Exercise 12.1

4. Hint: $X'/Y = (Y+Y')/Y$. Solution: $u(x,y) = c_1 e^{-y-\lambda(x+y)}$.

9. Solution:

a) If $(\lambda - 1)/k = 0$, then $u = e^{-t}(A_1 x + A_2)$.
b) If $(\lambda - 1)/k = -\alpha^2 < 0$, then $u = (A_1 \cosh \alpha x + A_2 \sinh \alpha x)\, e^{-(1-k\alpha^2)t}$.
c) If $(\lambda - 1)/k = \alpha^2 > 0$, then $u = (A_1 \cos \alpha x + A_2 \sin \alpha x)\, e^{-(1+\lambda\alpha^2)t}$.

Exercise 12.2

2. Solution:

$$k\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t},\ \ 0 < x < L,\ \ t > 0$$
$$u(0,t) = u_0,\ \ u(L,t) = u_1,\ \ t > 0.$$
$$u(x,0) = 0,\ \ 0 < x < L$$

11. Solution:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0,\ \ 0 < x < 4,\ \ 0 < y < 2$$
$$\left.\frac{\partial u}{\partial x}\right|_{x=0} = 0, u(4,y) = f(y),\ \ 0 < y < 2.$$
$$\left.\frac{\partial u}{\partial y}\right|_{y=0} = 0, u(x,2) = 0,\ \ 0 < x < 4$$

Exercise 12.3

5. Solution: $u(x,t) = \dfrac{e^{-ht}}{L}\displaystyle\int_0^L f(x)dx + \dfrac{2e^{-ht}}{L}\sum_{n=1}^{\infty}\left(\int_0^L f(x)\cos\frac{n\pi}{L}x dx\right)e^{-kn^2\pi^2 t/L^2}\cos\frac{n\pi}{L}x$.

Exercise 12.4

4. Solution: $u(x,t) = 2\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^3}\cos nat \sin nx$.

Exercise 12.5

8. Solution: $u(x,y) = \displaystyle\sum_{n=1}^{\infty} A_n (n\pi\cosh n\pi y + \sinh n\pi y)\sin n\pi x$, $n = 1, 2, 3, \ldots$

where $A_n = \dfrac{2}{n\pi\cosh n\pi + \sinh n\pi}\displaystyle\int_0^1 f(x)\sin n\pi x dx$.