

# Wireless Communication Systems

## @CS.NCTU

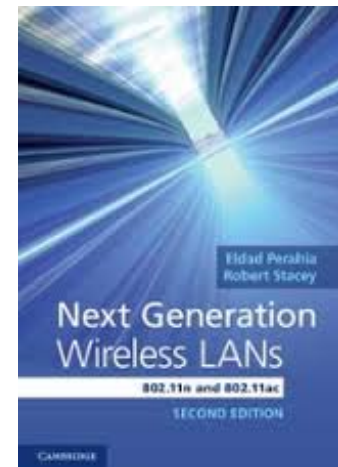
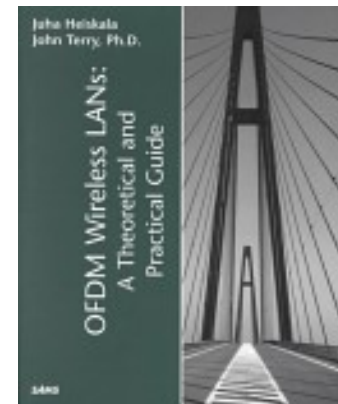
Lecture 3: 802.11 PHY and OFDM

Instructor: Kate Ching-Ju Lin (林靖茹)

# Reference

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1. OFDM Tutorial  
online:  
<http://home.iitj.ac.in/~ramana/ofdm-tutorial.pdf>
2. OFDM Wireless LANs: A Theoretical and Practical Guide  
By John Terry, Juha Heiskala
3. Next Generation Wireless LANs: 802.11n and 802.11ac  
By Eldad Perahia



# Agenda

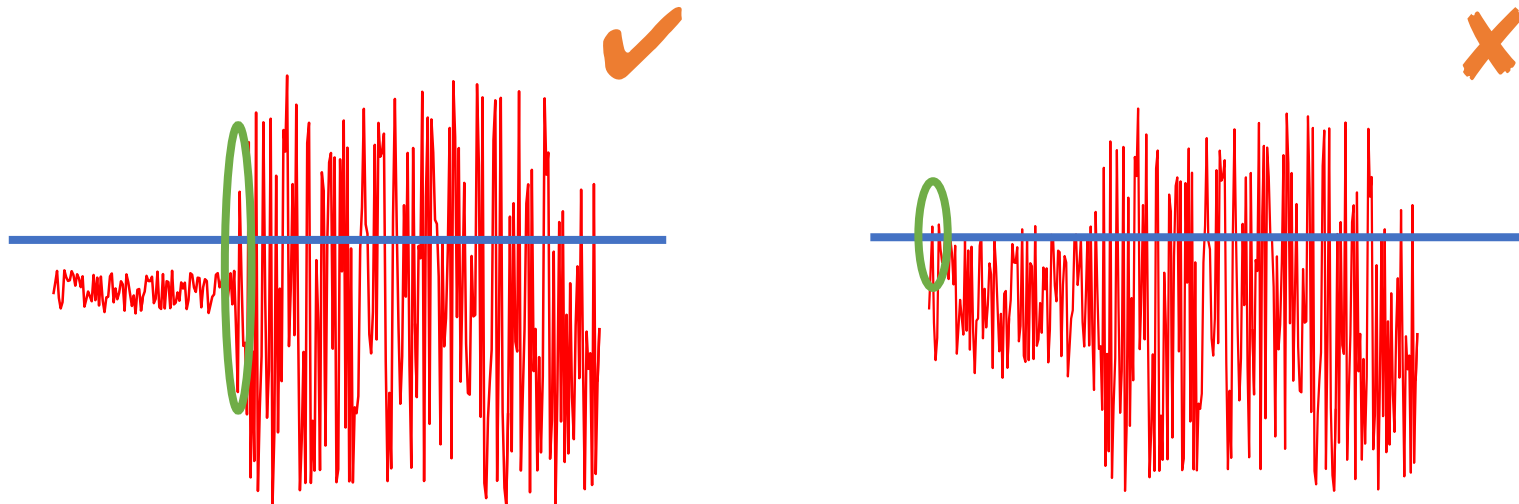
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- Packet Detection
- OFDM  
(Orthogonal Frequency Division Modulation)
- Synchronization

# What is Packet Detection

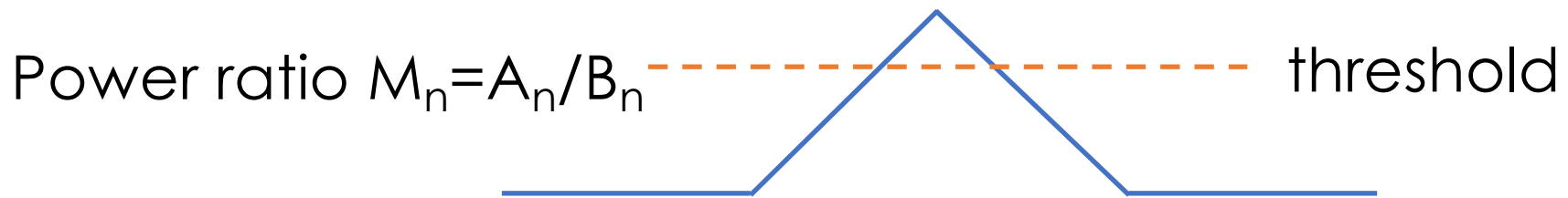
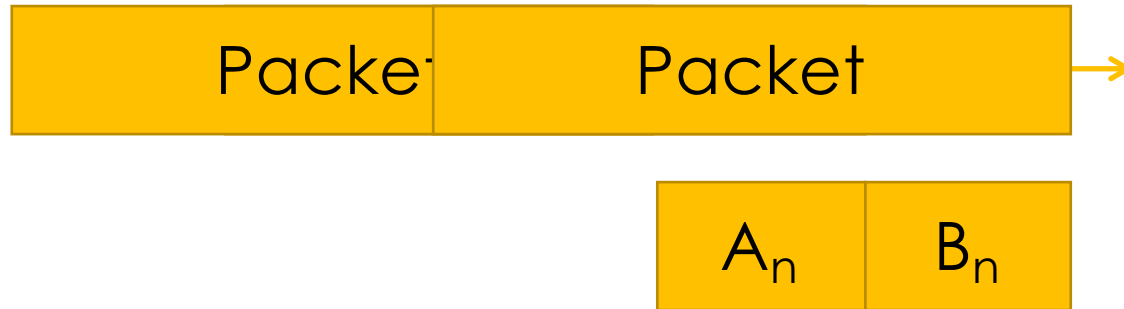
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- Detect where is the starting time of a packet
- It might be easy to detect visually, but how can a device automatically find it?
  - Simplest way: find the energy burst using a threshold
  - Difficulty: hard to determine a good threshold



# Packet Detection

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- Double sliding window packet detection
- Optimal threshold depends on the receiving power

# Packet Detection in 802.11

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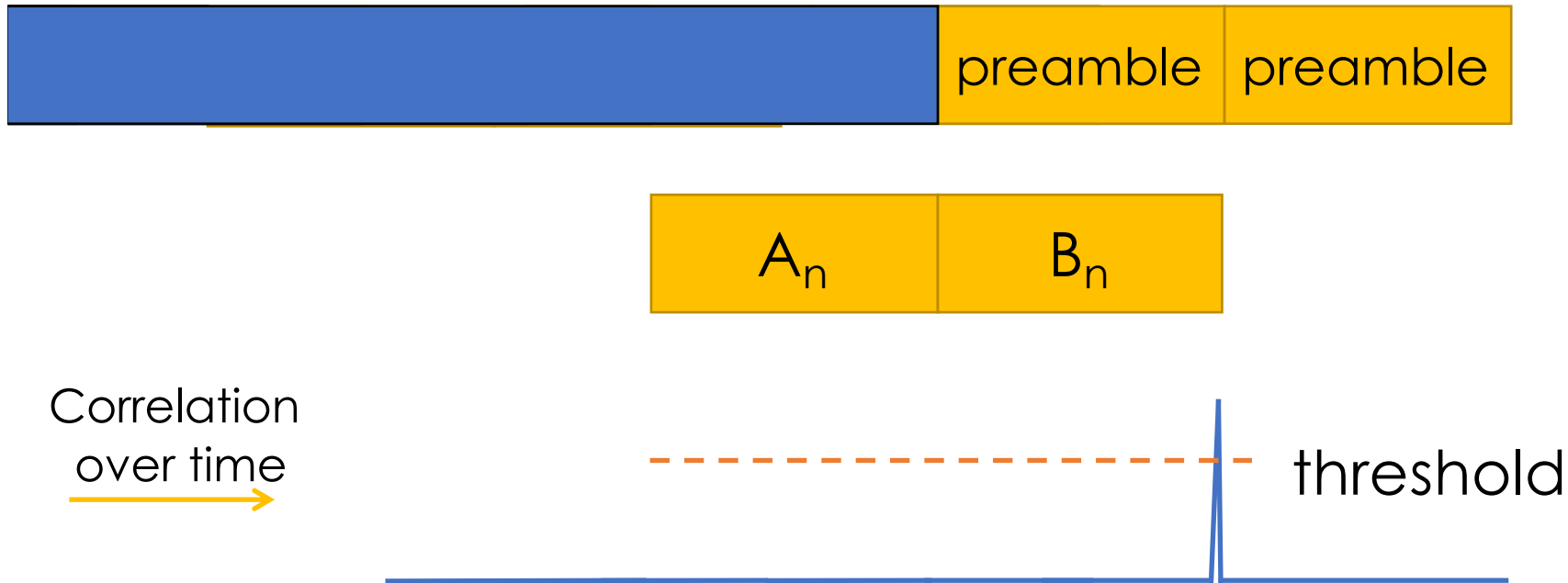
- Each packet starts with a preamble
  - First part of the preamble is exactly the same with the second part



- Use cross-correlation to detect the preamble
  - Use double sliding window to calculate the auto-correlation of the signals received in two windows
  - Leverage the key properties: 1) noise is uncorrelated with the preamble, and 2) data payload is also uncorrelated with the preamble

# Packet Detection in 802.11

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- Noise is uncorrelated with noise
- Noise is uncorrelated with preamble
- Get a peak exactly when the double windows receives the entire preamble
- Data is again uncorrelated with noise

# Agenda

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- Packet Detection
- OFDM  
(Orthogonal Frequency Division Modulation)
- Synchronization



# Narrow-Band Channel Model

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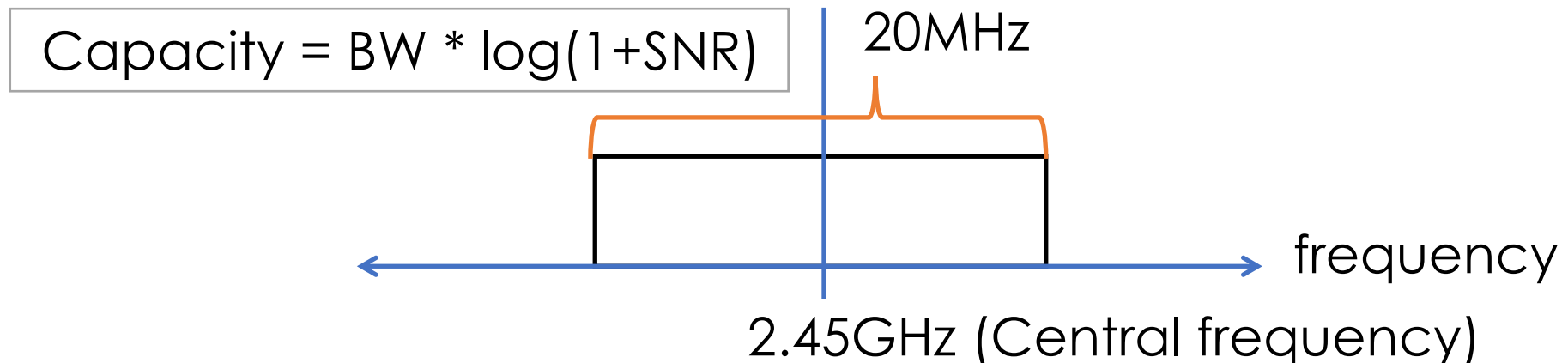
- Signal over wireless channels
  - $y[n] = Hx[n]$
- $H = a^* \exp^{2j\pi f \delta}$  is the channel between Tx and Rx
  - $a$ : received amplitude,  $\delta$ : propagation delay
- How to decode  $x[n]$ ?
  - $x[n] = y[n]/H$
  - How to learn  $H$ ?
  - Re-use the **known** preamble to learn  $H$ 
    - since  $y[n] = Hp[n]$ , we get  $H = y[n]/p[n]$

The procedure of finding  $H$  is called **channel estimation**

# Why OFDM?

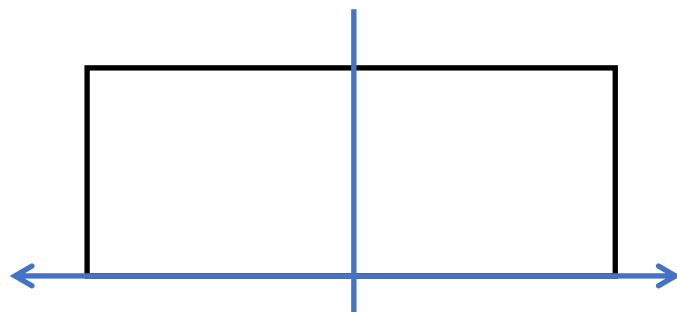
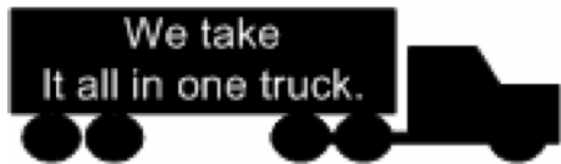
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- Signal over wireless channels
  - $y[n] = Hx[n] \rightarrow$  Decoding:  $x[n] = y[n]/H$
- Work only for narrow-band channels, but not for wide-band channels, e.g., 20 MHz for 802.11
  - Channels of different narrow bands will be different!



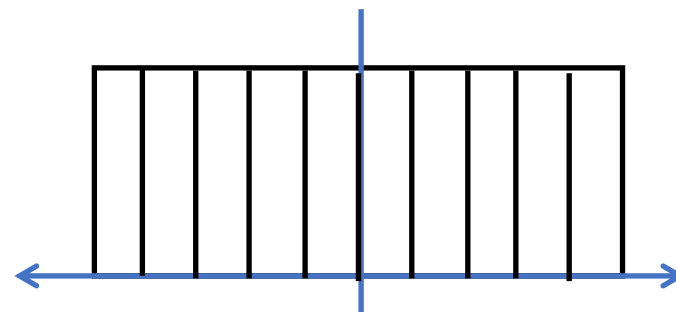
# Basic Concept of OFDM

Wide-band channel



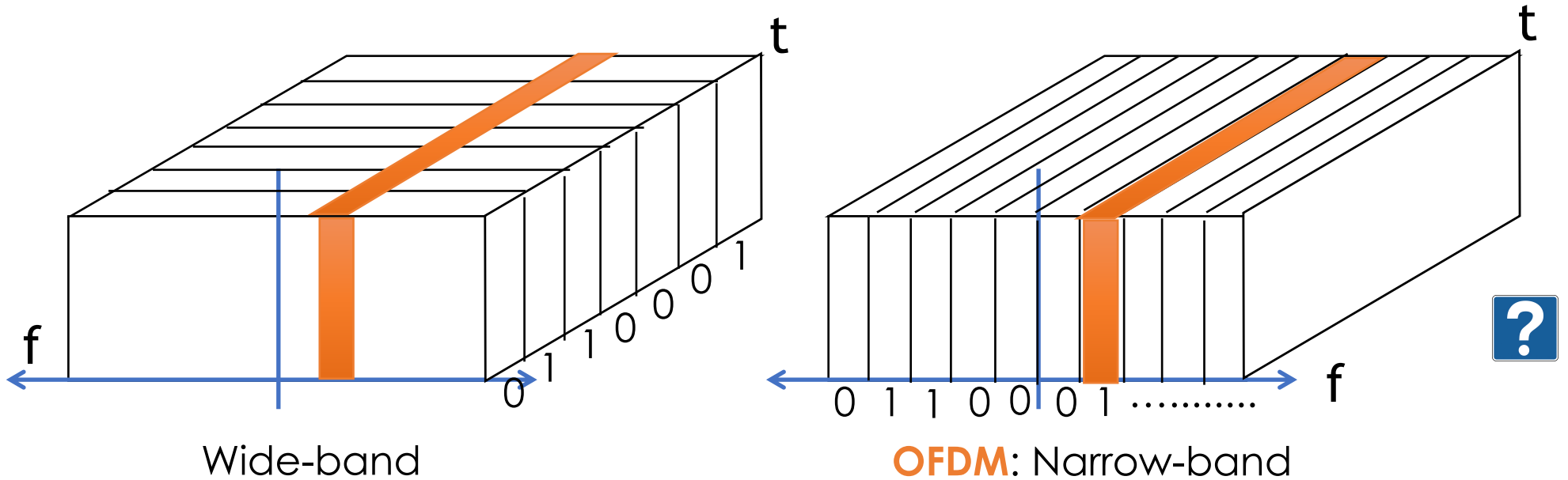
Send a sample using the entire band

Multiple narrow-band channels



Send samples concurrently using multiple **orthogonal sub-channels**

# Why OFDM is Better?

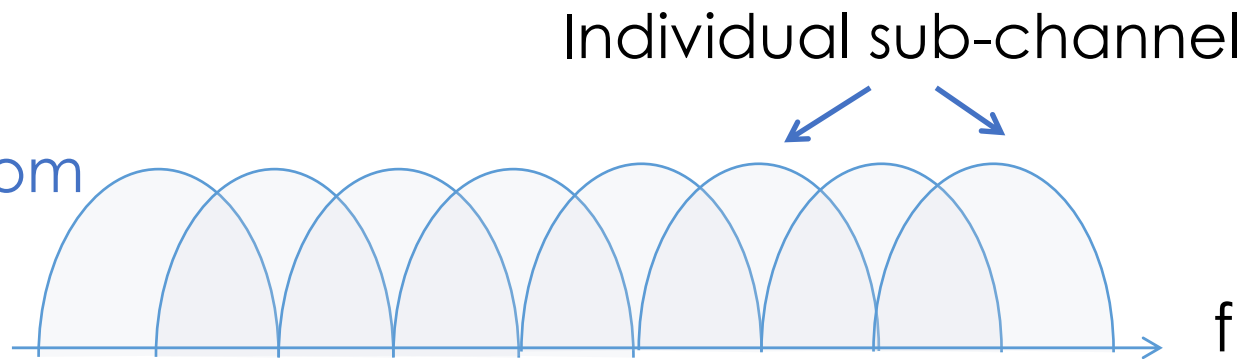


- Multiple sub-channels (sub-carriers) carry samples sent at a lower rate
  - Almost same bandwidth with wide-band channel
- Only some of the sub-channels are affected by interferers or multi-path effect

# Importance of Orthogonality

- Why not just use FDM (frequency division multiplexing)
  - Not orthogonal

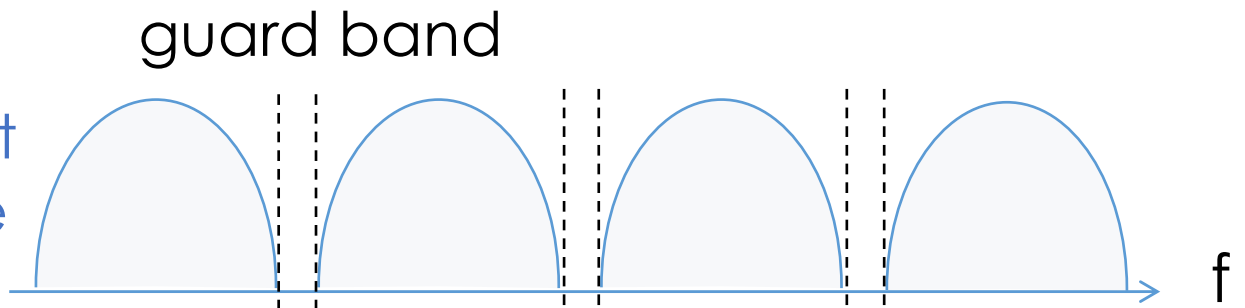
Leakage interference from adjacent sub-channels



- Need **guard bands** between adjacent frequency bands → extra overhead and lower utilization

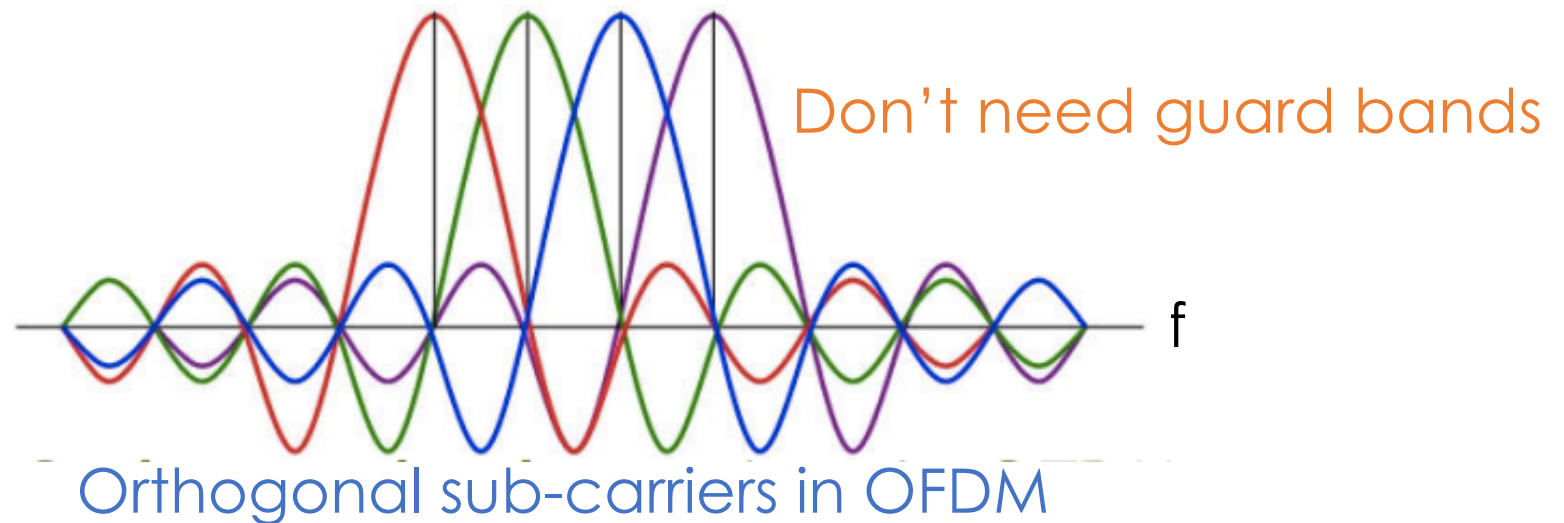
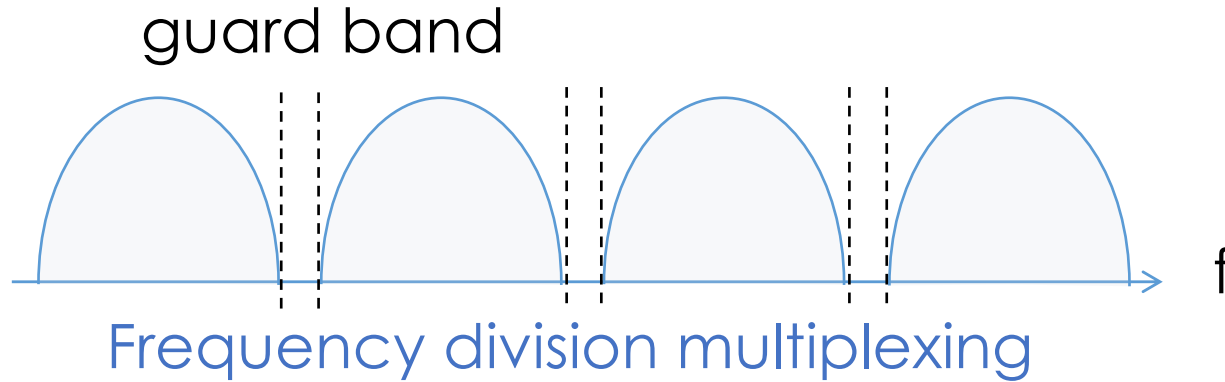


Guard bands protect leakage interference



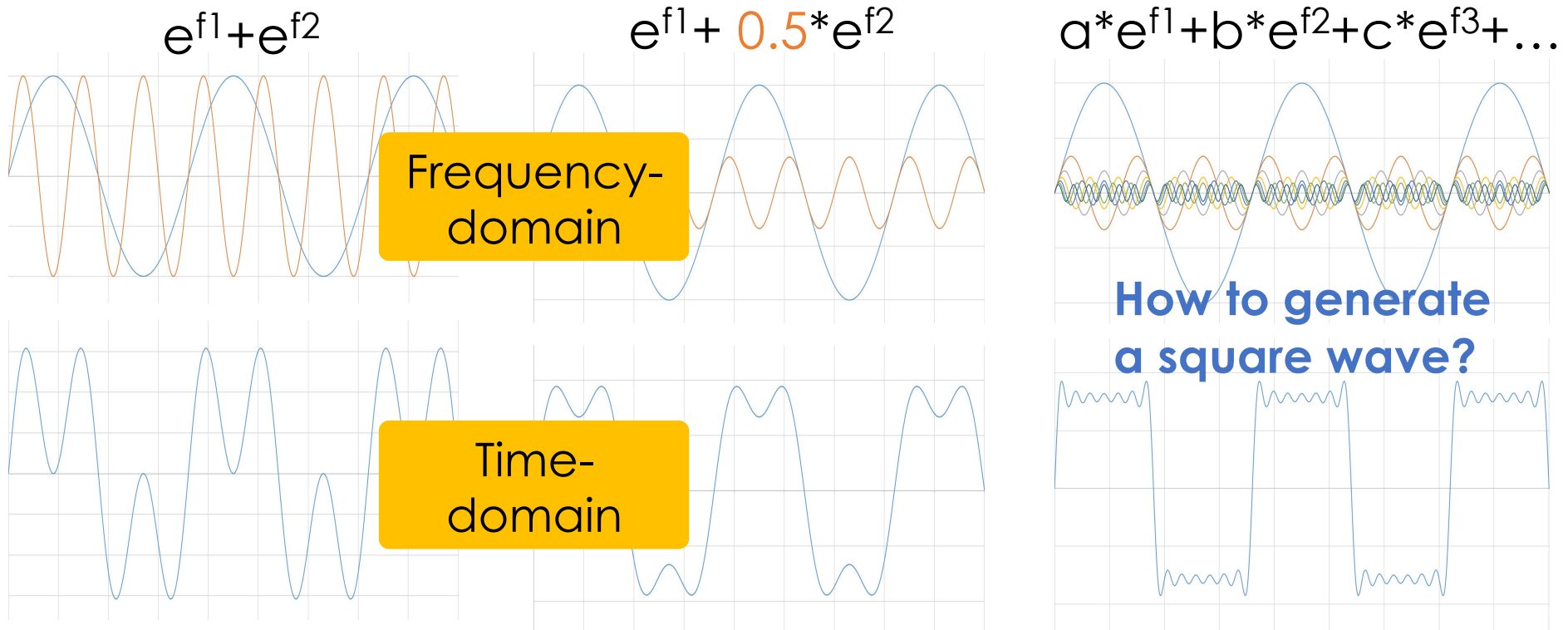
# Difference between FDM and OFDM

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# Key to Achieve Orthogonality: FFT

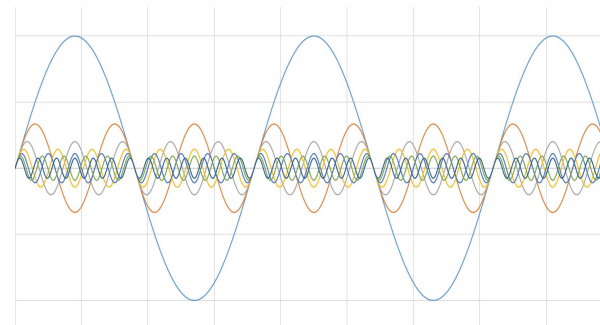
- Fast Fourier Transform (FFT)
- **All Signal Are the Sum of Sines**
  - Fourier's theorem: **ANY** waveform in the time domain can be represented by the weighted sum of sines



# Primer of FFT/iFFT

- iFFT: from frequency-domain signals to time-domain signals
- FFT: from time-domain signals to frequency-domain signals

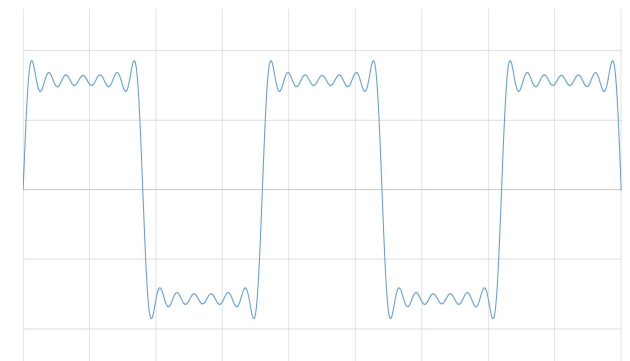
**Frequency-domain signal:**  
Amplitude of each freq.  
a, b, c, d, ...



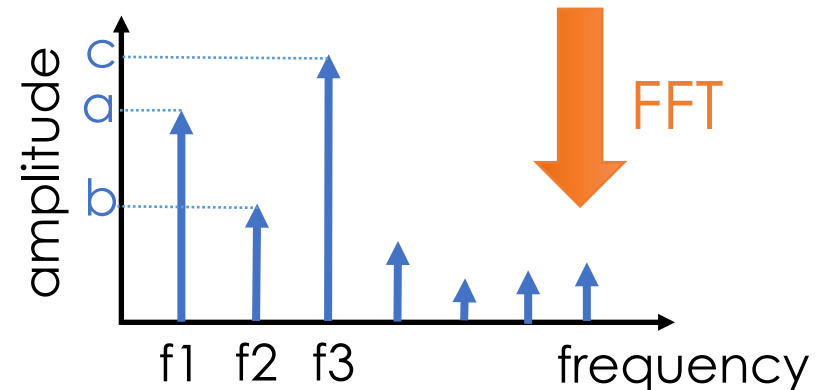
iFFT



**time-domain signal**



How can we know the frequency-domain components (**a, b, c, ...**) from this time-domain signal?

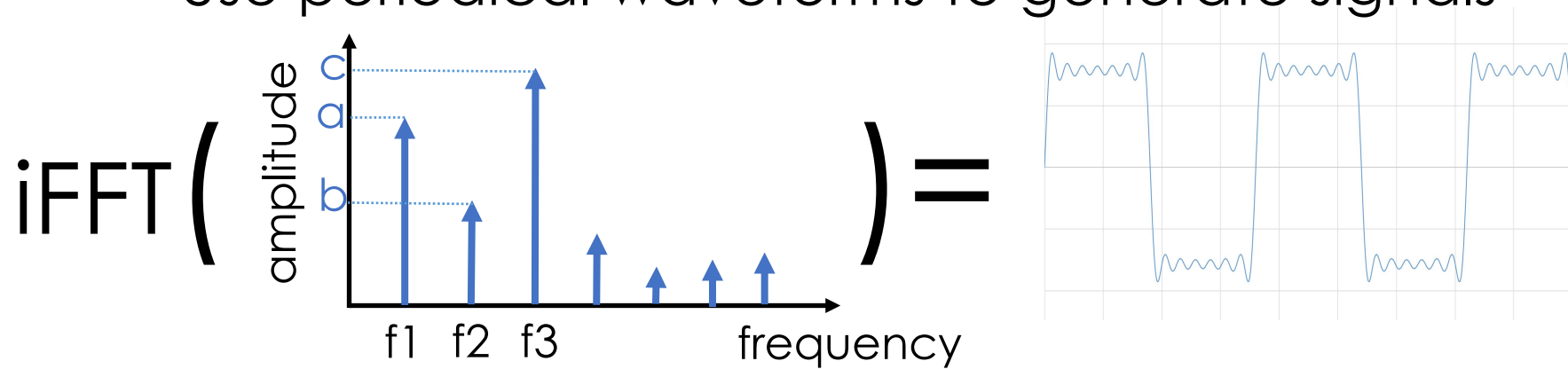




# Primer of FFT/iFFT

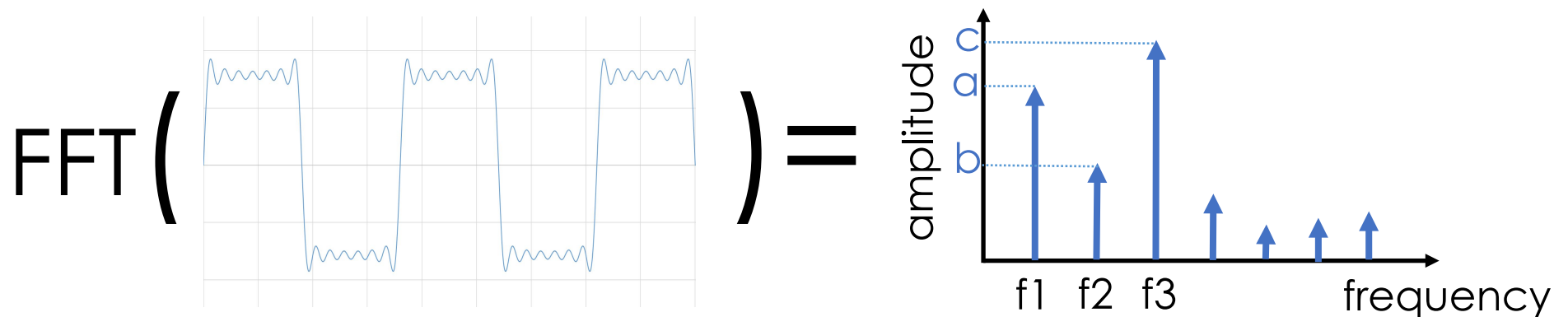
- iFFT: from frequency to time

– Use periodical waveforms to generate signals

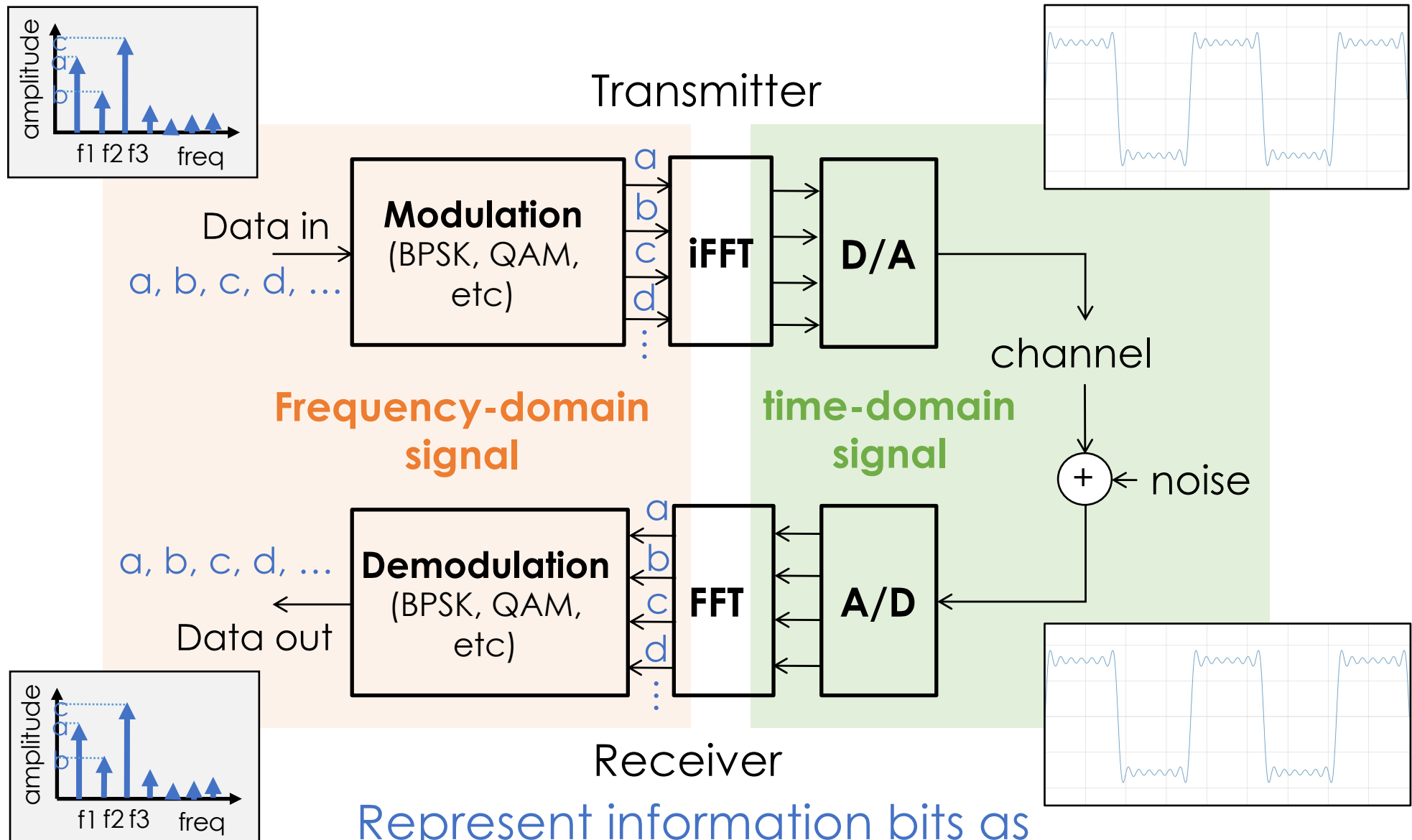


- FFT: from time to frequency

– Extract frequency components of any signal



# OFDM Transmitter and Receiver



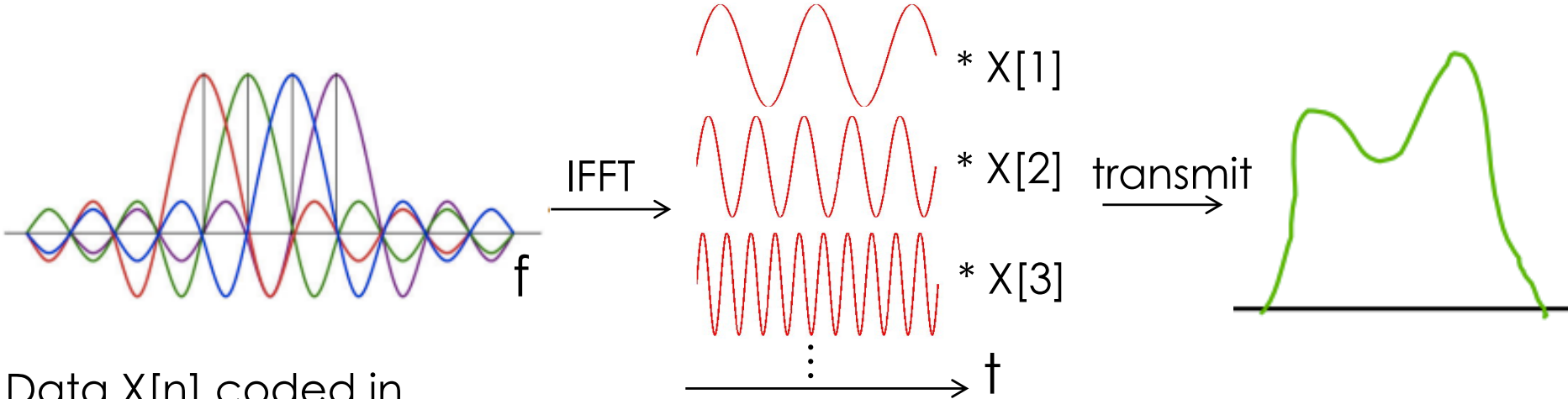
Represent information bits as the amplitudes of orthogonal subcarriers

# OFDM Basic



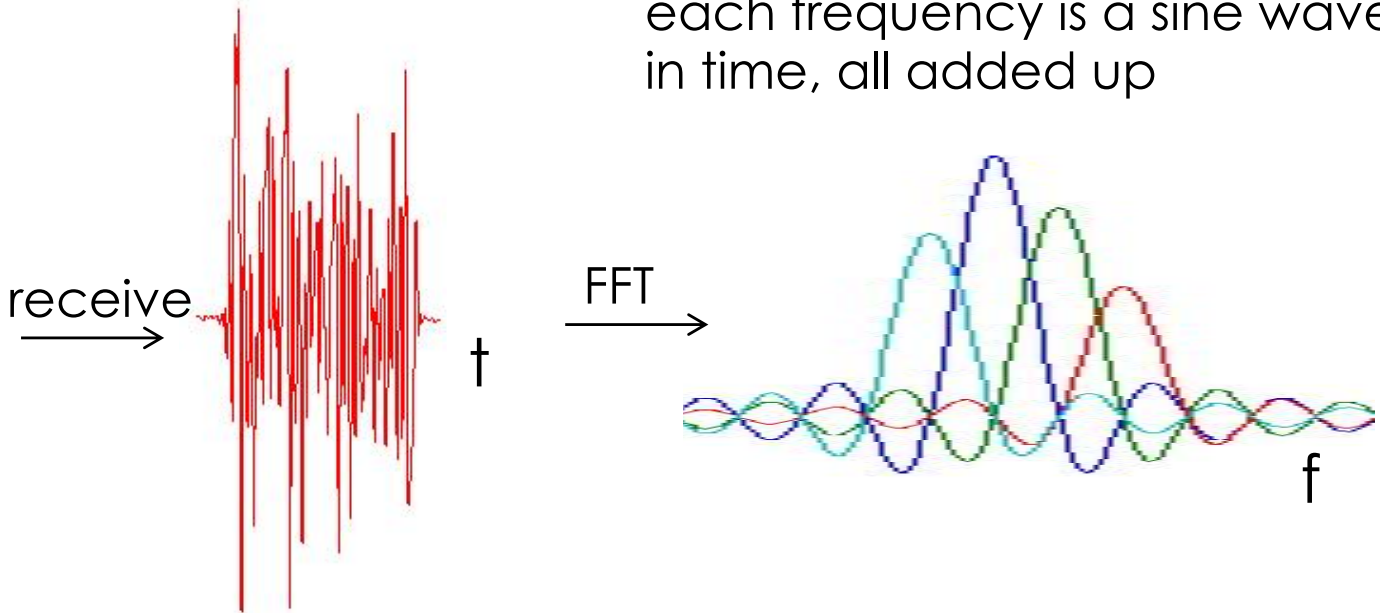
1. Partition the wide band to multiple narrow sub-carriers  $f_1, f_2, f_3, \dots, f_N$
2. Represent information bits as the **frequency-domain signal** (amplitude of each sub-carrier)
  - Example: if we want to send 1, -1, 1, 1, we let 1, -1, 1, 1 be the frequency-domain signals
3. Use iFFT to convert the information to the **time-domain sent over the air**
  - Example: Transmit  $1 \cdot e^{f_1} + (-1) \cdot e^{f_2} + 1 \cdot e^{f_3} + 1 \cdot e^{f_4}$
4. Rx uses FFT to extract information
  - Example:  $[1 \ -1 \ 1 \ 1] = \text{FFT}(1 \cdot e^{f_1} + (-1) \cdot e^{f_2} + 1 \cdot e^{f_3} + 1 \cdot e^{f_4})$

# Orthogonal Frequency Division Modulation



Data  $X[n]$  coded in frequency domain

Transformation to time domain: each frequency is a sine wave in time, all added up



Time domain signal

Frequency domain signal

Decode each subcarrier separately

$X'[N]$  = amplitude of each sub-carrier

# Orthogonality of Sub-carriers

Time-domain signals:  $x(t)$

Frequency-domain signals:  $X[k]$

IFFT

Encode: frequency-domain samples  $\rightarrow$  time-domain samples

$$x(t) = \frac{1}{N} \sum_{k=-N/2}^{N/2-1} X[k] e^{j2\pi kt/N}$$

k-th subcarrier

FFT

Decode: time-domain samples  $\rightarrow$  frequency-domain sample

$$X[k] = \sum_{t=-N/2}^{N/2-1} x(t) e^{-2j\pi kt/N}$$

Orthogonal  $\rightarrow$   
inner product = 0

Orthogonality of any two bins :  $\sum_{k=-N/2}^{N/2-1} e^{j2\pi kt/N} e^{-j2\pi pt/N} = 0, \forall p \neq k$

# Orthogonality between Subcarriers

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- Subcarrier frequencies ( $k/N$ ,  $k=-N/2, \dots, N/2-1$ ) are chosen so that the subcarriers are orthogonal to each other
  - No guard band is required
- Two signals are orthogonal if their inner product equals zero

$$\sum_{k=-N/2}^{N/2-1} e^{j2\pi kt/N} e^{-j2\pi pt/N} = \sum_{k=-N/2}^{N/2-1} e^{2j\pi(k-p)t/N}$$

$$= N\delta(k, p) = \begin{cases} N & \text{if } p = k \\ 0 & \text{if } p \neq k \end{cases}$$

$$X[k] \perp X[p], k \neq p$$

# Serial to Parallel Conversion

- Say we use BPSK and 4 sub-carriers to transmit a stream of samples

1, 1, -1, -1, 1, 1, 1, -1, 1, -1, -1, -1, -1, 1, -1, -1, -1, 1, 1, -1, -1, -1, 1, 1

- Serial-to-parallel conversion of samples

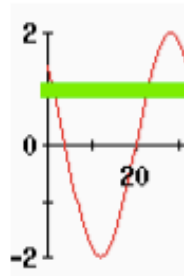
	Frequency-domain signal					Time-domain signal			
	c1	c2	c3	c4					
symbol1	1	1	-1	-1	IFFT →	0	2 - 2i	0	2 + 2i
symbol2	1	1	1	-1		2	0 - 2i	2	0 + 2i
symbol3	1	-1	-1	-1		-2	2	2	2
symbol4	-1	1	-1	-1		-2	0 - 2i	-2	0 + 2i
symbol5	-1	1	1	-1		0	-2 - 2i	0	-2 + 2i
symbol6	-1	-1	1	1		0	-2 + 2i	0	-2 - 2i

- Send time-domain samples after parallel-to-serial conversion

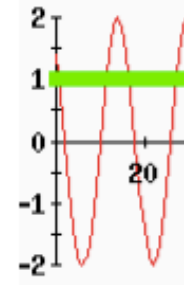
0, 2 - 2i, 0, 2 + 2i, 2, 0 - 2i, 2, 0 + 2i, -2, 2, 2, 2, -2, 0 - 2i, -2, 0 + 2i, 0, -2 - 2i, 0, -2 + 2i, 0, -2 + 2i, 0, -2 - 2i, ...

f1-4

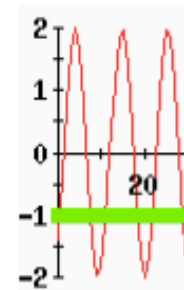
symbol1	1	1	-1	-1
symbol2	1	1	1	-1
symbol3	1	-1	-1	-1
symbol4	-1	1	-1	-1
symbol5	-1	1	1	-1
symbol6	-1	-1	1	1



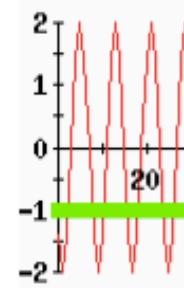
f1



f2



f3

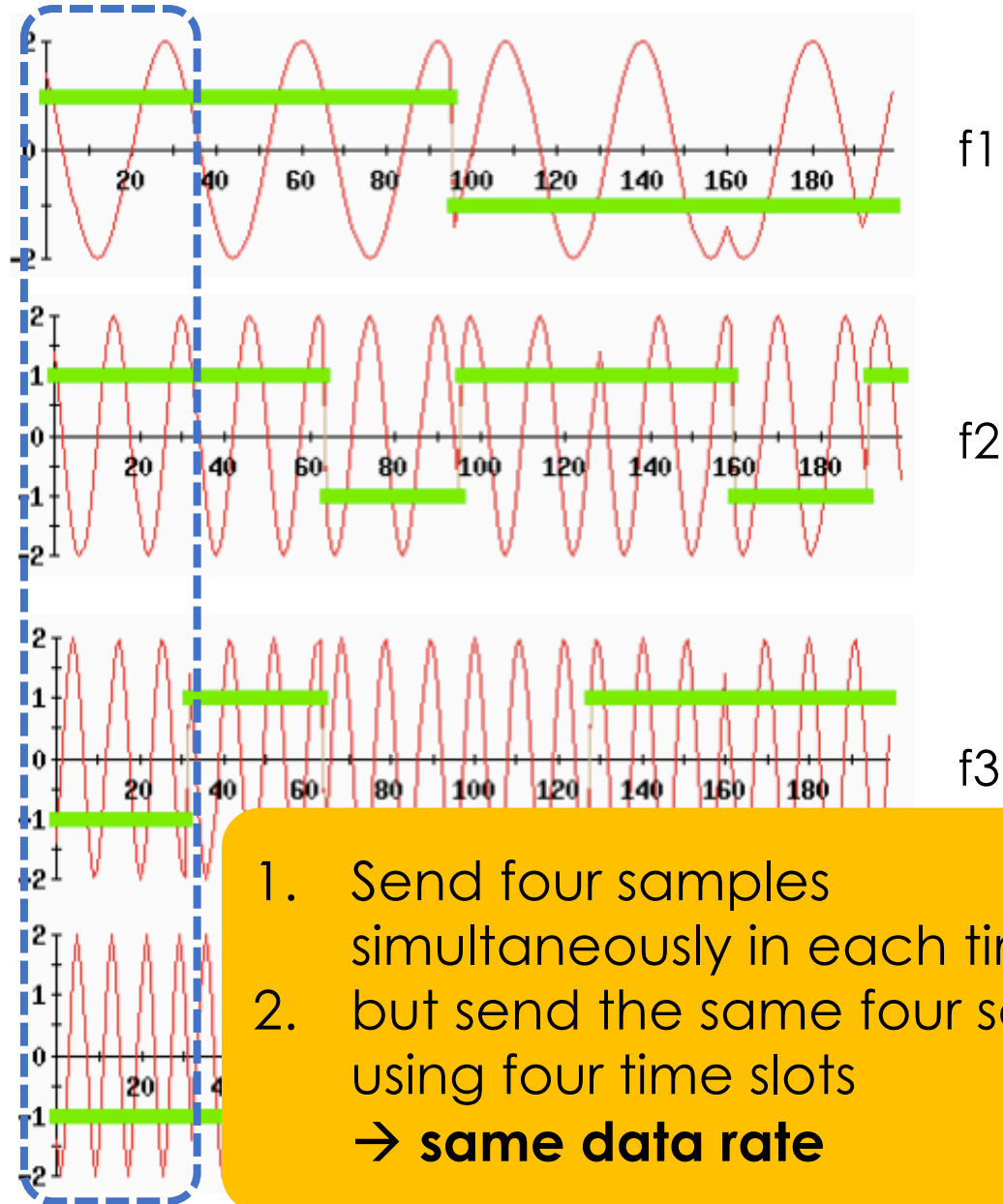


f4



t1-4   t5-8   t9-12   t13-16   t17-20   t21-24

symbol1	1	1	-1	-1
symbol2	1	1	1	-1
symbol3	1	-1	-1	-1
symbol4	-1	1	-1	-1
symbol5	-1	1	1	-1
symbol6	-1	-1	1	1



1. Send four samples simultaneously in each time-slot
2. but send the same four samples using four time slots  
→ **same data rate**

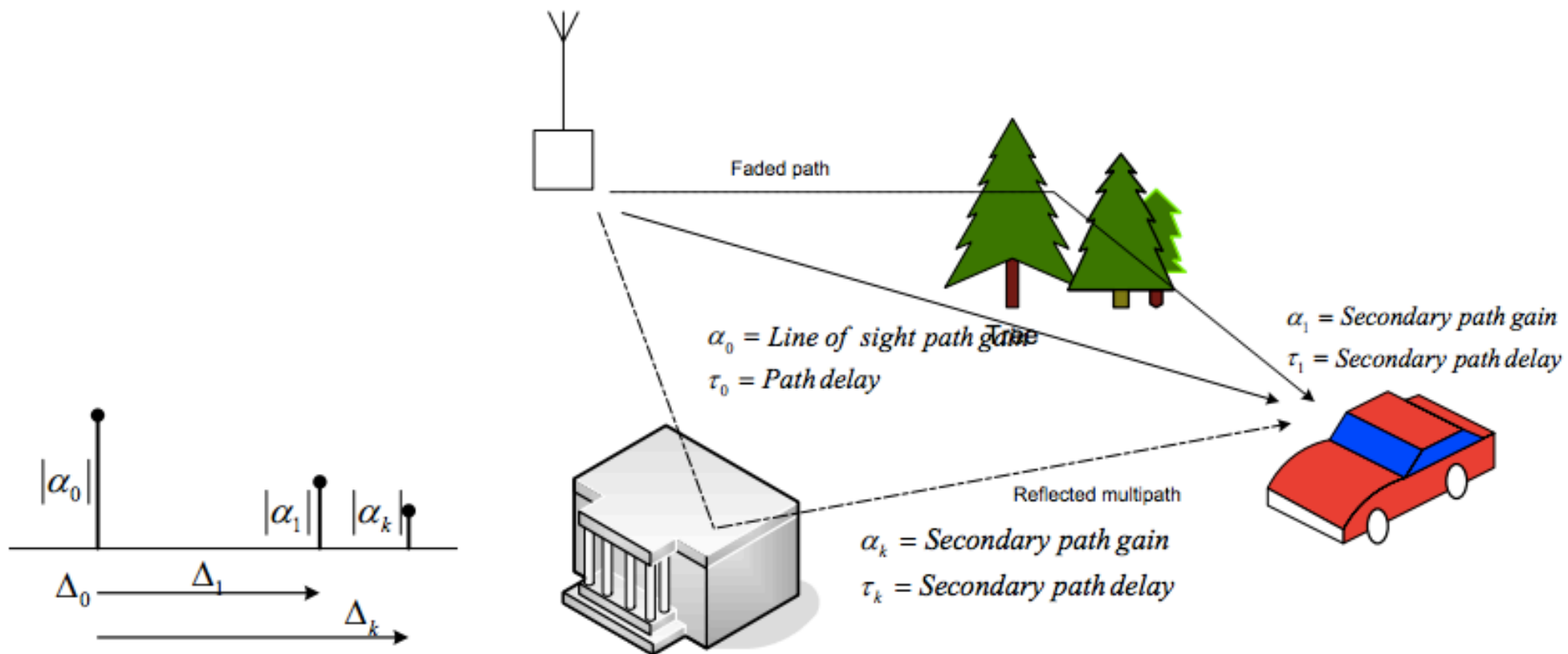
Send the combined signal as the time-domain signal

# Why OFDM?

combat **multipath** fading



# Multi-Path Effect



$$y(t) = h(0)x(t) + h(1)x(t - 1) + h(2)x(t - 2) + \dots$$

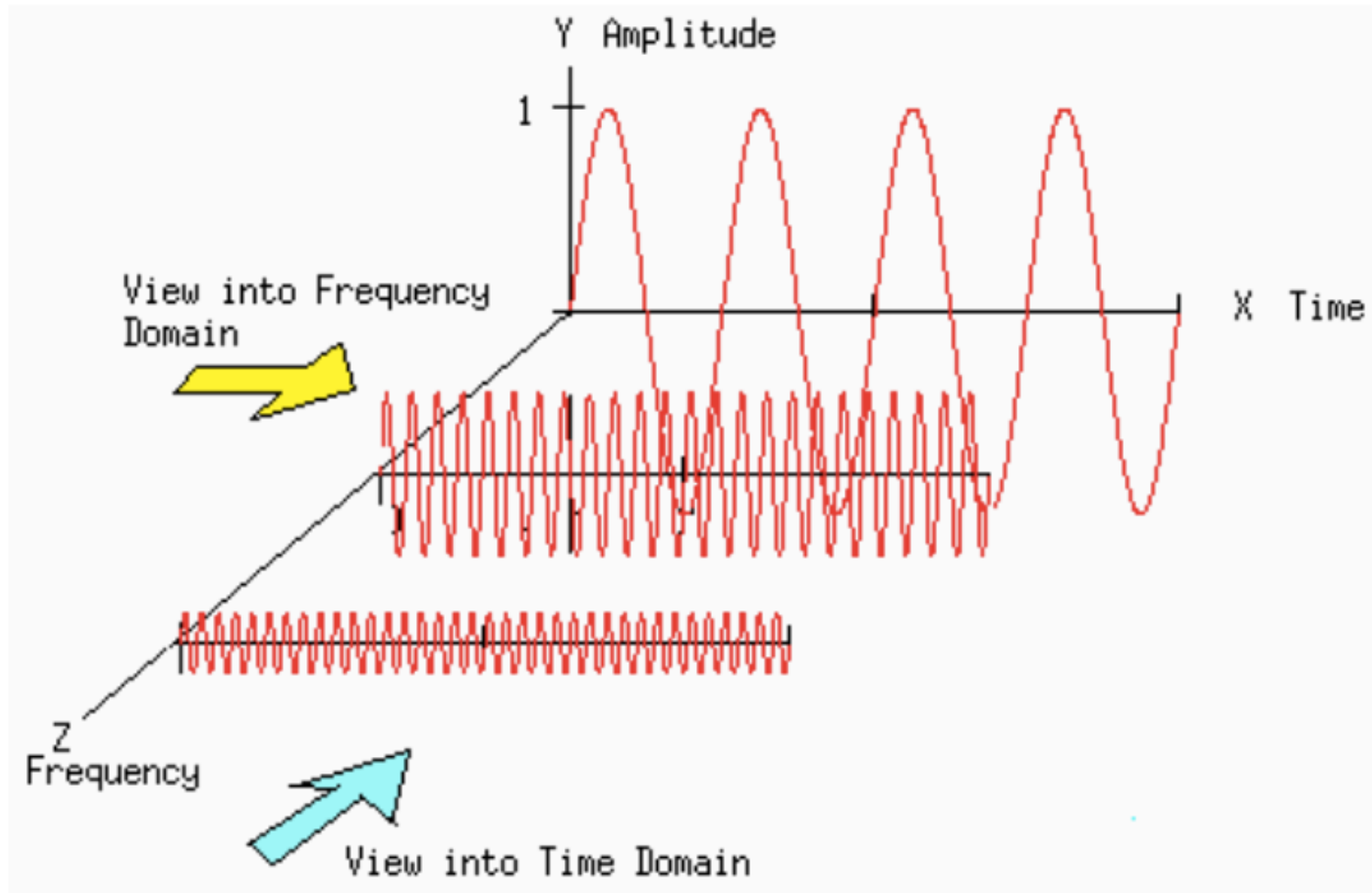
$$= \sum_{\Delta} h(\Delta)x(t - \Delta) = h(t) \otimes x(t)$$

time-domain

convolution

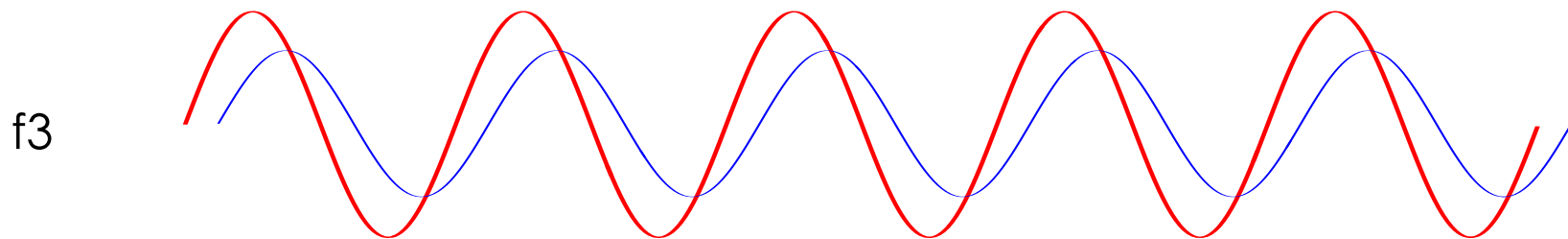
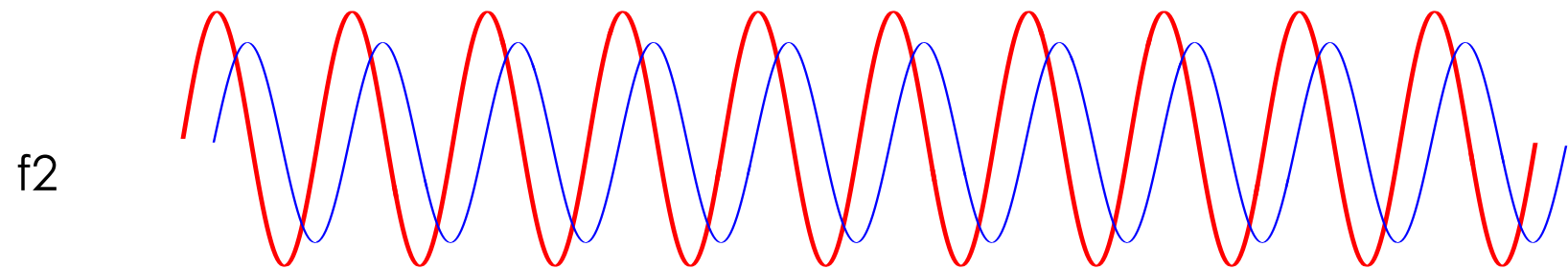
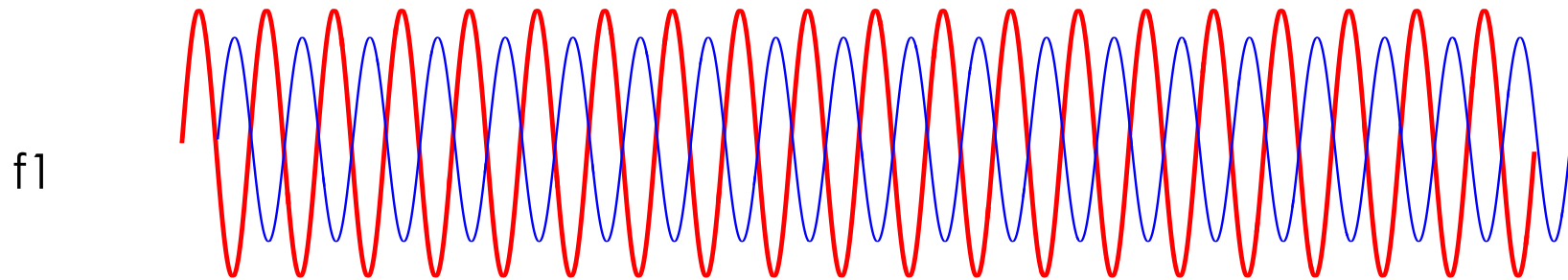
$$\Leftrightarrow Y(f) = H(f)X(f)$$

frequency-domain



Current symbol + delayed-version symbol  
 → Signals are destructive in only certain frequencies

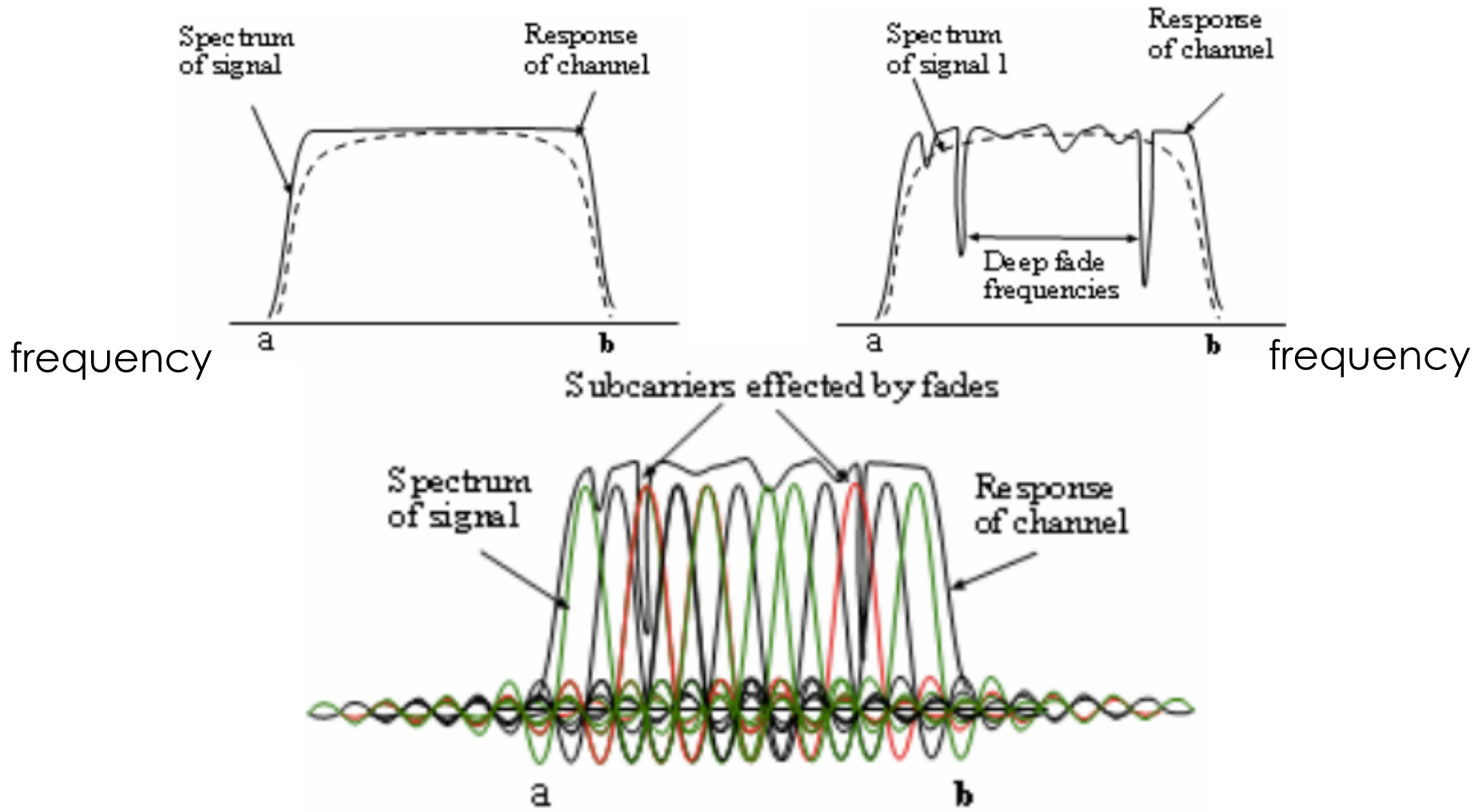
— direct — delay



Current symbol + delayed-version symbol

→ Signals are destructive in only certain frequencies

# Frequency Selective Fading



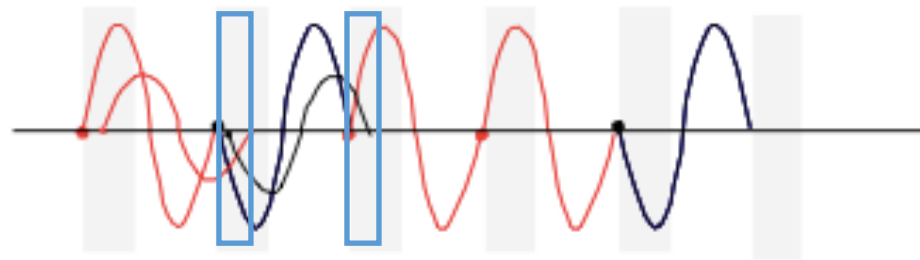
Frequency selective fading: Only some sub-carriers get affected

Can be recovered by proper coding!

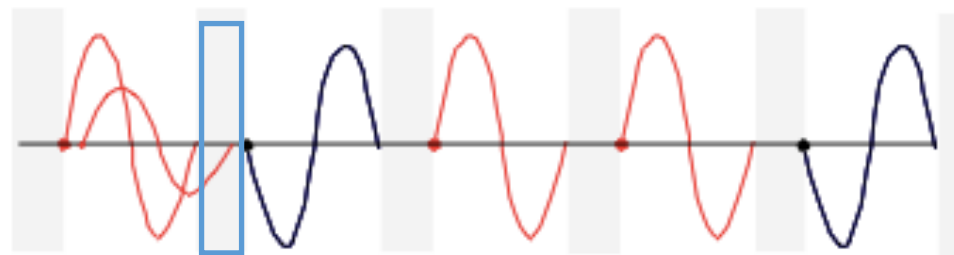
# Inter Symbol Interference (ISI)

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- The delayed version of a symbol overlaps with the adjacent symbol



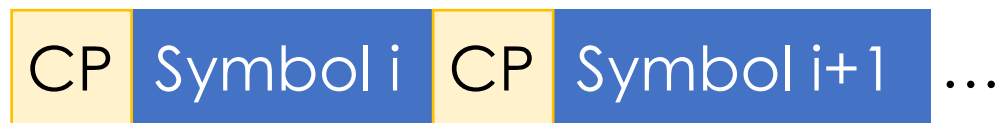
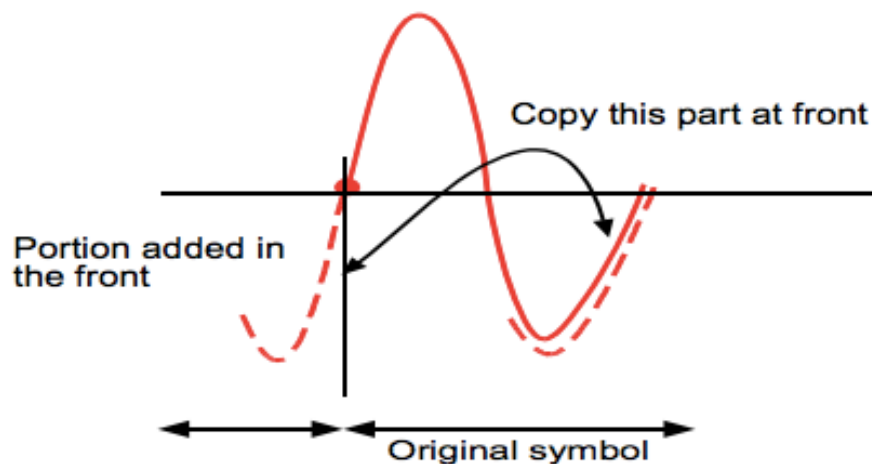
- One simple solution to avoid this is to introduce a guard-band



Guard band

# Cyclic Prefix (CP)

- However, we don't know the delay spread exactly
  - The hardware doesn't allow blank space because it needs to send out signals continuously
- Solution: **Cyclic Prefix**
  - Make the symbol period longer by copying the tail and glue it in the front



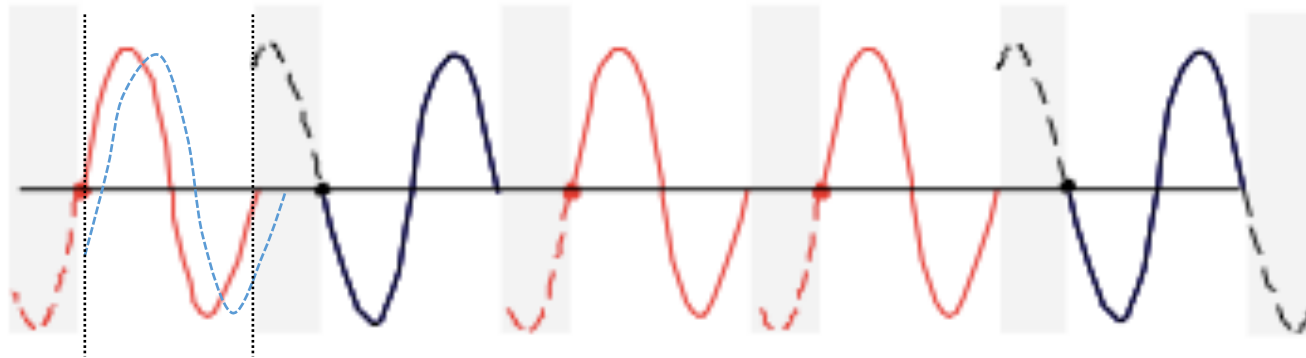
In 802.11, each symbol with 64 samples

CP:data = 1:4

→ CP: last 16 samples



# Cyclic Prefix (CP)



- Because of the usage of FFT, the signal is periodic

$$\text{FFT}(\text{delayed version}) = \exp(-2j\pi\Delta_f) * \text{FFT}(\text{original signal})$$

delayed version                      original signal

- Delay in the time domain corresponds to phase shift in the frequency domain
  - Can still obtain the correct signal in the frequency domain by compensating this rotation

# Cyclic Prefix (CP)

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## w/o multipath

$$y(t) \rightarrow \text{FFT}(\text{original signal}) \rightarrow Y[k] = H[k]X[k]$$

## w multipath

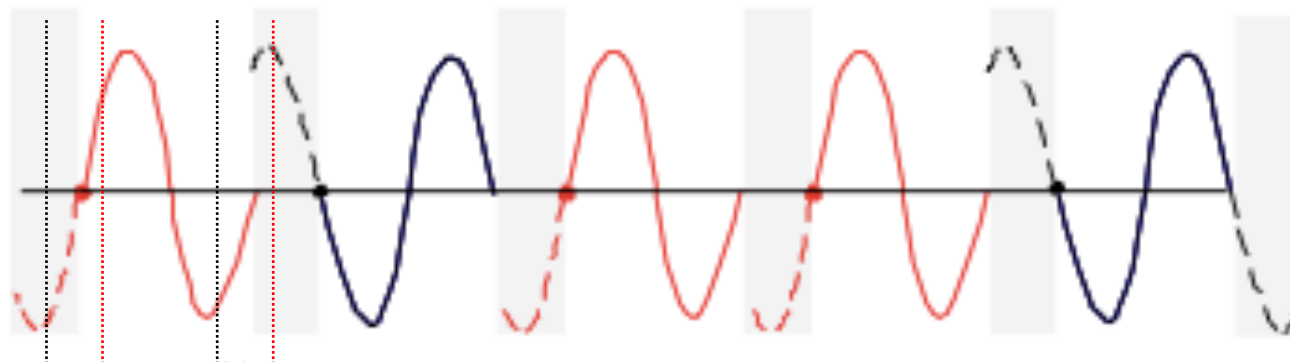
$$y(t) \rightarrow \text{FFT}(\text{original signal} + \text{delayed-version signal}) \rightarrow Y[k] = (H[k] + \exp(-2j\pi\Delta_k)H[k])X[k]$$
$$= (H[k] + H_2[k])X[k]$$
$$= H'[k]X[k]$$

Lump the phase shift in H

# Side Benefit of CP

- Allow the signal to be decoded even if the packet is detected not that accurately

decodable **undecodable**

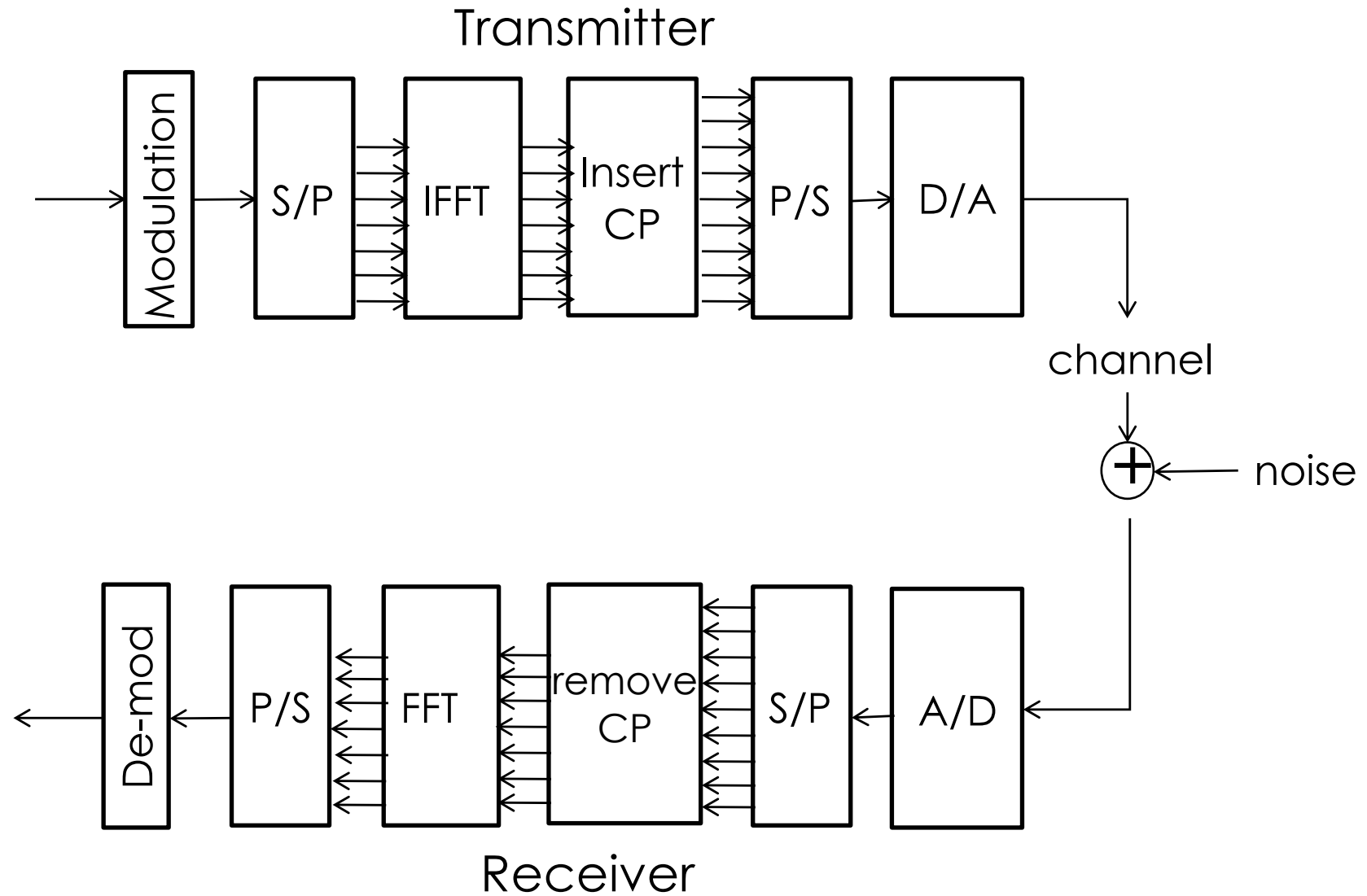


The last sample you actually use for FFT

The point you think the first symbol ends

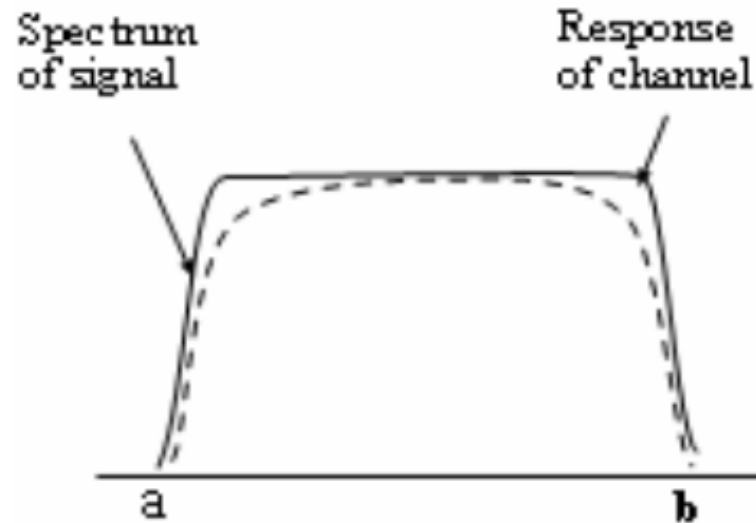
Check the parameter **FFT\_OFFSET** in the WARP code. Try to modify it!

# OFDM Diagram



# Unoccupied Subcarriers

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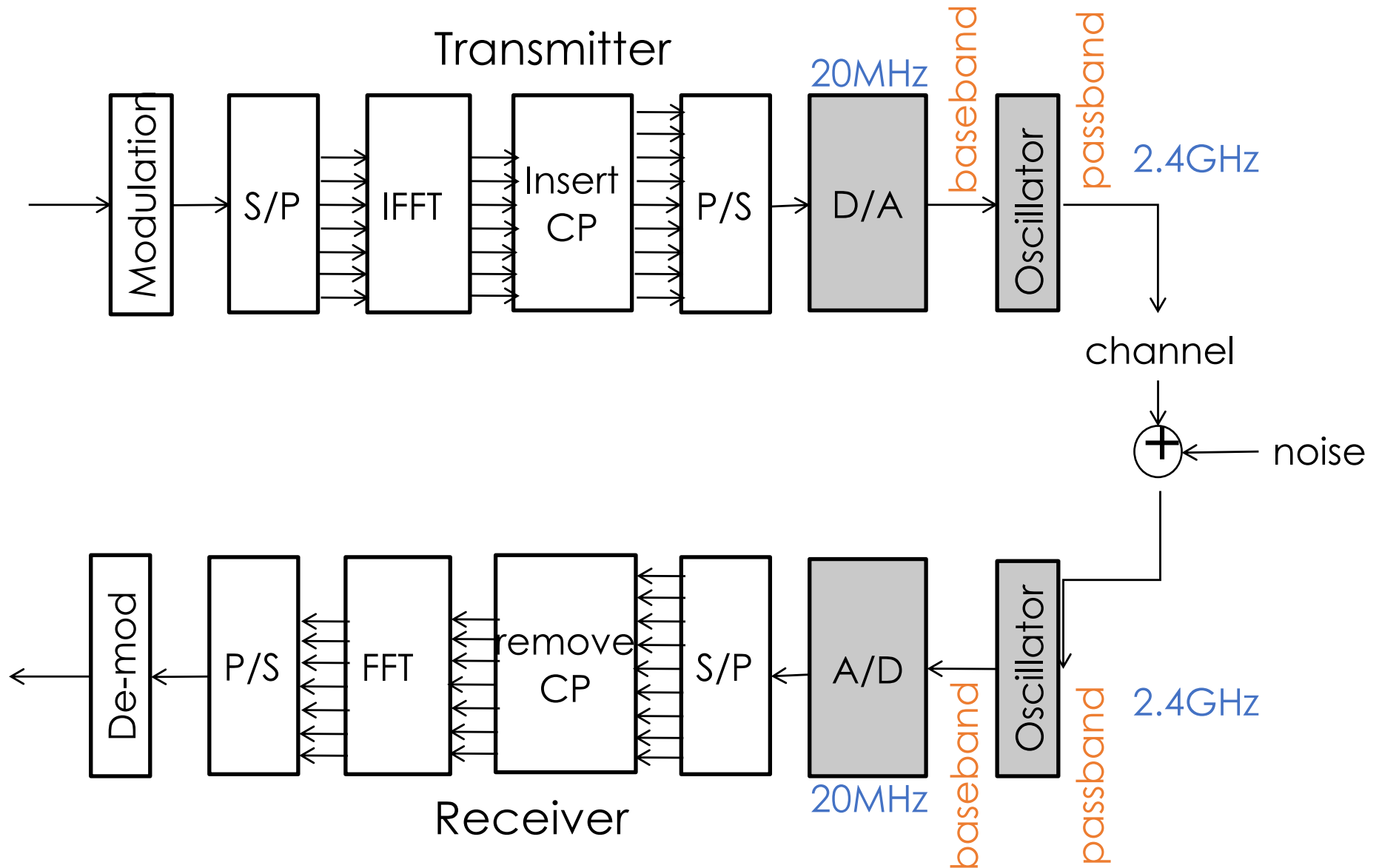
- Edge sub-carriers are more vulnerable
  - Frequency might be shifted due to noise or multi-path
- Leave them unused
  - In 802.11, only 48 of 64 bins are occupied bins
- Is it really worth to use OFDM when it costs so many overheads (CP, unoccupied bins)?

# Agenda

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- Packet Detection
- OFDM  
(Orthogonal Frequency Division Modulation)
- Synchronization

# OFDM Diagram



# Overview

- Carrier Frequency Offset (CFO)

- $f_{tx}^c \neq f_{rx}^c$  (e.g., TX: 2.45001 GHz, RX: 2.44998 GHz)

- CFO:  $\Delta_f = f_{tx} - f_{rx}$

- Time-domain signals:

$$y'(t) = y(t) * \exp(2j\pi\Delta_f t)$$

real                      theoretical

Error accumulates  
over time

- Sample Frequency Offset (SFO)

- Sampling rates in Tx and Rx are slightly different (e.g., TX: 20.0001 MHz, RX: 19.99997 MHz)

- SFO :  $\delta = \frac{T_{rx} - T_{tx}}{T_{tx}}$

Phase rotates  **$2j\pi\delta k\phi$**  in the  
k-th subcarrier

- Freq.-domain signals:  $Y'[k] = Y[k] * \exp(2j\pi\delta k\phi)$

constant



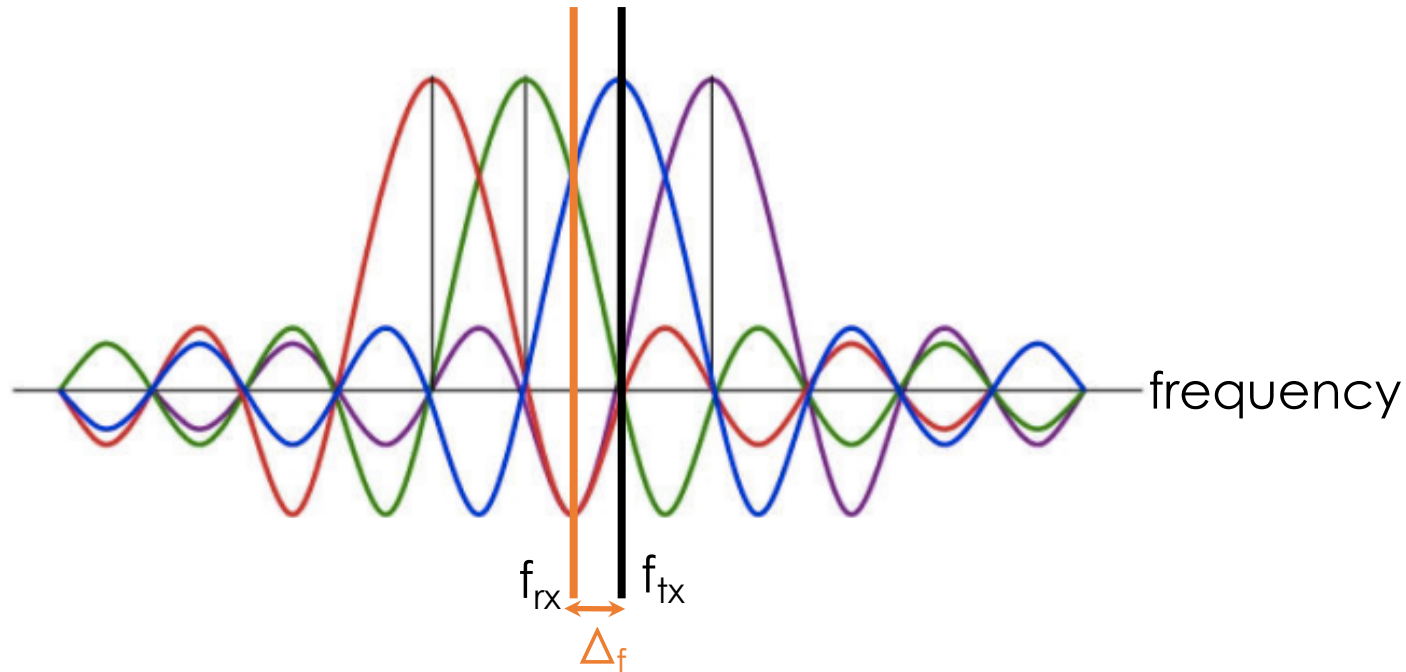
# Overview

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- Carrier Frequency Offset (CFO)
  - Calibrate in time-domain
  - How: Use the preamble
- Sample Frequency Offset (SFO)
  - Calibrate in frequency-domain
  - How: Use the pilot subcarriers

# Carrier Frequency Offset (CFO)

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- The oscillators of Tx and Rx are not perfectly synchronized
  - Carrier frequency offset (CFO)  $\Delta_f = f_{tx} - f_{rx}$
  - Leading to **inter-carrier interference (ICI)**
- OFDM is sensitive to CFO

# CFO Estimation

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- Up/Down conversion at Tx/Rx
  - Up-convert baseband signal  $s(t)$  to passband signal

$$r(t) = s(t)e^{j2\pi f_{tx}t} \otimes h(t, \tau)$$

- Down-convert passband signal  $r(t)$  back to

$$\begin{aligned} y_n &= r(nT_s)e^{-j2\pi f_{rx}t} \\ &= s(nT_s)e^{j2\pi f_{tx}t}e^{-j2\pi f_{rx}t} \otimes h(nT_s, \tau) \\ &= s(nT_s)e^{j2\pi \Delta_f nT_s} \otimes h(nT_s, \tau) \end{aligned}$$

Error caused by CFO, accumulated with time  $nT_s$

# CFO Correction in 802.11

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- Reuse the preamble to calibrate CFO
- The first half part of the preamble is identical to the second half part
  - The two transmitted signals are identical:  $s_n = s_{n+N}$
  - But, the received signals contain different errors

$$y_n = (s_n \otimes h) e^{j2\pi\Delta_f n T_s} \quad \rightarrow \text{Additional phase rotation } \Delta_f n T_s$$

$$y_{n+N} = (s_n \otimes h) e^{j2\pi\Delta_f (n+N) T_s} \quad \rightarrow \text{Additional phase rotation } \Delta_f (n+N) T_s$$

Find  $\Delta_f$  by taking  $y_{n+N} / y_n$

# CFO Correction in 802.11

$$\begin{aligned}
 y_n y_{n+N}^* &= (s_n \otimes h) e^{j2\pi\Delta_f n T_s} (s_n \otimes h) e^{-j2\pi\Delta_f (n+N) T_s} \\
 &= e^{-j2\pi\Delta_f N T_s} |(s_n \otimes h)|^2
 \end{aligned}$$

- To learn CFO  $\Delta_f$ , find the angle of  $(y_n y_{n+N}^*)$

$$\angle \left( \sum_n y_n y_{N+n}^* \right) = -2\pi\Delta_f N T_s$$

$$\Rightarrow \tilde{\Delta}_f T_s = \frac{-1}{2\pi N} \angle \left( \sum_n y_n y_{N+n}^* \right)$$

- Calibrate the signals to remove phase rotation

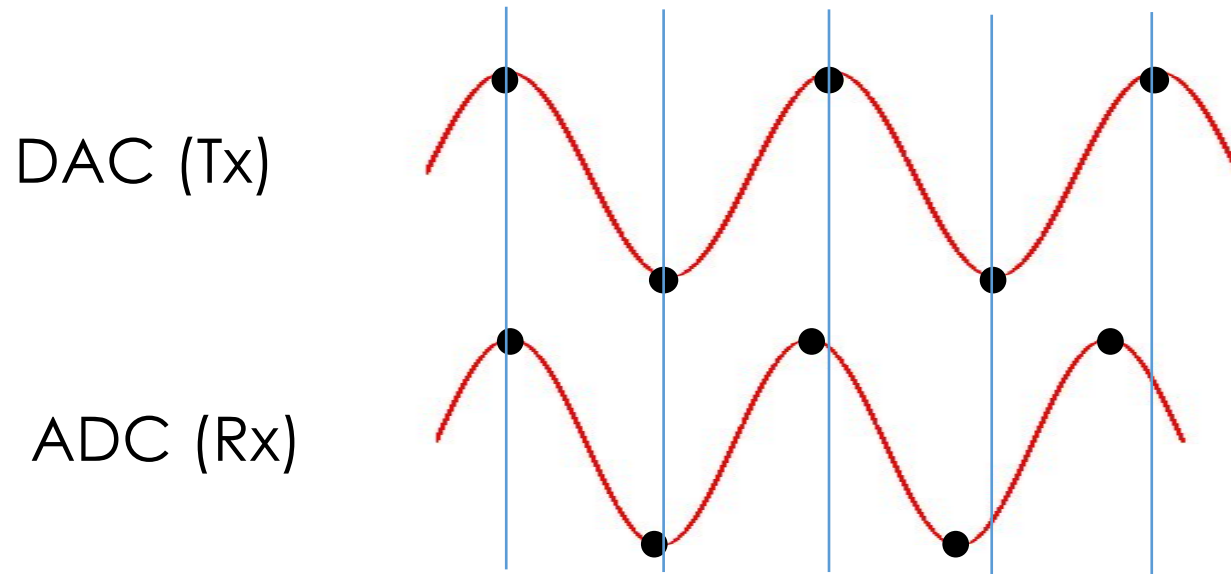
$$y_n e^{-j2\pi\tilde{\Delta}_f n T_s} = \underbrace{(s_n \otimes h) e^{j2\pi\Delta_f n T_s}}_{\text{Received signals}} \underbrace{e^{-j2\pi\tilde{\Delta}_f n T_s}}_{\text{calibration}} \approx (s_n \otimes h)$$

Received signals

calibration

# Sampling Frequency Offset (SFO)

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- DAC (at Tx) and ADC (at Rx) never have exactly the same sampling period ( $T_{tx} \neq T_{rx}$ )
  - Tx and Rx may sample the signal at slightly different timing offset

$$\text{SFO} : \delta = \frac{T_{rx} - T_{tx}}{T_{tx}}$$

# Phase errors due to SFO

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- Assuming no residual CFO, the k-th subcarrier in the received symbol i becomes

$$Y_{i,n} = H_k X_{i,k} e^{j2\pi\delta k\phi}$$

See proof in  
the next slide

- All subcarriers experience the same sampling offset, but applied on different frequencies  $k$ 
  - $\phi$  is a constant
  - Each subcarrier is rotated by a constant phase shift  $2\pi\delta\phi$
  - Lead to Inter Carrier interference (ICI), which causes loss of the orthogonality of the subcarriers

# Proof of phase errors due to SFO

## Time-domain

Up-convert:  $r(t) = s(t)e^{j2\pi f_{tx}t} \otimes h(t, \tau) + n(t)$

Down-convert:  $y_{i,n} = r(t)e^{-j2\pi f_{rx}t} \Big|_{t=(iN_S+N_{CP}+n)T_{rx}}$

## Frequency-domain

FFT  $\longrightarrow$

$$Y_{i,k} = H_k X_{i,k} e^{j2\pi(\Delta_f T_{FFT} + \delta k)\phi}$$

Residual CFO      SFO  
↑                      ↑

$N_{CP}$  : Number of samples in CP

$N_{FFT}$  : FFT window size

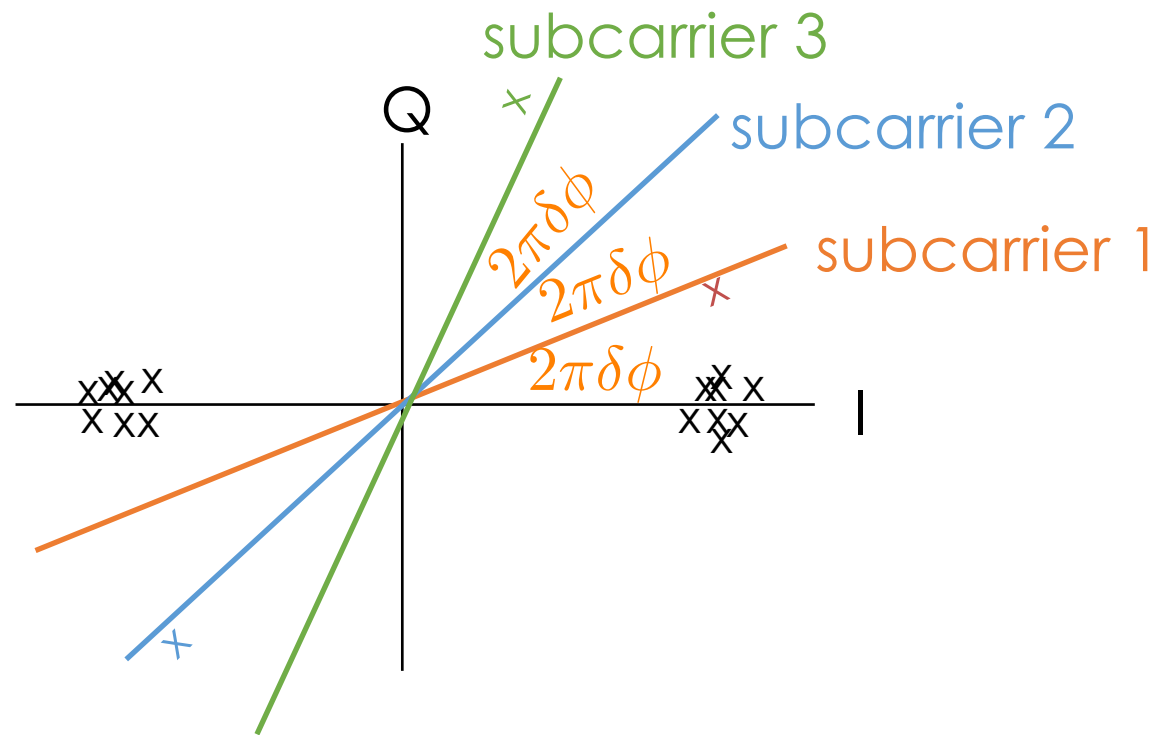
$N_S = N_{FFT} + N_{CP}$  : Symbol size

$\phi = 0.5 + \frac{iN_S + N_{CP}}{N_{FFT}}$  : a constant indicating the initial phase error of symbol  $i$



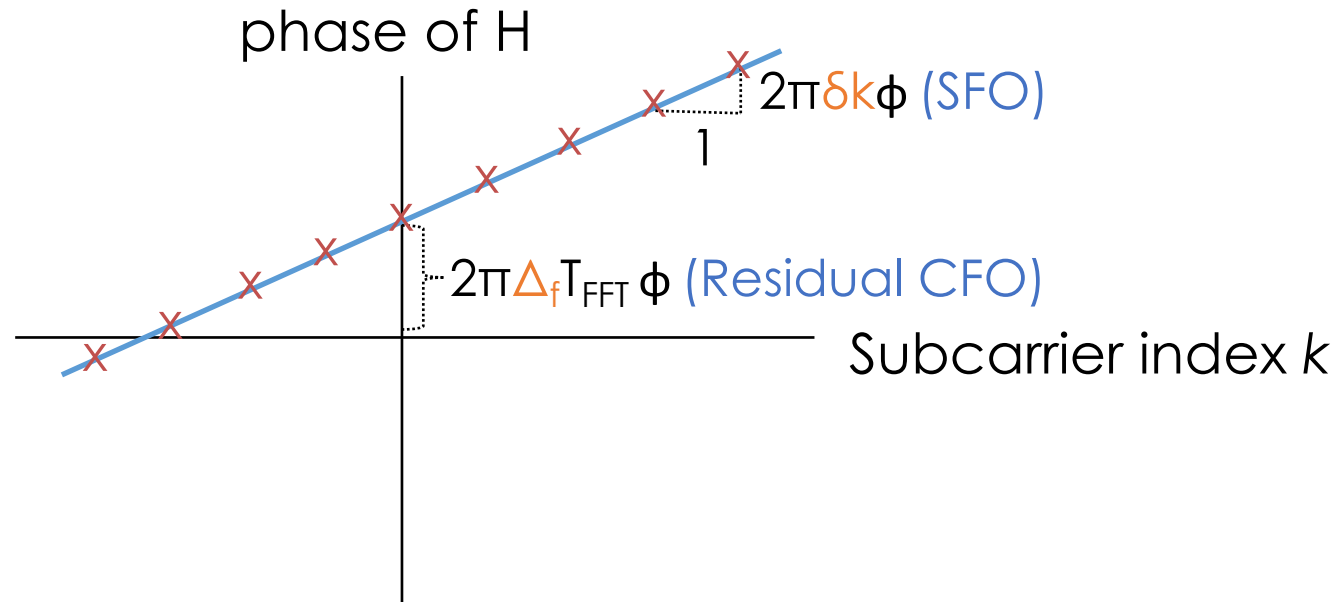
# Sample Rotation due to SFO

Incremental phase errors in different subcarriers  
→ Signals keep rotating in the I-Q plane



Ideal BPSK signals (No rotation)

# Phase Errors due to SFO and CFO

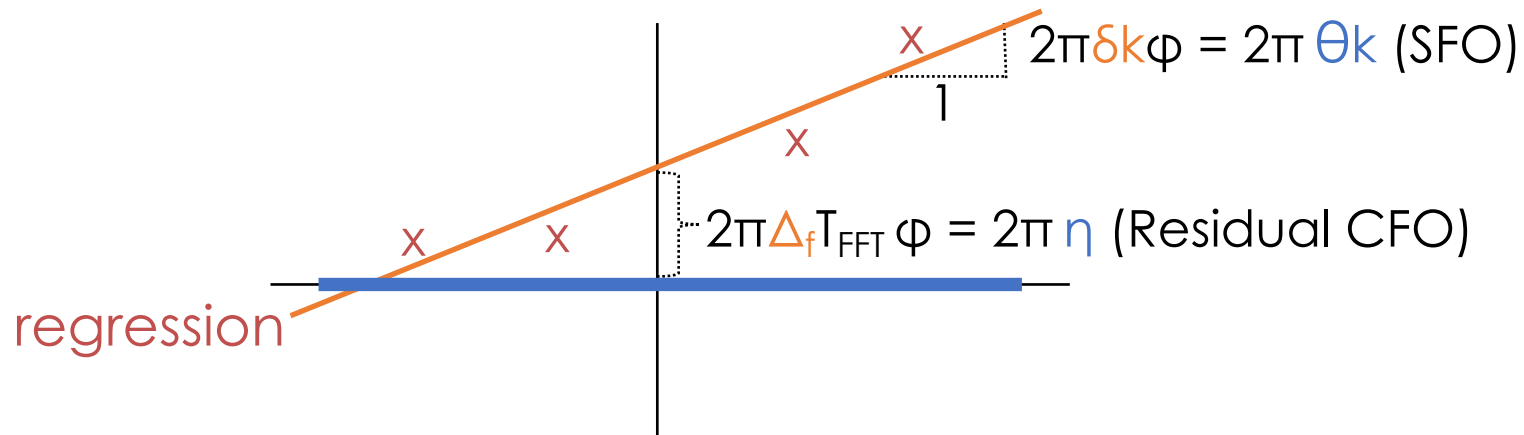


- Subcarrier  $i$  of the received frequency domain signals in symbol  $n$

$$Y_{i,k} = H_k X_{i,k} e^{j2\pi(\Delta_f T_{FFT} + \delta k)\phi}$$

- SFO: slope; residual CFO: intersection of y-axis

# Data-aided Phase Tracking



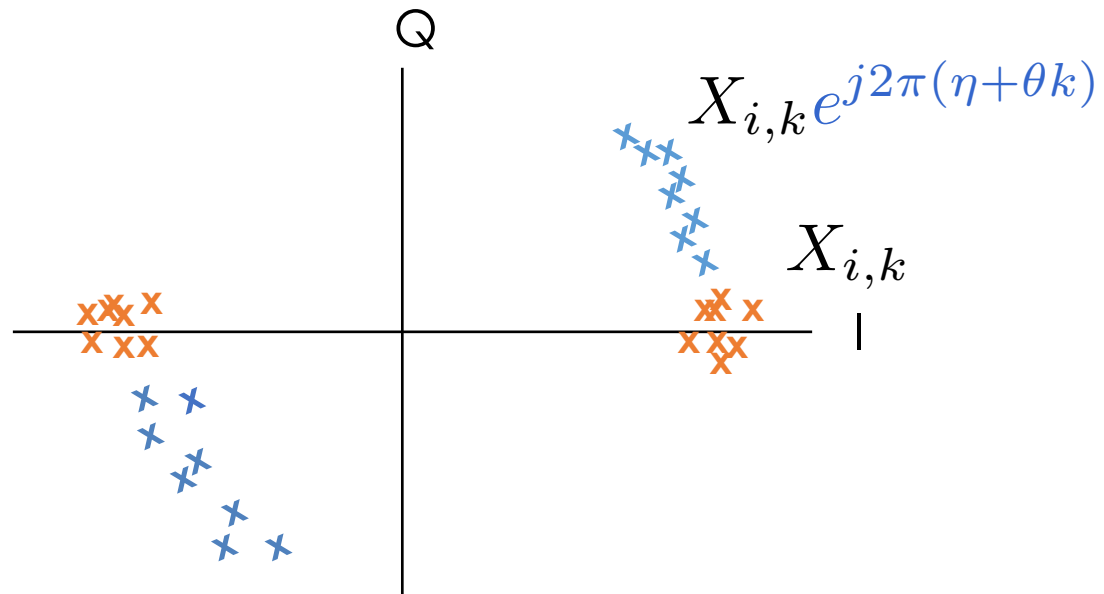
- WiFi reserves 4 known **pilot bits** (subcarriers) to compute  $H_k e^{j2\pi(\eta+\theta k)} = Y_k / X_k$
- Estimate **SFO**  $\theta_k$  and **CFO**  $\eta$  by finding the **linear regression** of the phase changes experienced by the pilot bits
- Update the channel by  $H'_k = H_k e^{j2\pi(\eta+\theta k)}$  for every symbol  $k$ , and then decode the remaining non-pilot subcarriers

$$Y_{i,k} = H_k X_{i,k} e^{j2\pi(\eta+\theta k)} = H'_k X_{i,k}$$

$$\Rightarrow \hat{X}_{i,k} = Y_{i,k} / H'_k$$

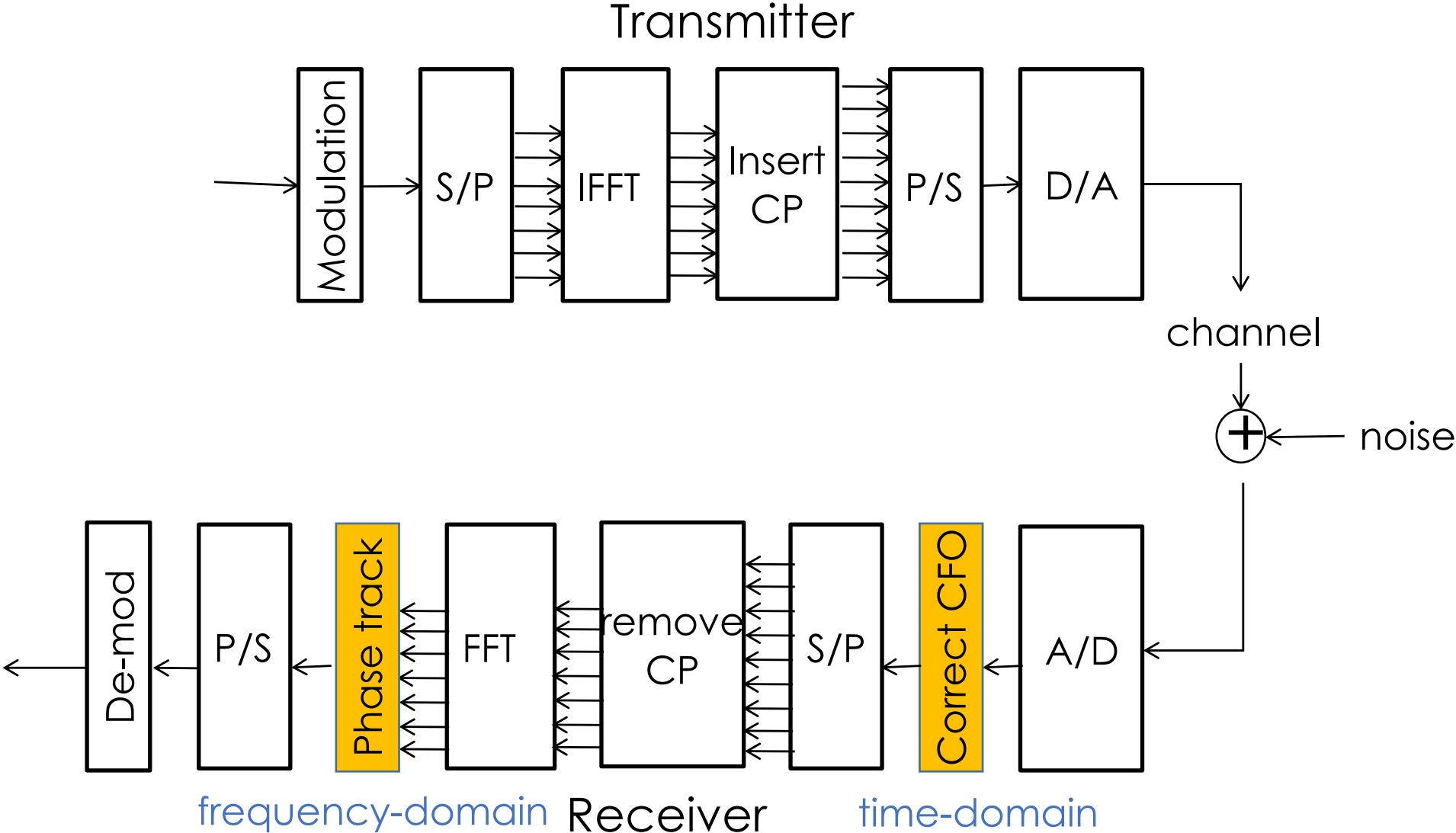
# After Phase Tracking

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Decoded signals in the I-Q plane after phase tracking

# OFDM Diagram



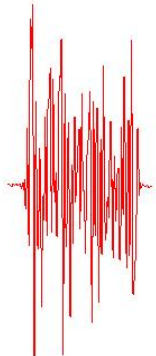
# Quiz

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- Say we want to send  $(1, -1, 1, 1, -1)$ ,

and transmit  over the air

$(1, -1, 1, 1, -1)$  is the (a) frequency-domain or  
(b) time-domain signal?



is the (a) frequency-domain or  
(b) time-domain signal

- What is the **Multipath Effect**? Why does it cause **Deep Fading**?