The Upper Bound of Capacity for A Concurrent-transmission-based Ad-hoc Network with Single Channel

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***Abstract*—The upper bound of capacity for the ad hoc networks is one of the most important issues. We investigate the effect of the transmission range on a concurrent-transmission-based ad hoc network with single channel. Thereby, for a given network topology, we find what the upper bound of capacity is and which transmission range makes it. The problem is formulated and we propose two approaches to solve it. One is the brute force approach and the other is the greedy approach. Simulation results show that the upper bound of capacity we found is approximative to the optimal one. The difference between them is even less than 1%.**

***Index Terms*—802.11, ad hoc networks, capacity, concurrent transmission, transmission range.**

I. Introduction

Wireless ad hoc networks are receiving more and more attention from the research community. It can be quickly deployed to provide wireless communication services in the areas without a pre-established infrastructure. Due to the convenience, many applications are proposed and thus encounter the challenges of spectrum deficiency.

Concurrent transmission are important technologies to enhance system capacity in ad-hoc networks. Based on the concurrent transmission technologies, users can establish multiple transmission links under the existing links. All links can concurrently transmit on the same channel without any interference. For example, [1] used transmission power control (TPC) technology to avoid the interference between concurrent transmission links. Furthermore, multiple-input and multiple-output (MIMO) technic is used to cancel the interference from the other active links [2]. Moreover, [3] proposes a concurrent-transmission medium access control (MAC) protocol to establish a slave link when a master link is transmitting.

Although many methods are proposed to establish concurrent-transmission links, the system capacity of concurrent-transmission-based ad-hoc networks still has not been investigated.

According to the literatures, no existing work addresses the upper bound of network capacity (with a given network topology) is and which transmission range makes it. Since a too short transmission range makes links disconnect and too long transmission range makes interference serious, both of them reduces the network capacity. Therefore, a best transmission range can maximize the network capacity. To the best of our knowledge, we are the first one who addresses this problem.

So, in this paper, we would find the upper bound of capacity for a concurrent-transmission-based ad hoc network and the optimal transmission range making it by any topology. This network can allow the links with concurrent transmission (a precise definition of concurrent transmission will be described latter). So, the number of concurrent transmission links is more, the capacity of this network is larger. Therefore, we investigate the maximal number of concurrent transmission links (so does the upper bound of capacity) of this network. We formulate the problem. Then, we propose two approaches to solve it. These approaches include an optimal method and a greedy method to find the maximal number of concurrent transmission links.

The rest of this paper is organized as follows. Section II formally defines this problem. Section III presents our proposed solutions. Simulation results are given in Section IV. Section V concludes this paper.

II. Related Work

Naturally, the capacity is one of the most important issues. In [4], it says that the capacity of the network is $\Theta(W\sqrt{An})$, where n is number of nodes and W is the capacity of a link, if each node is with optimal traffic pattern, optimal transmission range, and optimal displacement in a disk of area. In [5], it shows that the previous bound holds even when the nodes are allowed to approach arbitrarily close to each other and even when the transmission rate is given by the SINR conducted with Shannon's logarithm function. It also establishes a maximum throughput capacity of $\Theta(1/\lg n)$ per node in the absence of power control. In [6], it determines the asymptotic scaling for the per user throughput in a large hybrid ad hoc network. This network includes both ad hoc nodes and infrastructure nodes. It shows that when $m \lesssim \sqrt{n/\lg n}$, $\sqrt{n/\lg n} \lesssim m \lesssim n/\lg n$, and $m \gtrsim n/\lg n$, the per user throughput is of order $W/\sqrt{n/\lg n}, Wm/n$, and $W/\lg n$, respectively (where n is the number of ad hoc nodes, m is the number of infrastructure nodes, and W the capacity of a wireless link). In [7], it develops closed form bounds for the transmission capacity of CDMA ad hoc networks for both perfect and imperfect interference cancellation. In [8], it derives the transmission capacity of ad hoc networks, where nodes employing multiple antenna diversity techniques.

In addition, the wireless popularity results in the problem of overcrowded spectrum. In [9], a link-layer protocol named *Slot Seeded Channel Hopping (SSCH)* is proposed to increase the capacity of the IEEE 802.11 network by utilizing frequency diversity. In [10], it proposes to spread the traffic to intermediate relay nodes to improve the capacity of mobile ad hoc networks. In [11], it uses the topology control to avoid the interference between multiple links. This increases the opportunities of concurrent transmission. So far, by any topology, what the upper bound of capacity in this concurrent-transmission-based network is and which transmission range makes it are still an open issue.

So, in this paper, we find the upper bound of capacity for a concurrent-transmission-based ad hoc network and the optimal transmission range making it by any topology. Since the number of concurrent transmission links is more, the capacity of this network is larger. Therefore, we investigate the maximal number of concurrent transmission links (so does the upper bound of capacity) of this network. We formulate the problem and solve it by two procedures. The details are described in the following sections.

III. Problem Formulation

We consider a wireless network with N nodes. These nodes can form $|\mathbf{L}|$ logical links using a common channel, where $\mathbf{L}$ is denoted as the logical link set in this network. So, it can be expressed by a complete direct graph G(V,E), where $|V| = N$ and $|E| = |\mathbf{L}| = 2(N-1)^2$. A protocol interference model [4] is involved as the transmission model for this paper. Then, we let T_m and R_m be the transmitter and receiver for each link m ('l_m' for short), respectively. For brevity, we also use T_m and R_m to be their positions. Through the interference model, we define a link l_m is *valid for concurrent transmission* if the link is with successful transmission from T_m to R_m. A link is with successful transmission should be satisfied by following rules:

1) The distance between T_m and R_m is less or equal to a given *transmission range* ('r' for short), i.e., $d(T_m - R_m) \leq r$.
2) For every other valid links l_n, the distance from its transmitter T_n to this receiver R_m is larger or equal to the *interference range* $I(r)$, where $I(r) = (1+\Delta)r$. In many reference, Δ is defined as a real nonnegative number. So, we have $d(T_n - R_m) \geq I(r)$, $l_n \neq l_m, \forall l_m, l_n \in$ valid links.

So, consider two active links, l_1 and l_2. We assume that link l_1 and l_2 are valid links, if

$$d(T_1 - R_1) \leq r, \tag{1}$$
$$d(T_2 - R_2) \leq r, \tag{2}$$
$$d(T_2 - R_1) \geq (1+\Delta)r, \tag{3}$$
$$d(T_1 - R_2) \geq (1+\Delta)r \tag{4}$$

However, link l_2 can interfere link $l_1$if

$$d(T_2 - R_1) < (1+\Delta)r \tag{5}$$

and link l_1 can interfere link l_2 if

$$d(T_1 - R_2) < (1+\Delta)r \tag{6}$$

Then, we let $\mathbf{S}_i$ be a link subset from the link set $\mathbf{L}$, i.e., $\mathbf{S}_i \subset \mathbf{L}$. For example, $\mathbf{S}_1 = \{l_1, l_2, l_4\}$ and $\mathbf{S}_2 = \{l_2, l_3, l_5\}$ if $\mathbf{L}$=$\{l_1, l_2, l_3, l_4, l_5\}$. Then, we let $\mathbf{S}$ be the collection of all link subsets of $\mathbf{L}$, i.e. $\mathbf{S} = \{..., \mathbf{S}_i, \mathbf{S}_j, ...\}$, where $\mathbf{S}_i \neq \mathbf{S}_j$, for each $\mathbf{S}_i, \mathbf{S}_j \subset \mathbf{L}$. Since each link from $\mathbf{L}$ can be included or not, we have $|\mathbf{S}| = 2^{|\mathbf{L}|} = 2^{2(N-1)^2}$. On the other hand, the transmission range (r), which is a real number (i.e., $r \in \Re$), affects the total number of valid links. Since the transmission range is too short, communication between transmitter and receiver will be disconnected. In contrast, if the transmission range r is too long, one link may be interfered by other links. Both of these two cases would decrease the total number of valid links (thus the capacity of the network). Therefore, we are interested in how the maximal number of concurrent transmission links is and by which transmission range r^*. We formulate this problem as follows:

$$\max_{\forall r \in \Re, \mathbf{S}_i \in \mathbf{S}} F(r) \text{ and } r^* = \arg \max_{\forall r \in \Re, \mathbf{S}_i \in \mathbf{S}} F(r)$$

s.t.

$$F(r) = \max_{\forall \mathbf{S}_i \in \mathbf{S}} f_r(\mathbf{S}_i)$$

$$f_r(\mathbf{S}_i) = \begin{cases} |\mathbf{S}_i|, & \text{if } d(T_m - R_m) \leq r, d(T_n - R_m) \geq I(r), \\ & \forall l_m, l_n \in \mathbf{S}_i, l_m \neq l_n. \\ 0, & otherwise. \end{cases}$$

The objective function $\max_{\forall r \in \Re, \mathbf{S}_i \in \mathbf{S}} F(r)$ is to find the maximal number of concurrent transmission links and $arg\ max_{\forall r \in \Re, \mathbf{S}_i \in \mathbf{S}} F(r)$ is to find the optimal transmission range r^* such that the number of concurrent transmission links is the maximal. Function $f_r(\mathbf{S}_i)$ answers the size of the valid link set $\mathbf{S}_i$. Note that if the link is invalid, it will not be counted. Function $F(r)$ answers the maximal valid link subset by a given transmission range r. Note that if we could find the optimal transmission range r^*, the upper bound of network capacity will be $\max_{\forall \mathbf{S}_i \in \mathbf{S}} f_r(\mathbf{S}_i)$.

IV. Our Proposed Approaches

In this section, we propose a solution to find the upper bound of capacity and the optimal transmission range r^*. The solution is divided into two procedures. The first procedure is looking for the *maximal number of concurrent transmission* *links (MCL)* by given a transmission range r. The second procedure is to adjust the given transmission range r to maximize MCL obtained by the first procedure. For the first procedure, we propose two approaches, one is the *optimal approach* and the other is the *greedy approach*. For the second procedure, all transmission ranges at fixed interval will be tried by first procedure. By this way, the best transmission range can be found and the precision of the best transmission range depends on the size of intervals. Since the second procedure is easier to attain, we will focus on the solutions for the first procedure. The details will be described as follows.

A. Brute Force Approach

According to the problem definition in the section II, we know that if the number of nodes is N, the total number of link subsets is $|\mathbf{S}| = 2^{2(N-1)^2}$. So, by a brute force approach, it tries all cases of $\mathbf{S}$ and find the one S_i with maximal number of valid links while satisfying links validity. This approach can obtain the S_i with MCL. However, the time complexity of this approach is exponential of N due to $|\mathbf{S}| = 2^{2(N-1)^2}$. Therefore, the computation overhead is too high. It is not suitable for a large network.

B. Greedy Approach

Here we propose a greedy approach which is with lower time complexity. We first define a links subset named *conflict links* of l_m if the links interfere l_m. For example, link l_2 is a conflict link of l_1 if (5) is true. And link l_1 is a conflict link of l_2 if (6) is true. So, a link set collecting all the conflict links of l_m is called the *conflict link set* of l_m, denoted as $\mathbf{C}_m$. According to this, we know that a valid link will not appear in other valid links' conflict link sets. All the conflict link sets are made from a limited link set, i.e., $\mathbf{L}$. We have

$$\mathbf{C}_{MCL} = \cup_{\forall l_m \in \mathbf{L}_{MCL}} \mathbf{C}_m. \tag{7}$$

$$|\mathbf{C}_{MCL}| \leq |\mathbf{L}| = 2(N-1)^2, \tag{8}$$

where $\mathbf{L}_{MCL}$ are the links making MCL. So, by the observation, for each link l_m from $\mathbf{L}_{MCL}$, the conflict link set of l_m would be as smaller as possible such that the number of valid links would be maximal. According to this idea, we design the greedy approach by recursive selecting the links with smaller conflict link set. Here, we describe the detailed algorithm as follows.

In line (1) to (3), it collects the conflict links as a conflict link set $\mathbf{C}_m$ for each link l_m. Note that l_m's conflict link set $\mathbf{C}_m$ includes l_m itself. In line (4) to (11), it finds the link l_m, from link set $\mathbf{L}$, which has the smallest conflict link set $\mathbf{C}_m$ until $\mathbf{L}$ is empty. In line (6), it adds the selected l_m to the output link set $\mathbf{S}_G$. Then, in line (7), it removes all links in this conflict links $\mathbf{C}_m$ from link set $\mathbf{L}$. Line (8) to (10) updates the conflict link set for each link contained in current link set $\mathbf{L}$.

The time complexity is analyzed below. In link (1) to (3), each link will check all links in $\mathbf{L}$ if interfering it. This costs O($|\mathbf{L}| \times |\mathbf{L}|$). In line (5), to pick up the smallest conflict link set costs O($|\mathbf{L}|$), since there are $|\mathbf{L}|$ conflict link sets

Algorithm 1 Greedy Approach

Input: $\mathbf{L}$=$\{l_1, l_2, ..., l_m, l_n, ...l_{2(N-1)^2}\}$,$\mathbf{S}_G = \phi$
Output: $\mathbf{S}_G$

1: **for** each $l_m \in \mathbf{L}$ **do**
2: $\quad$ $\mathbf{C}_m = \{l_i | l_i \in \mathbf{L}, l_i$ is a conflict link of $l_m\}$
3: **end for**
4: **while** $\mathbf{L} \neq \phi$ **do**
5: $\quad$ $m^* \leftarrow \arg\min_{\forall m, l_m \in \mathbf{L}} \{|\mathbf{C}_m|\}$
6: $\quad$ $\mathbf{S}_G \leftarrow \mathbf{S}_G \cup l_{m^*}$
7: $\quad$ $\mathbf{L} \leftarrow \mathbf{L} - (\mathbf{C}_{m^*} \cap \mathbf{L})$
8: $\quad$ **for** each $l_n \in \mathbf{L}$ **do**
9: $\quad\quad$ Update $\mathbf{C}_n = \{l_i | l_i \in \mathbf{L}, l_i$ is a conflict link of $l_n\}$
10: $\quad$ **end for**
11: **end while**

at most. In line (8) to (10), similar to line (1) to (3), costs O($|\mathbf{L}|^2$). In short, line (4) to (11) costs O($|\mathbf{L}|$)×O($|\mathbf{L}|$+$|\mathbf{L}| \times |\mathbf{L}|$)=O($|\mathbf{L}|^3$). So, the total time complexity from line (1) to (11) is O($|\mathbf{L}|^2$)+O($|\mathbf{L}|^3$)=O($|\mathbf{L}|^3$).

V. Performance Evaluation

In this section, we evaluate the performance of the average maximal number of concurrent transmission links (MCL) against the size of the transmission range. The transmission range will be simulated at intervals of 50 (m). We simulate the results by both of the proposed greedy approach and the optimal approach, where optimal approach is conducted by brute force approach in first procedure.

We develop the simulation program in C++. We deploy the nodes randomly in a square field with 1 km in length. The instance of each result is calculated by 50 times of randomly deployment. That means each instance is an average MCL results by 50 times of randomly deployment with the same transmission range. In addition, we also observe the effect of the number of nodes (i.e., nodes density) on the average MCL.

Fig. 1 shows the results of the effect of the transmission range on MCL with 9 nodes, 16 nodes, 25 nodes and 36 nodes, respectively. Because of the brute force approach's high complexity, it is not possible to simulate for more than 60 nodes. In Fig. 1, the result shows that as the transmission range increases, the average MCL first increases and then decreases. The former increase is because the links are connectable gradually. The later decrease is because the interference among links is more serious if a larger transmission range is conducted.

The result also shows that the upper bound of average capacity of $N = 9$, $N = 16$, $N = 25$, and $N = 36$ are 2.8, 5.06, 7.8, and 11.4, respectively. The best transmission ranges are 300, 200, 150, and 100 m, accordingly. That means the transmission range setting as 300, 200, 150, and 100 m for $N = 9$, $N = 16$, $N = 25$, and $N = 36$ with random deployment will reach the peak capacities.

Consequently, all of their upper bounds, i.e. peak capacities, of the average MCL increases as the number of nodes increases. This is because the number of possible transmission pairs increases as the number of nodes increasers. For example,

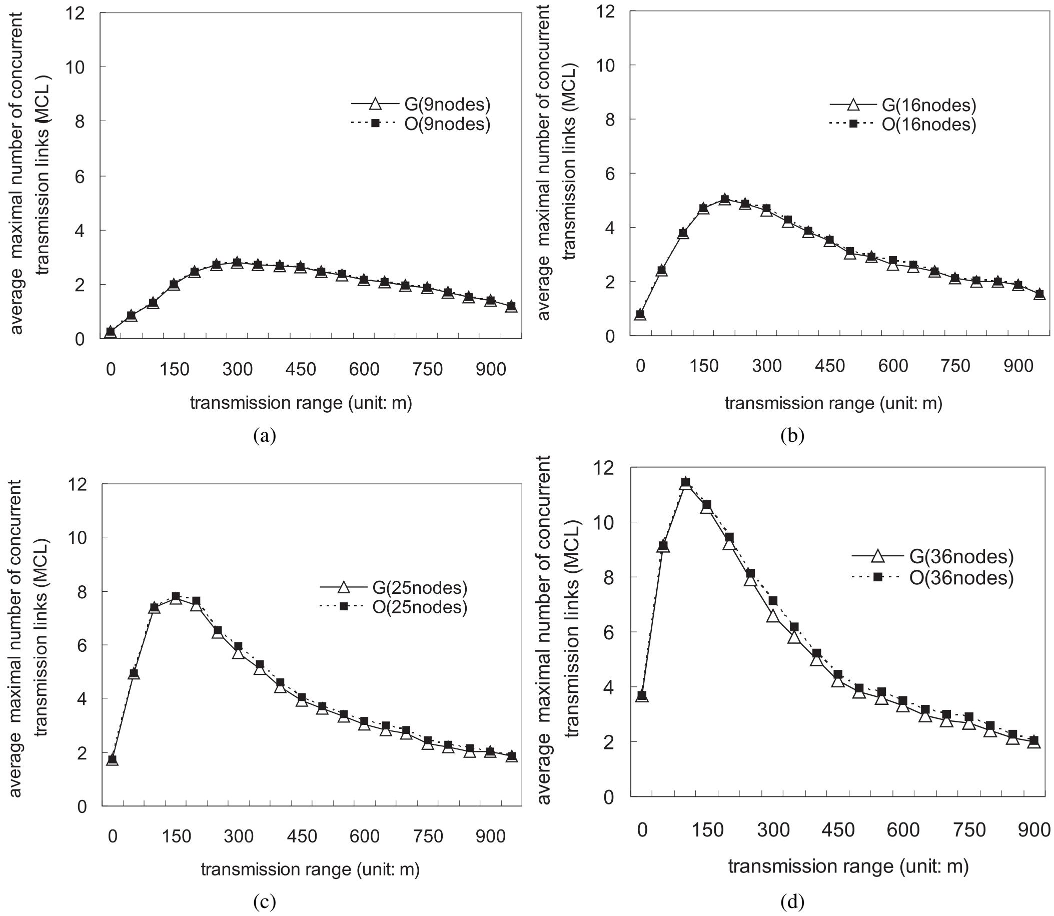

Fig. 1. Effects of the transmission range on the average number of concurrent transmission links (MCL): (a) $N = 9$, (b) $N = 16$, (c) $N = 25$, and (d) $N = 36$. Note that '*G*' represents the greedy approach and '*O*' represents the optimal approach.

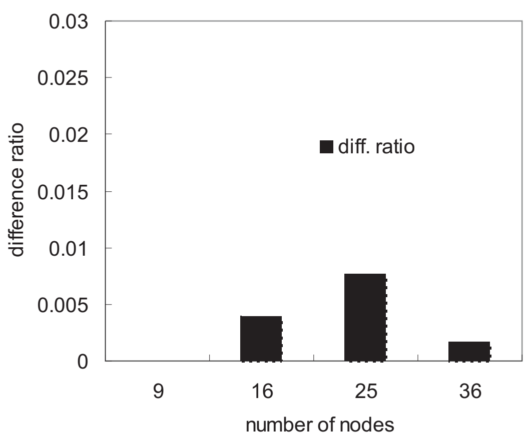

Fig. 2. Effect of the difference ratio on the number of nodes.

the most number of possible transmission pairs is $N/2$ if the number of node is N. In addition, as the number of nodes increases (thus the node density increases), the best transmission range gets shorter. This is because the increase of node density makes the distance between nodes shorter. That means each transmitter-receiver pair can communicate by a shorter transmission range and a longer transmission range will seriously cause interference to other links. So, the best transmission range for link connection reduces.

In addition, Fig. 1 also shows that the performance of the proposed greedy approach approximates to that of the optimal approach. For accuracy, we define the *difference ratio* of these two methods as

$$\frac{\text{MCL of optimal method - MCL of greedy method}}{\text{MCL of optimal method}}, \quad (9)$$

where MCL of both two methods is computed at the best transmission range r. Fig. 2 shows that the effects of difference ratio on the number of nodes. Here the best transmission range for $N = 9$, $N = 16$, $N = 25$, and $N = 36$ are 300, 200, 150, and 100 m, respectively. As can be seen, the difference ratios are quite small, the one with maximal difference ratio, i.e., $N = 25$, is even lower than 0.01. That means that the performance of our proposed greedy method approximates to that of the optimal method. Therefore, we recommend using our proposed method for finding maximal capacity upper bound on a concurrent-transmission-based ad hoc network with single channel, especially for a large network.

VI. Conclusions

In this paper, we investigate the effect of transmission range on the network capacity and find the optimal transmission range and the upper bound of capacity for a concurrent-transmission-based ad hoc network with single channel. We propose two approaches, called brute force approach and the greedy approach, to solve this problem. Simulation results show that the upper bound of capacity found by our proposed approach is approximative to that of the optimal one; the difference between them is even smaller than 1%.

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