Exercise 11.1

8. Hint: show that for $m \neq n$, $\int_0^{\pi/2} \cos(2n+1)x \cos(2m+1)x \, dx = 0$. The norm is

$$\|\cos(2n+1)x\| = \frac{1}{2}\sqrt{\pi}.$$

25. Hint: For (b), we must show that not all real-value functions can be represented as a linear combination of sin nx, n = 1, 2, 3, …, on the interval $[-\pi, \pi]$. For example, $f(x) = 1$ cannot be represented as a linear combination of { sin nx } because it is orthogonal to sin nx for any n on the interval $[-\pi, \pi]$.

Exercise 11.2

7. Solution: $f(x) = \pi + \sum_{n=1}^{\infty} \frac{2}{n}(-1)^{n+1} \sin nx$.

$f(x)$ is continuous on the interval.

10. Solution: $f(x) = \frac{1}{\pi} + \sum_{n=1}^{\infty}\left[\frac{2(-1)^{n+1}}{\pi(4n^2-1)}\cos 2nx + \frac{4n}{\pi(4n^2-1)}\sin 2nx\right]$.

$f(x)$ is discontinuous at $x = 0$ and converges to ½ there.

21. Hint: Let $x = \pi/2$ and substitute it into the Fourier series expansion of $f(x)$:

$$\frac{3\pi}{2} = f\left(\frac{\pi}{2}\right) = \pi + \sum_{n=1}^{\infty} \frac{2}{n}(-1)^{n+1} \sin\frac{n\pi}{2}.$$

Exercise 11.3

16. Solution: $f(x) = \sum_{n=1}^{\infty}\left(\frac{2(-1)^{n+1}}{n\pi} + \frac{4}{n^3\pi^3}\left[(-1)^n - 1\right]\right)\sin n\pi x$.

27. Solution: Cosine expansion: $f(x) = \frac{2}{\pi} + \sum_{n=1}^{\infty}\frac{4(-1)^n}{\pi(1-4n^2)}\cos 2nx$.

Sine expansion: $f(x) = \sum_{n=1}^{\infty}\frac{8n}{\pi(4n^2-1)}\sin 2nx$

44. Solution: $x_p(t) = \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{n(10-n^2\pi^2)}\sin n\pi t$.