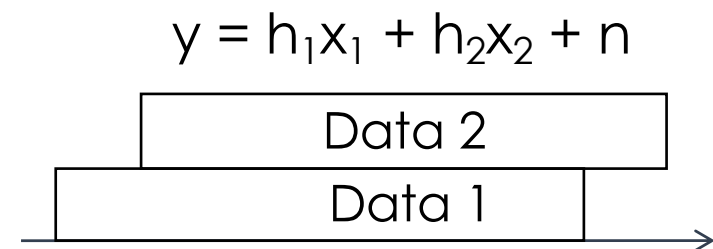
noise = -90 dBm

SNR \sim -70 - (-75) = 5 dB

Can still decode if selecting a very low bit-rate

SIC Decoding

- Successive Interference Cancellation (SIC)
 1. **Decode one** user first in the presence of interference
 $x'_2 = y/h_2 = x_2 + h_1x_1/h_2 + n/h_2$
 2. **Re-encode** the recovered data to remove the noise
(demodulate x'_2 and re-modulate it)
 3. **Subtract** the re-encoded data from the received signal
 $y' = y - h_2x_2 = h_1x_1 + n$
 4. Decode the second user
 $x'_1 = y'/h_1$



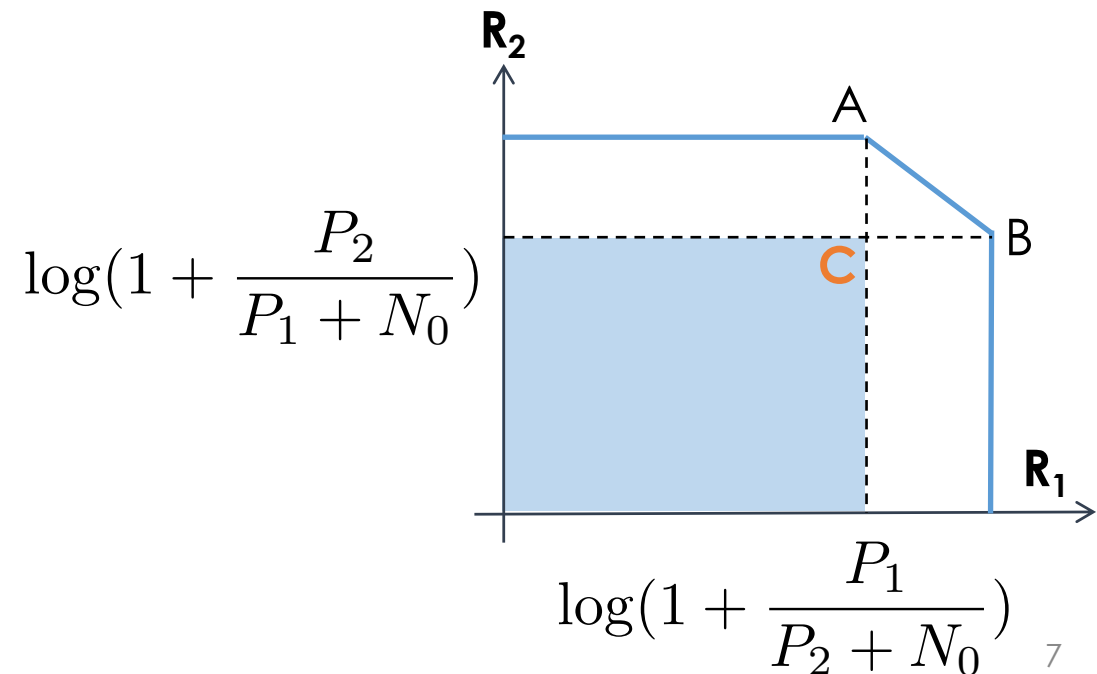
Capacity Region without SIC

- $y_1 = h_1x_1 + (h_2x_2 + n)$
- $y_2 = h_2x_2 + (h_1x_1 + n)$

➔

$$R_1 \leq \log\left(1 + \frac{P_1}{P_2 + N_0}\right)$$
$$R_2 \leq \log\left(1 + \frac{P_2}{P_1 + N_0}\right)$$

Maximal sum-rate: point C



Capacity Region of SIC



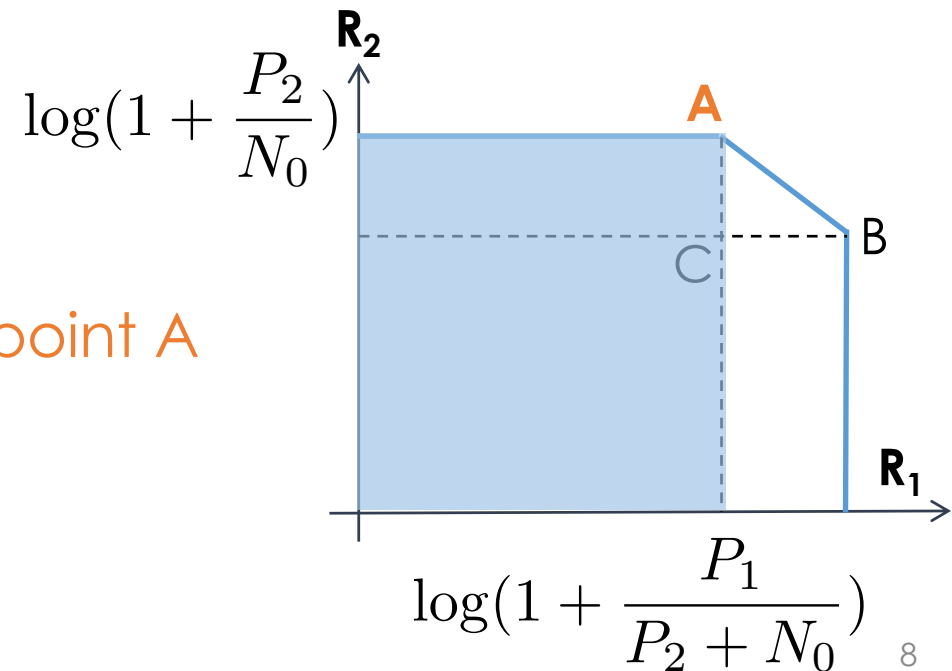
- Decoding order: user 1 \rightarrow user 2
 - If we decode u1 in the presence of interfering u2, and then decode u2

$$- y_1 = h_1 x_1 + (h_2 x_2 + n)$$

$$- y_2 = h_2 x_2 + n$$

\rightarrow Get single-user rate

\rightarrow Maximal sum-rate: point A



Capacity Region of SIC

- Decoding order: user 2 \rightarrow user 1
 - If we decode u2 in the presence of interfering u1, and then decode u1

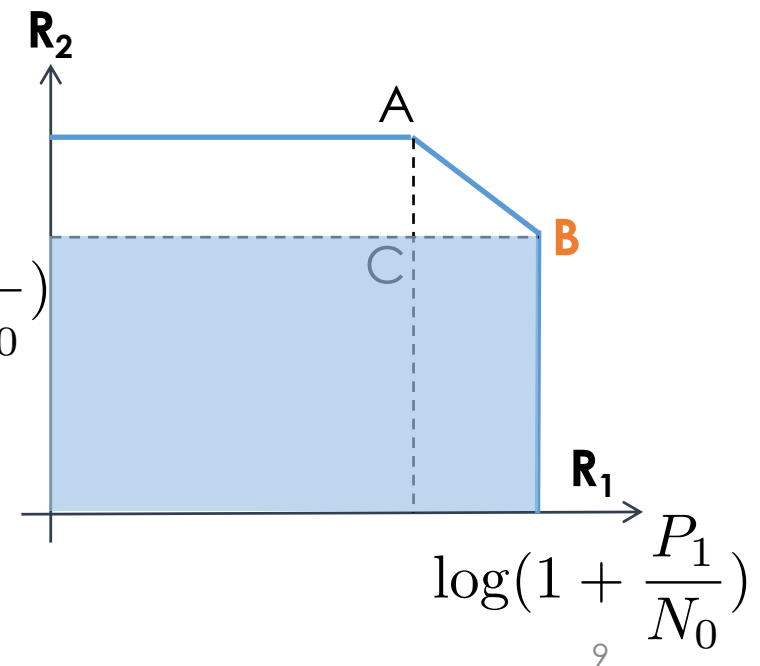
$$- y_2 = h_2 x_2 + (h_1 x_1 + n)$$

$$- y_1 = h_1 x_1 + n$$

\rightarrow Get single-user rate

$$\log\left(1 + \frac{P_2}{P_1 + N_0}\right)$$

\rightarrow Maximal sum-rate: point B



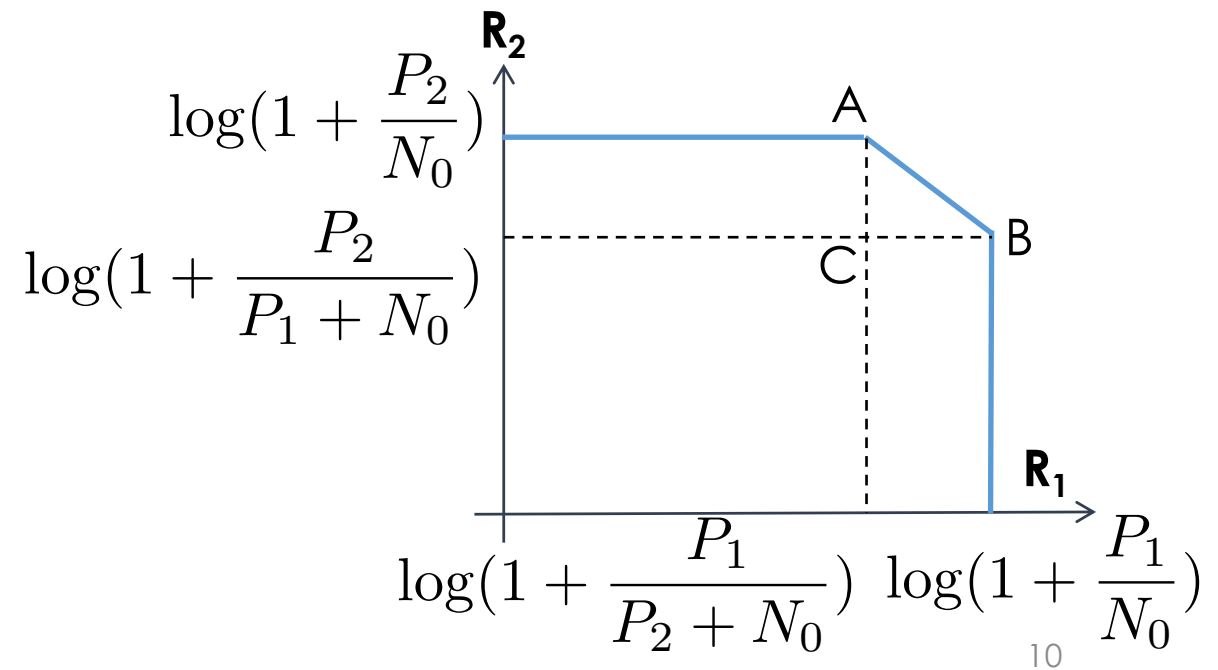
Capacity Region of SIC

- To ensure reliable decoding, the rates (R_1, R_2) need to satisfy three constraints:

$$R_1 + R_2 \leq \log\left(1 + \frac{P_1 + P_2}{N_0}\right)$$

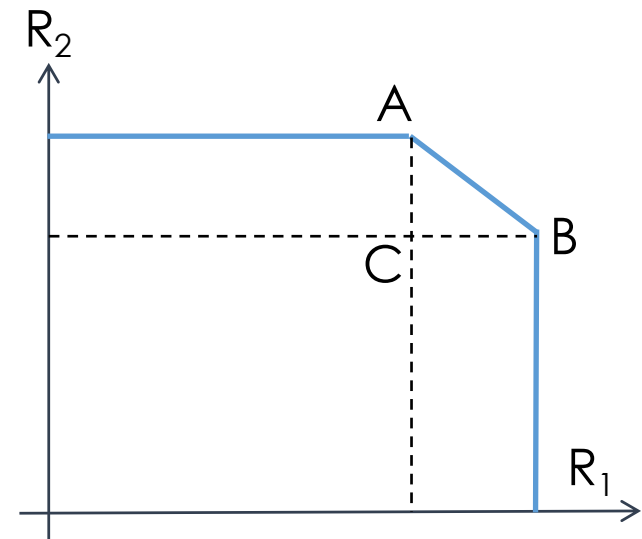
$$R_1 \leq \log\left(1 + \frac{P_1}{N_0}\right)$$

$$R_2 \leq \log\left(1 + \frac{P_2}{N_0}\right)$$



Capacity Region of SIC

- User 1 achieves its single-user bound (point B) while user 2 can get a non-zero rate
 - $R_2^* = \log\left(1 + \frac{P_1 + P_2}{N_0}\right) - \log\left(1 + \frac{P_1}{N_0}\right) = \log\left(1 + \frac{P_2}{P_1 + N_0}\right)$
 - Namely, decode u2 in the presence of interfering u1
- Segment AB contains all the optimal sum-rate, and can be achieved via time-sharing
 - Pareto optimal



Decoding Order

- If the goal is to maximize the sum-rate, any point on AB is equally fine
- If we want to ensure **max-min fairness** such that the weak user get its best possible rate
 - Decode the stronger user first
- To **minimize the total transmit power** or increase the capacity in an **interference-limited** system
 - Decode the stronger user first

With SIC,
the near-far problem ($SNR_2 < SNR_1$) becomes an advantage
→ a far user now becomes decodable if $SNR_2 \ll SNR_1$

SIC for Multiple Users

$$y = h_1x_1 + h_2x_2 + \dots + h_Nx_N + n$$

- Repeat the following procedure iteratively
 1. Decode any user $x_i = y/h_i$
 2. Re-encode x_i (demodulate and re-modulate)
 3. Subtract the re-encoded signal from y
- The user decoded earlier is interfered by more users

Use SIC in MIMO Decoding

- Standard Zero Forcing (ZF) decoding
 - SNR reduction due to channel correlation
$$\text{SNR}_{\text{ZF}} = \text{SNR}_{\text{orig}} * \sin^2(\theta)$$
 - In 2x2 system, both streams suffer from SNR reduction if they are both decoded using ZF
- Combine ZF with SIC
 - 2x2 example
 - Decode x_2 using ZF
 - Decode x_1 using SIC

Decode x_2 Using ZF

orthogonal vectors

$$+) \quad \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} h_{11} \\ h_{21} \end{pmatrix} x_1 + \begin{pmatrix} h_{12} \\ h_{22} \end{pmatrix} x_2 + \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}$$

* h_{21}
* $-h_{11}$

$$y_1 h_{21} - y_2 h_{11} = (h_{12} h_{21} - h_{22} h_{11}) x_2 + n'$$

$$\begin{aligned} x'_2 &= \frac{y_1 h_{21} - y_2 h_{11}}{h_{12} h_{21} - h_{22} h_{11}} \\ &= x_2 + \frac{n'}{h_{12} h_{21} - h_{22} h_{11}} \\ &= x_2 + \frac{n'}{\vec{h}_2 \cdot \vec{h}_1^\perp} \end{aligned}$$

Decode x_1 Using SIC

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} h_{11} \\ h_{21} \end{pmatrix} x_1 + \begin{pmatrix} h_{12} \\ h_{22} \end{pmatrix} x_2 + \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}$$

- Re-encode x_2
- Removing x_2 and we get

$$\begin{aligned} y_1 &= h_{11}x_1 + n_1 \\ y_2 &= h_{21}x_1 + n_2 \end{aligned}$$

- Use traditional SISO decoder

$$x_1 = \frac{y_1}{h_{11}} \text{ or } x_1 = \frac{y_2}{h_{21}}$$

ZF-SIC Decoding

- Combine ZF with SIC to improve SNR
 - Decode one stream and subtract it from the received signal
 - Repeat until all the streams are recovered
 - Example: after decoding x_2 , we have $y_1 = h_1x_1+n_1$
→ decode x_1 using standard SISO decoder
- Why it achieves a higher SNR?
 - The streams recovered after SIC can be projected to a smaller subspace → lower SNR reduction
 - In the 2x2 example, x_1 can be decoded as usual without ZF → no SNR reduction (though x_2 still experience SNR loss)

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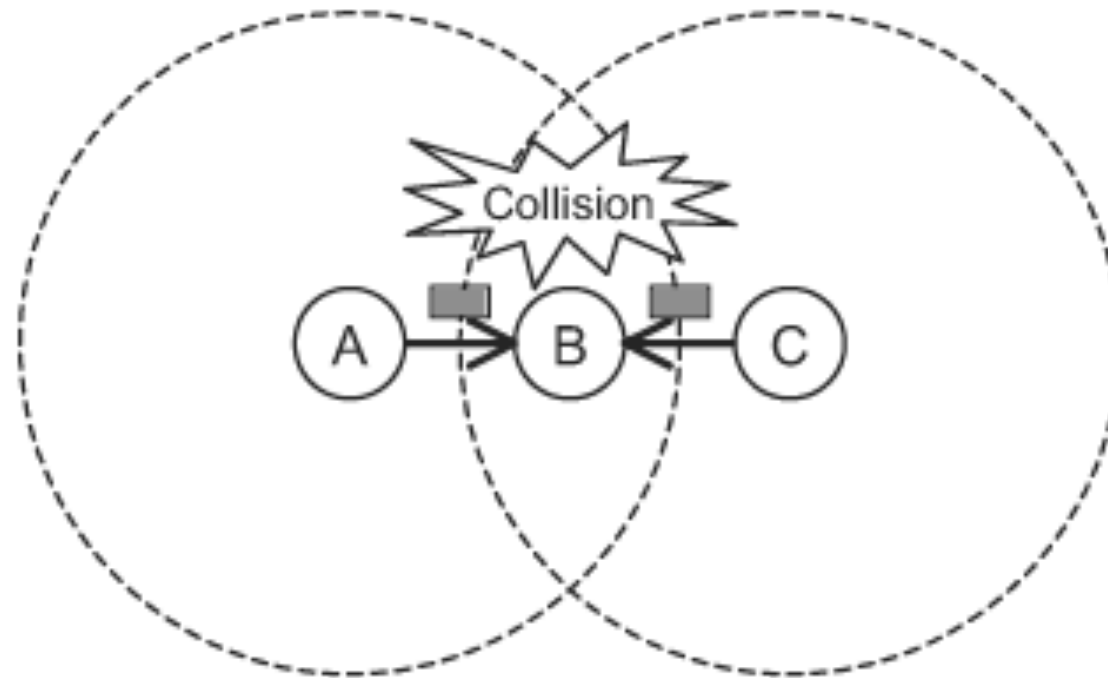
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Lecture 8: Successive Interference Cancellation

ZigZag Decoding (SIGCOMM'08)

Lecturer: Kate Ching-Ju Lin (林靖茹)

Hidden Terminal

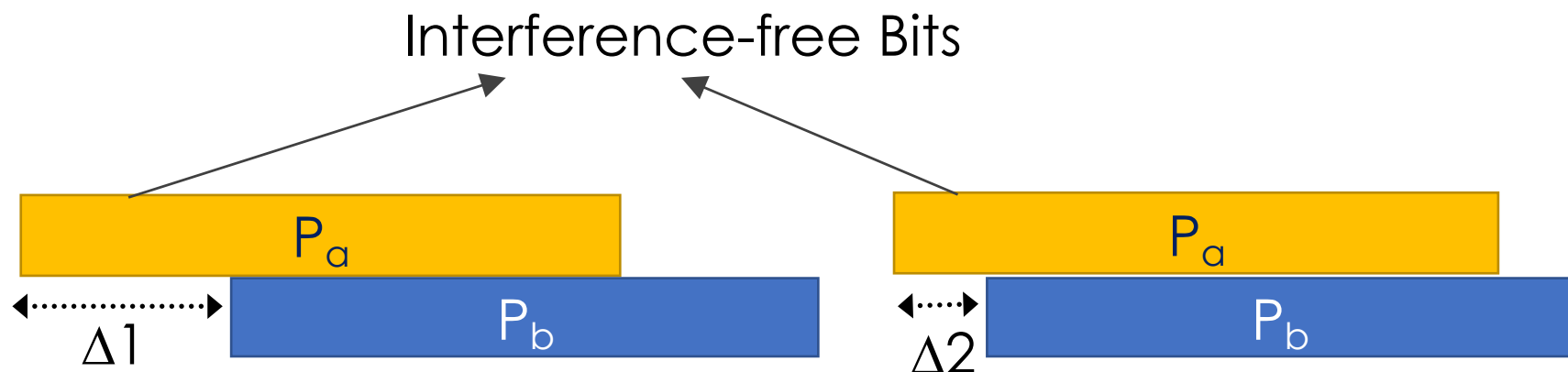


- Two nodes hidden to each other transmit at the same time, leading to collision

ZigZag

Exploits 802.11's behavior

- Retransmissions
 - Same packets collide again
- Senders use random jitters
 - Collisions start with interference-free bits



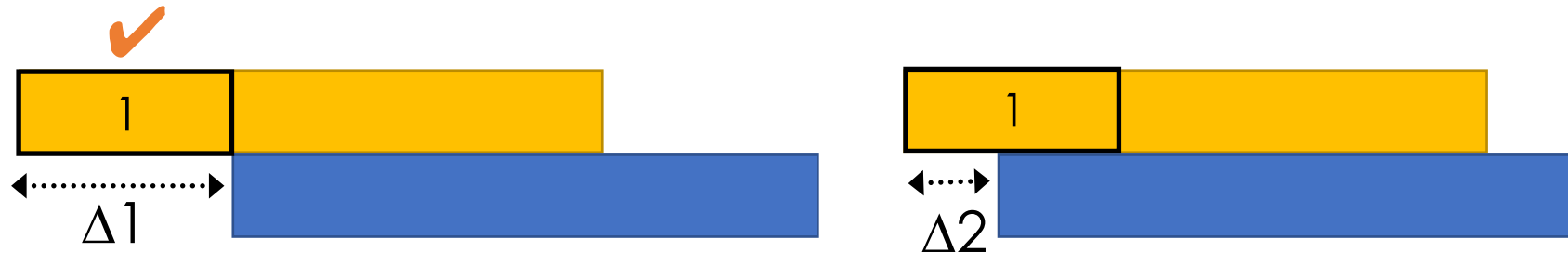
How does ZigZag Work?



$$\Delta 1 \neq \Delta 2$$

- Find a chunk that is **interference free** in one collision and has **interference** in the other

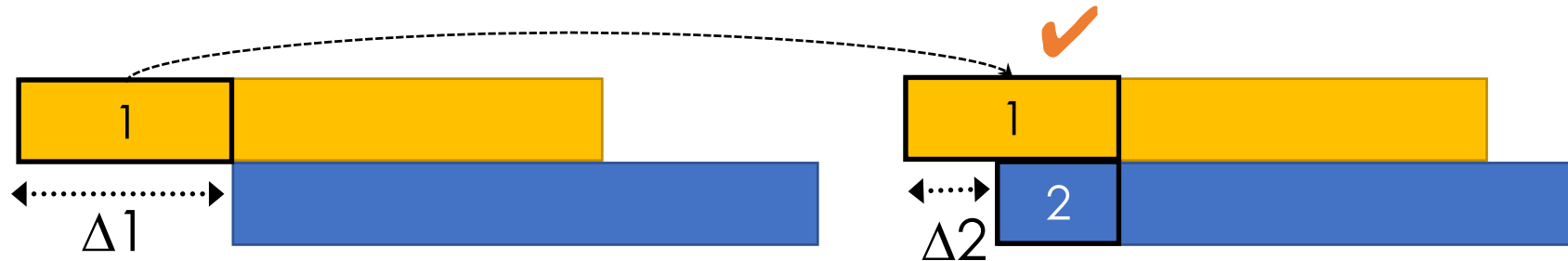
How does ZigZag Work?



$$\Delta 1 \neq \Delta 2$$

- Find a chunk that is **interference free** in one collision and has **interference** in the other
- Decode the interference-free chunk and subtract it from the other collision

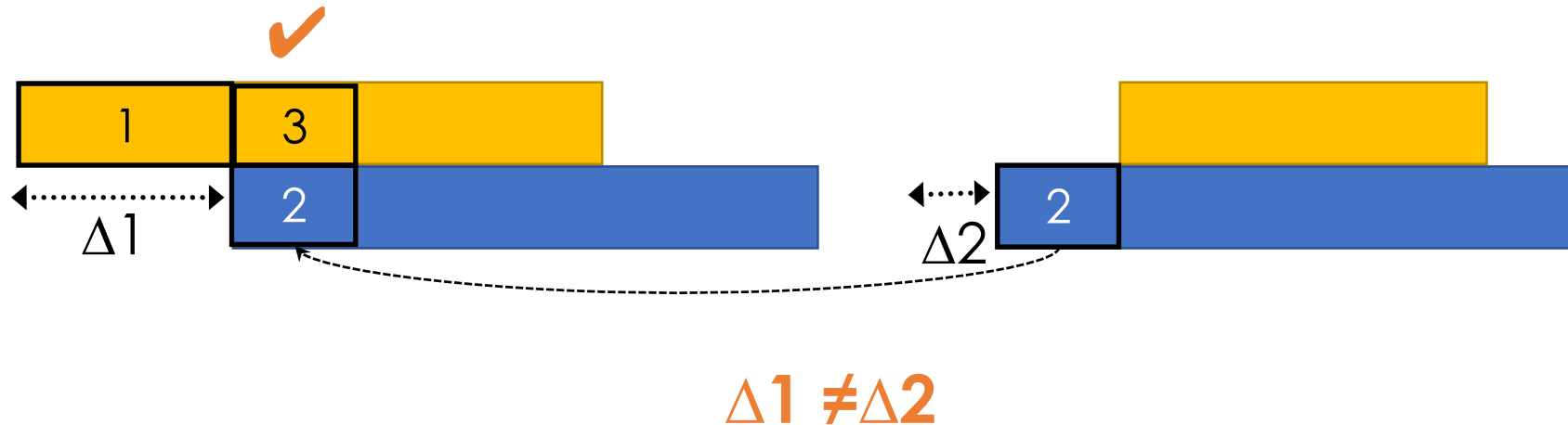
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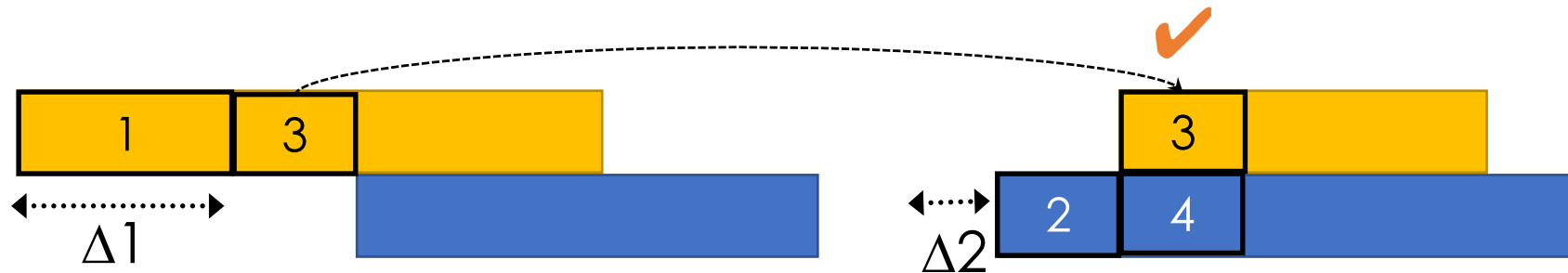
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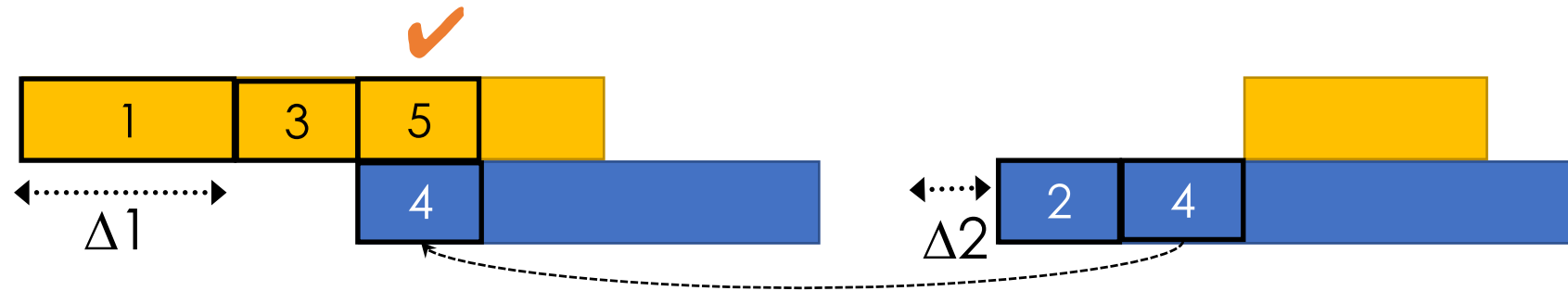
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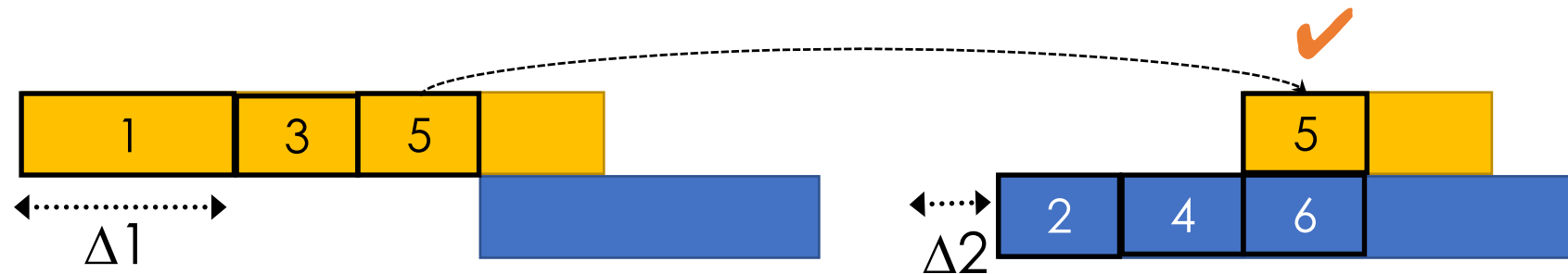
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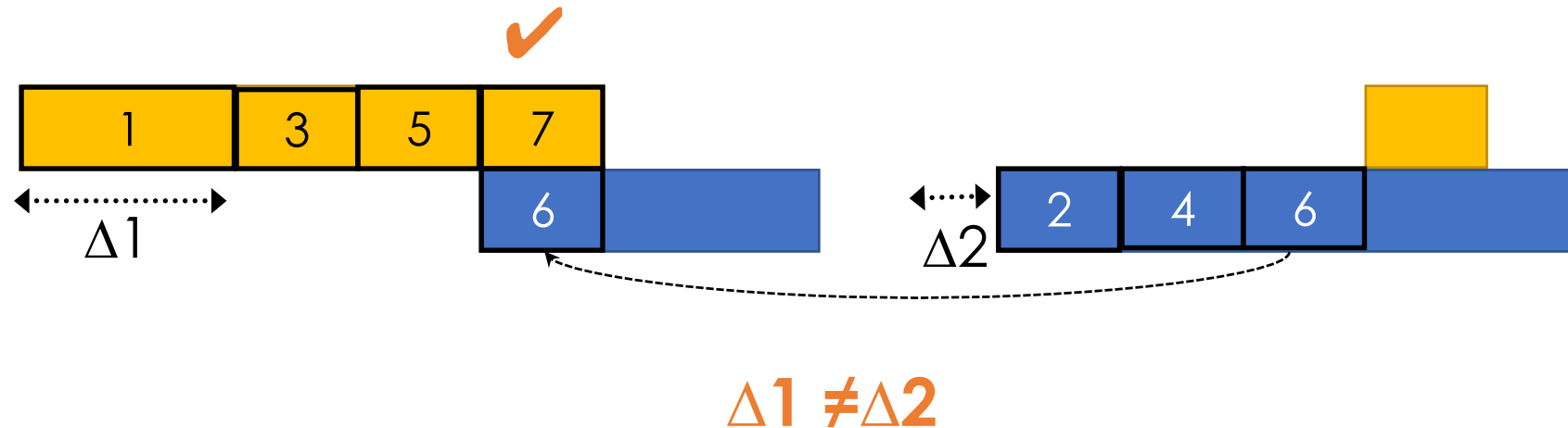
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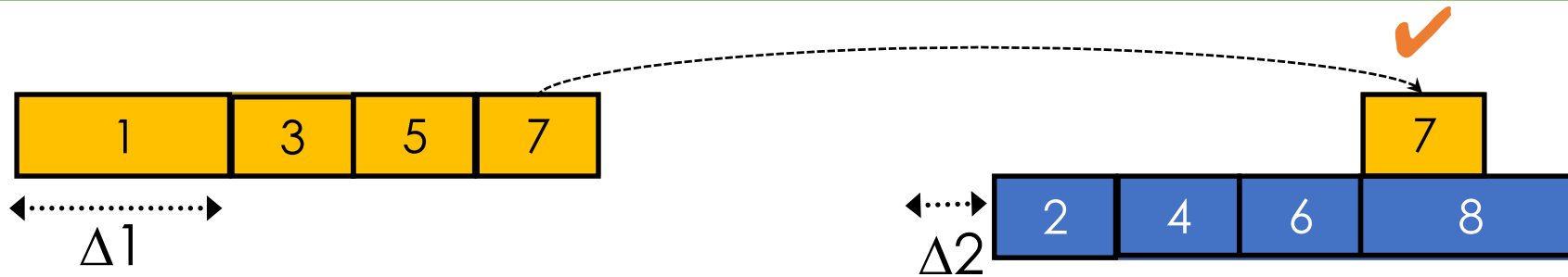
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How does ZigZag Work?



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How does ZigZag Work?



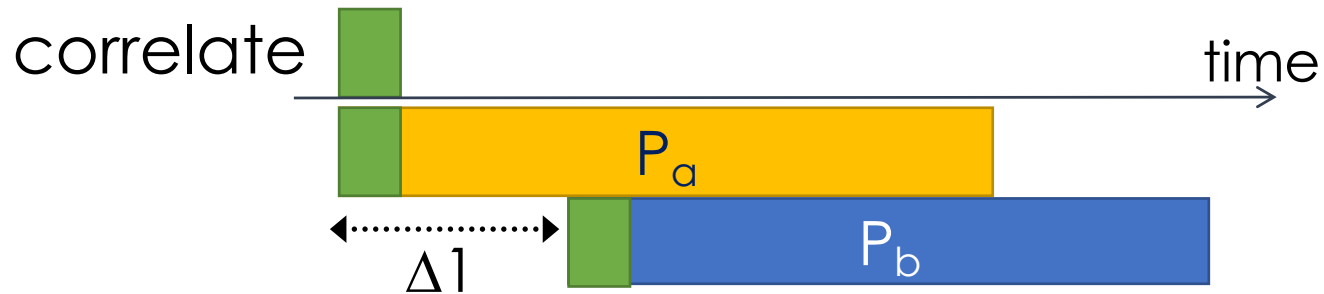
$$\Delta 1 \neq \Delta 2$$

- Deliver 2 packets in 2 timeslots
- As efficient as if the packets did not collide

Practical Issues

- How does the receiver know it is a collision and where they start?
- What if the channel has changes in the second collision?
- How to deal with error propagation?

Detecting Collisions

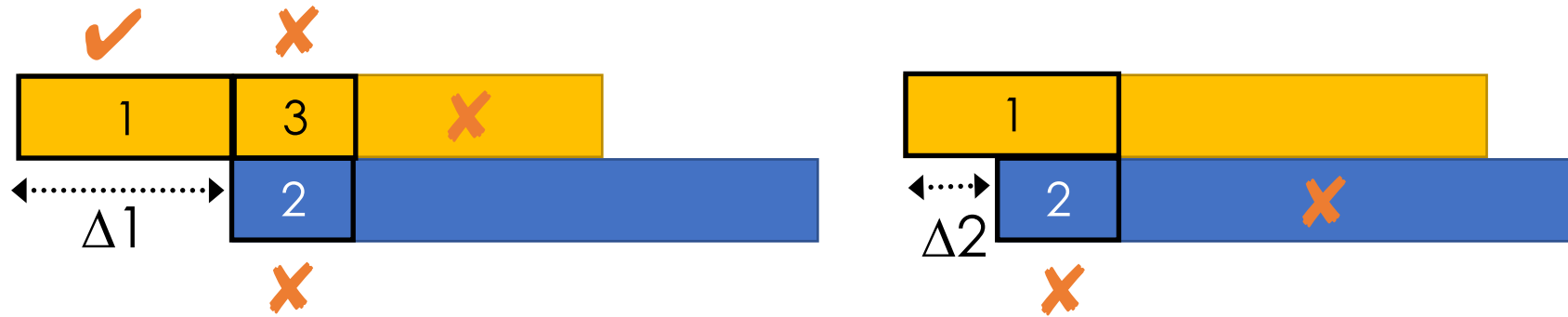


- Preamble correlation
 - Detect collision and the offset value Δ
 - Work despite interference because correlation with an independent signal (random data samples) is zero

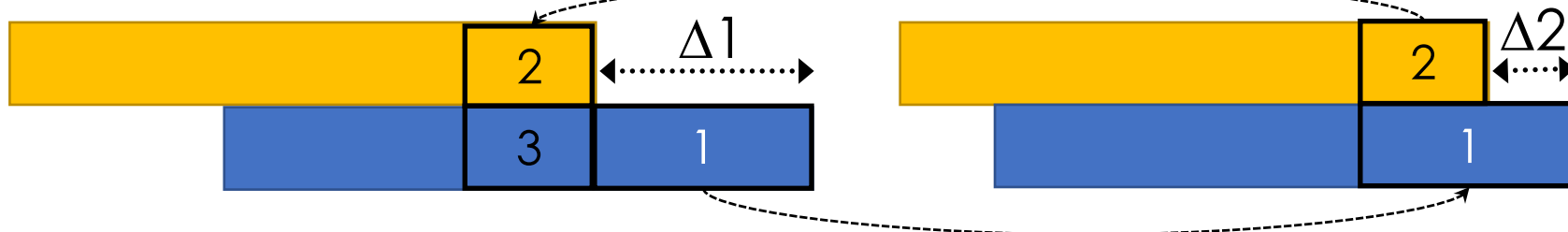
Signal Subtraction

- Channel's attenuation or phase may change between collisions
- Can't simply subtract a chunk across collisions
- Subtract as conventional SIC
 - **Decode** chunk in one collision into bits
 - Demodulate and **re-modulate** bits to get channel-free signal
 - Apply the **channel learned from the other collision** to encode the signal
 - **Subtract** it!

What if decoding errors happen?



- Error can propagate across chunks
- Cannot completely avoid the problem, but can reduce this probability via leveraging **time diversity**
 - Get two independent decodings: forward and backward



When will ZigZag Fail?

- The offsets in the two collisions happen to be the same
- A packet is sent at different bit-rates (modulation and coding schemes) in the two collisions
- Packets are modulated with OFDM
 - Symbols cannot be reliably converted the frequency domain when the colliding packets are not aligned in the symbol level
 - Lead to **inter-symbol interference**

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Lecture 8: Successive Interference Cancellation

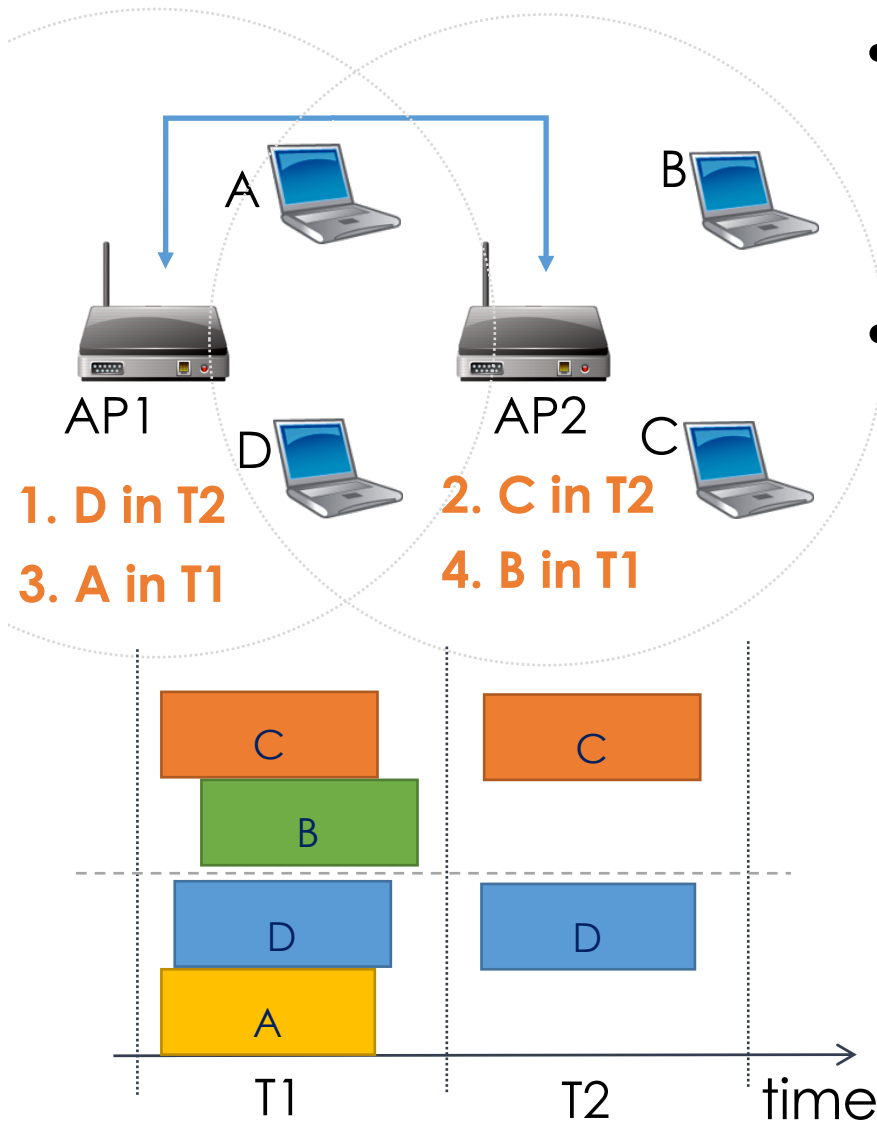
Symphony: Cooperative Packet Recovery
over the Wired Backbone(MOBICOM'13)

Lecturer: Kate Ching-Ju Lin (林靖茹)

Basic Ideas

- Allow multiple APs to cooperatively recover their collided packets
- Exchange decoded bits via the wired backbone
- Leverage the property that not all the APs will hear the same set of packets
 - An AP hears an interference-free packet can initiate SIC decoding

Example



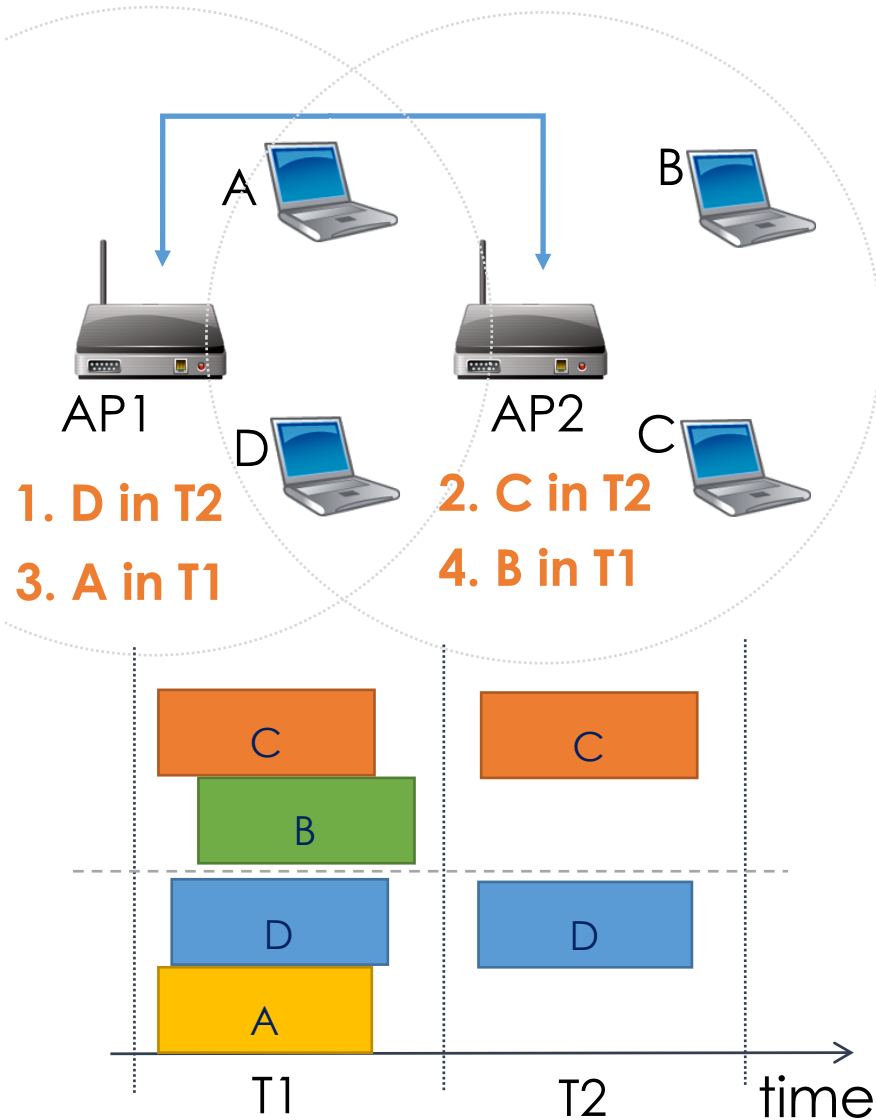
- Clients

- In T1, four nodes transmit
- In T2, C and D retransmit

- APs

1. AP1 decodes the interference-free packet from D in T2
2. AP1 forwards the bits of D to AP2 s.t. it can use SIC to recover C in T2 via SIC
3. AP1 uses SIC to subtract D in T1 and decode A
4. AP1 forwards the bits of A to AP2 s.t. it can recover B in T1

Example



- Deliver 4 packets in two slots
- TDMA: need 4 slots

Challenges

- Determine the decoding order so as to minimize the amount of traffic forwarded via the wired backbone
- Specify which clients should transmit in which time slots so as to maximize the number of transmissions
- Deal with imperfect time synchronization among APs and the latency over the backbone

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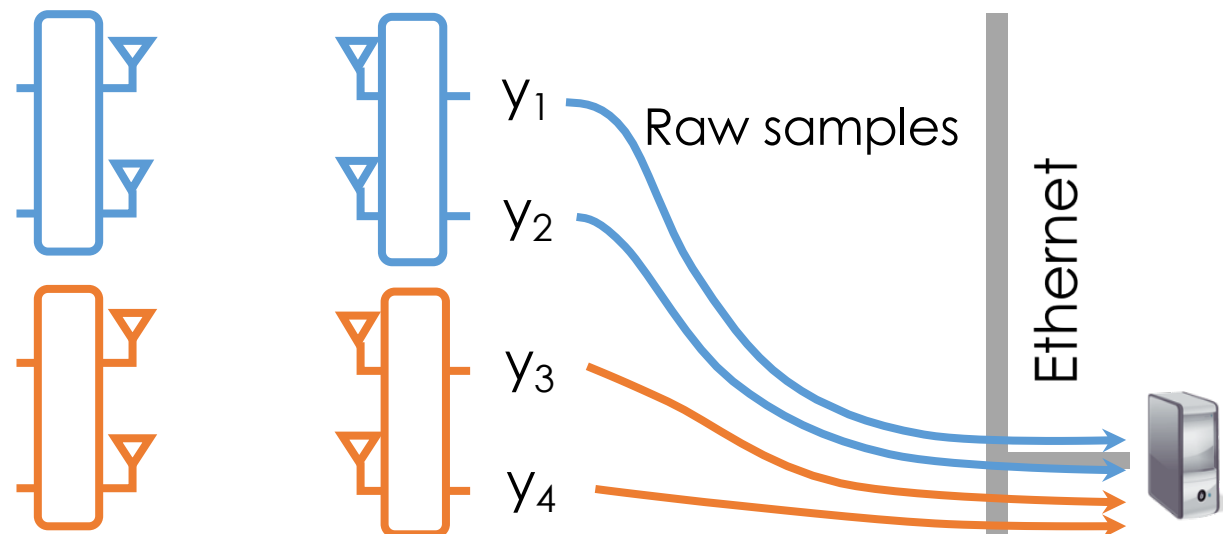
Lecture 5: Multi-User MIMO (MU-MIMO)

Interference Alignment and Cancellation (SIGCOMM'09)

Lecturer: Kate Ching-Ju Lin (林靖茹)

Naïve Cooperative MIMO

- Say we combine two 2-antenna APs as a 4-antenna virtual AP
- Naïve solution:
 - Connect the two APs to a server via Ethernet
 - Each physical AP sends every received raw signal (complex values) to the server over Ethernet

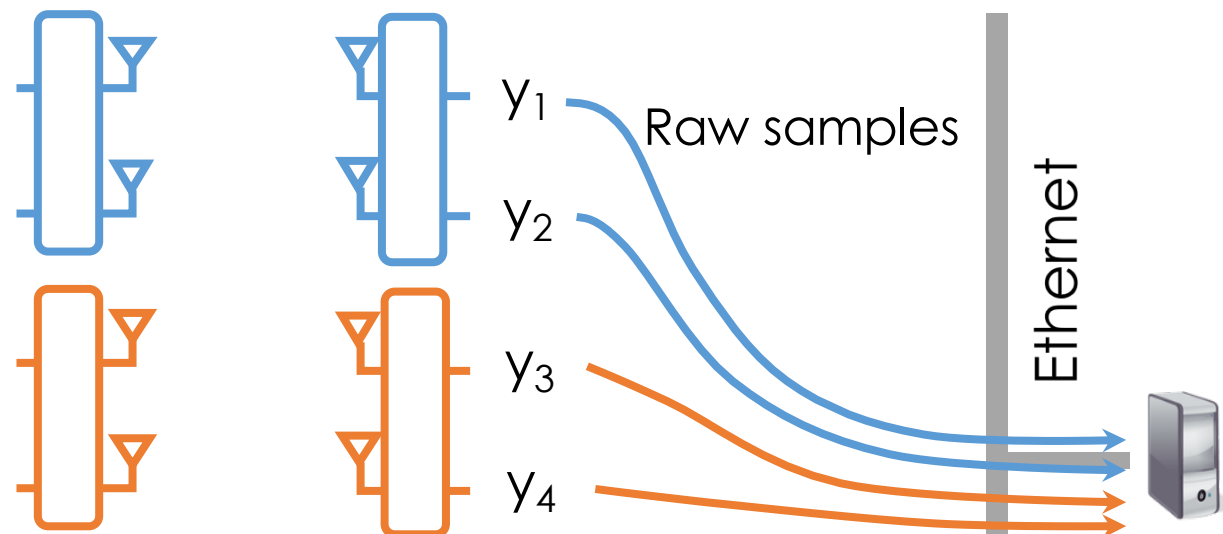


Naïve Cooperative MIMO

Impractical overhead:

For example, a 3 or 4-antenna system needs 10's of Gb/s

- Naïve solution:
 - Connect the two APs to a server via Ethernet
 - Each physical AP sends every received raw signal (complex values) to the server over Ethernet



How to Minimize Ethernet Overhead?

- High-level idea:

1. Decode some packets in certain AP
2. Forward the decoded packets through the Ethernet to other APs
3. Other APs decode the remaining packets
4. Repeat 1-3 until all packets are recovered

How to Minimize Ethernet Overhead?

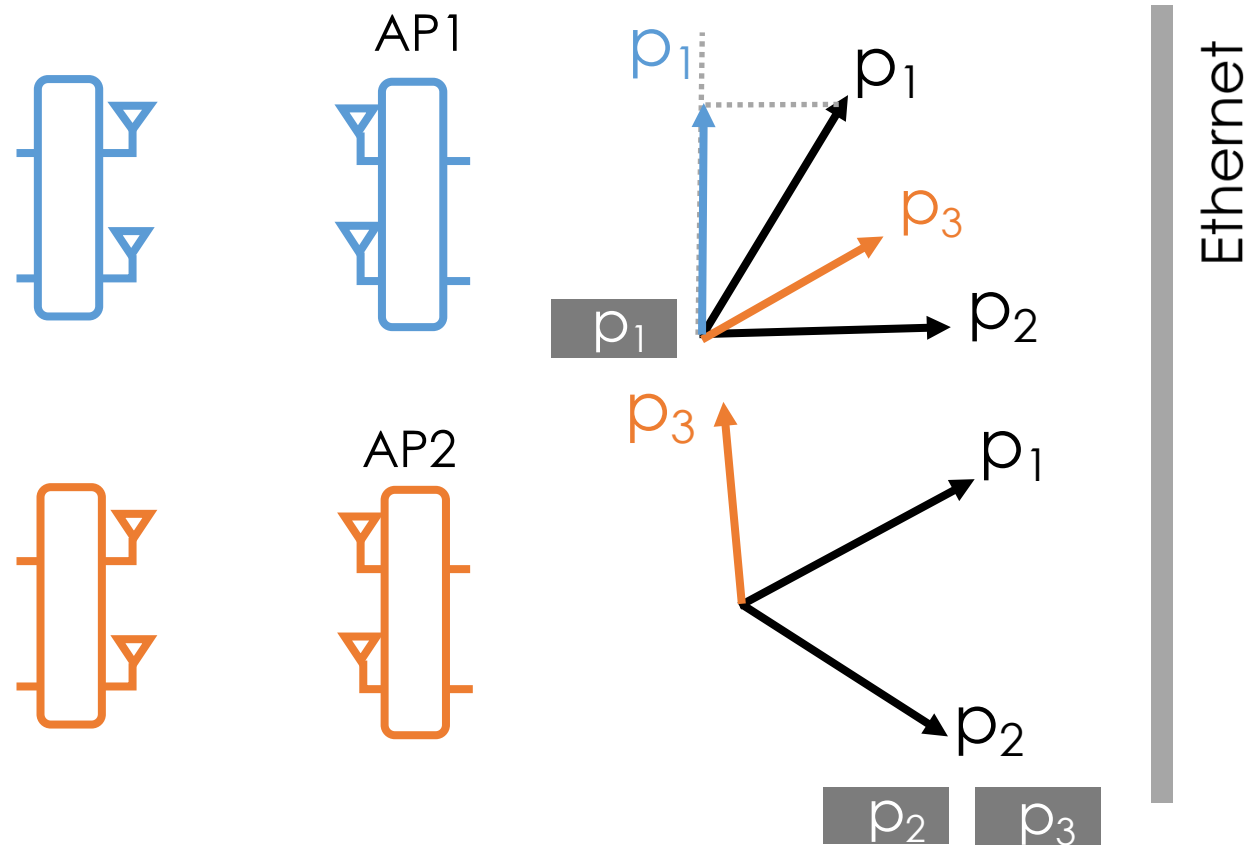
- Advantage:

- The size of data packets is much smaller than the size of raw samples → minimize overhead

- Challenge:

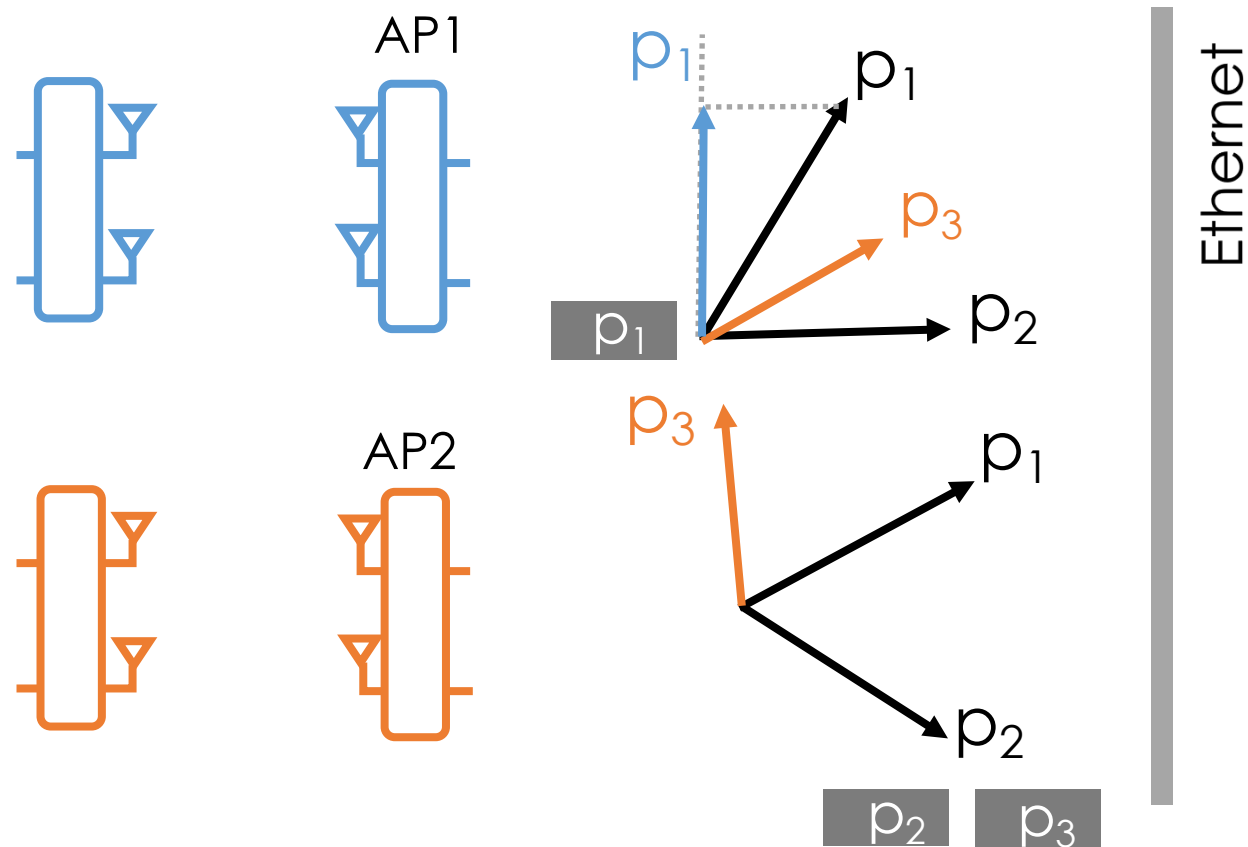
- In theory, an N-antenna AP cannot recover M concurrent transmissions if $M > N$
- How can an N-antenna AP recover its packet from M concurrent transmissions ($M > N$)?
→ **Interference Alignment and Cancellation**

Interference Alignment and Cancellation



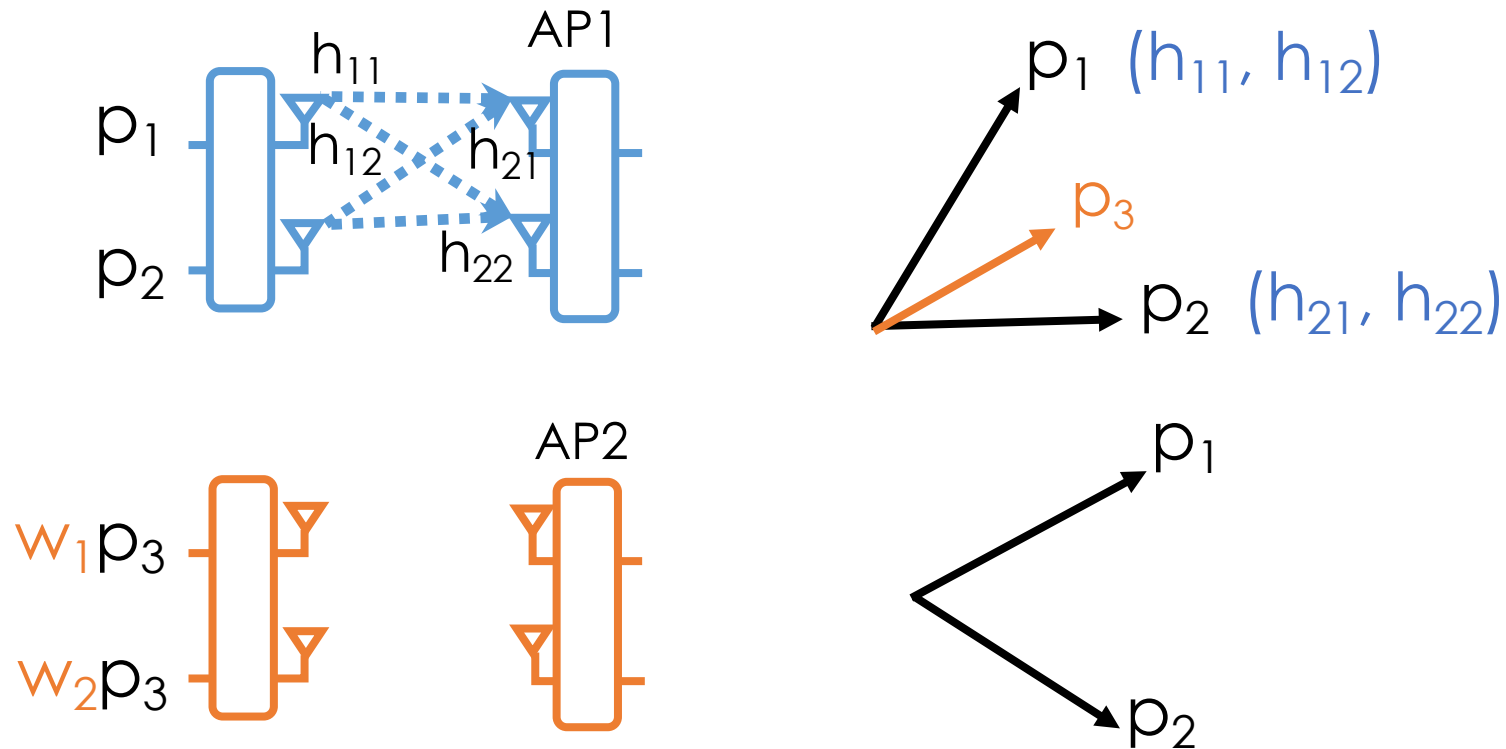
- **Align** p_3 with p_2 at AP1
- AP1 broadcasts p_1 on Ethernet
- AP2 subtracts/**cancel**s $p_1 \rightarrow$ decodes p_2, p_3

Interference Alignment and Cancellation



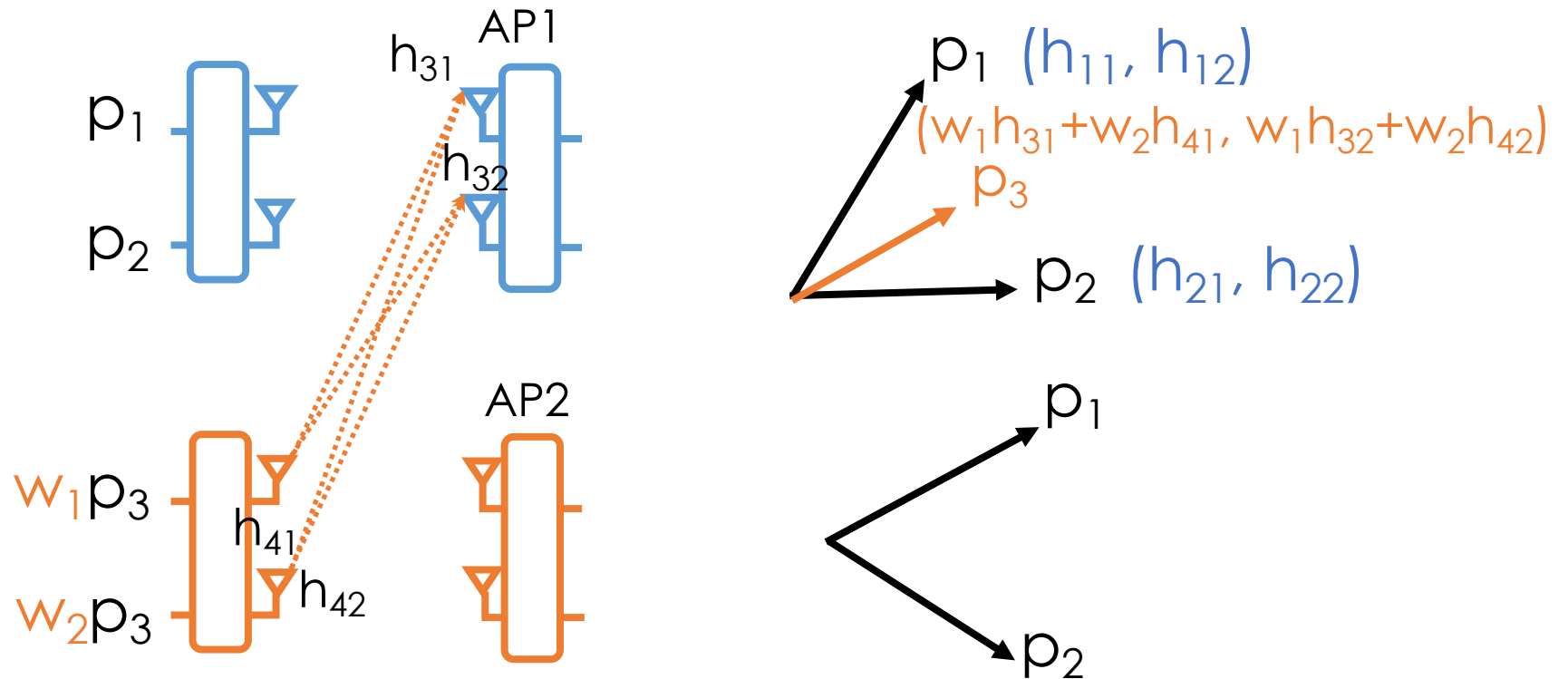
Only forward **1 data packet** through the Ethernet!

How to Align?



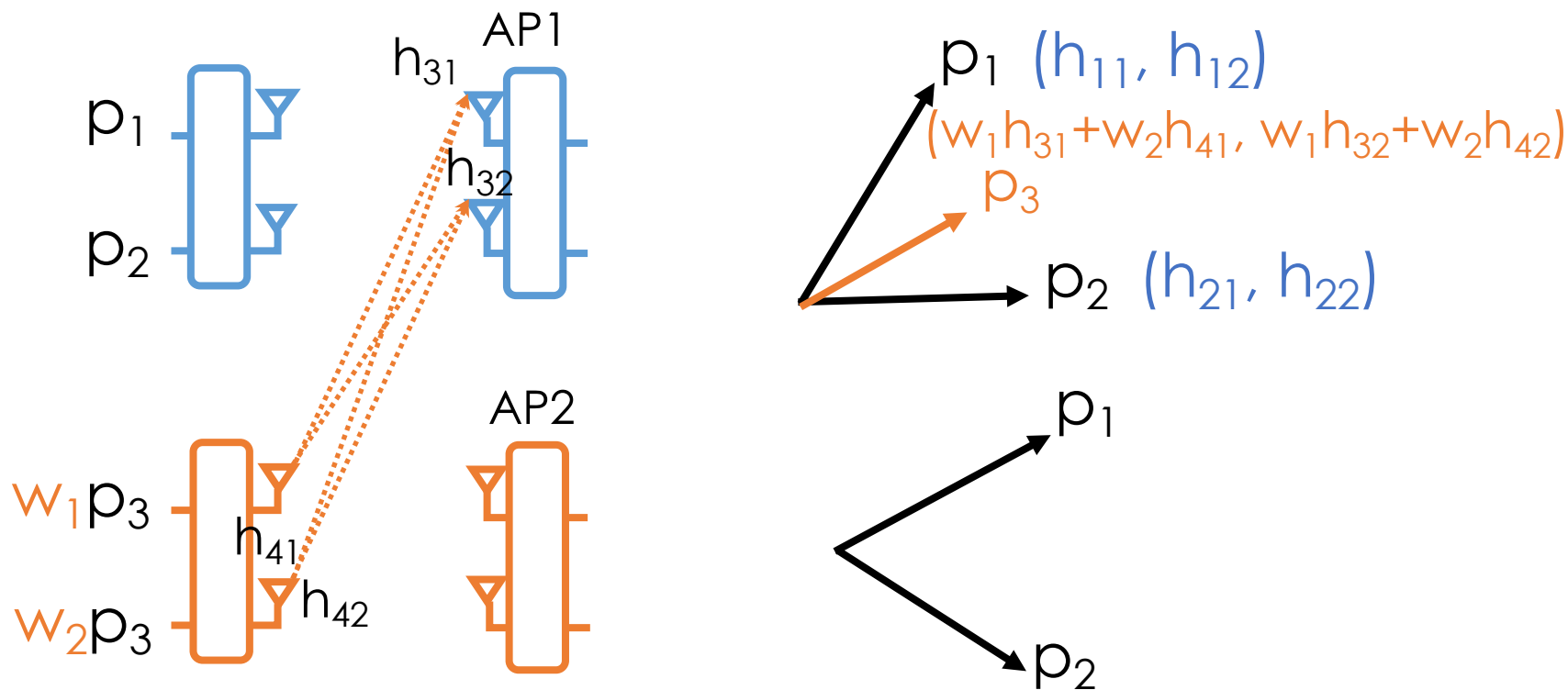
1. Learn the direction we need to align
 - Client 2 aligns p_3 along (h_{21}, h_{22}) at AP1

How to Align?



2. Precode p_3 by (w_1, w_2)
3. AP2 receives p_3 along the direction $(w_1h_{31} + w_2h_{41}, w_1h_{32} + w_2h_{42})$

How to Align?



4. Since AP1 tries to decode p1, we align the interference p3 along the direction of p2

→ Let $(w_1 h_{31} + w_2 h_{41}) / (w_1 h_{32} + w_2 h_{42}) = h_{21} / h_{22}$

Infinite number of solution?

No! **power constraint** $w_1^2 + w_2^2 = P_{\max}$

How to Remove Interference?

- For example, how can AP2 remove the interference from p_1 ?
- Cannot just subtract the bits of p_1 from the received packet
 - Should subtract interference signals as received by AP2
- How? → Similar to SIC
 - AP2 **re-modulates** p_1 's bits
 - AP2 **estimate the channel** from client 1 to AP2 and **apply the learned channel** on the re-modulated signals of p_1
 - **Subtract** it from the received signal y

How to Generalize to M-Antenna MIMO?

Theorem

In a M- antenna MIMO system, IAC delivers

- *2M concurrent packets on uplink*
- *$\max\{2M-2, 3M/2\}$ concurrent packets on downlink*

e.g., M=2 antennas $\left\{ \begin{array}{l} 4 \text{ packets on uplink} \\ 3 \text{ packets on downlink} \end{array} \right.$

[See the paper for the details!](#)