**Exercise 2.2**

8. Hint: By integration by parts, we have $ye^{y} - e^{y} + e^{-x} + \frac{1}{3}e^{-3x} = c$.

11. Hint: $\cos^2 x = (1+\cos 2x)/2$. The solution is $4\cos y = 2x + \sin 2x + c$.

24. Solution: $y = x$.

29. Solution: $y(x) = e^{\int_4^x e^{-t^2} dt}$.

46. Hint: Let $u = \sqrt{x}$. The solution is $y = 3^{2/3}\left(-\sqrt{x}\cos\sqrt{x} + \sin\sqrt{x} + c\right)$.

55. Hint: Use the table of integration. The solution is $y = \pm\sin(x + c)$ and $y = \pm 1$.

**Exercise 2.3**

6. Solution: $y = \frac{1}{2}x^2 - \frac{1}{2} + ce^{-x^2}$ for $-\infty < x < \infty$. The transient tern is $ce^{-x^2}$.

24. Solution: $y = [(x + c)(x + 1)]/(x - 1)$. The largest interval of definition can be $(-\infty, -1)$, $(-1, 1)$, or $(0, \infty)$, depending on where your initial condition is. There is no transient term.

34. Solution: $y = \frac{\ln x}{x+1} + \frac{e}{x+1}$, for $I = (0, \infty)$.

49. Solution: $y' = 4 - y$.

53. Solution: The left-hand derivative of the function at $x = 1$ is $1 = e$ and the right-hand derivative at $x = 1$ is $1 - 1/e$. Thus, $y$ is not differentiable at $x = 1$.

**Exercise 2.4**

4. Solution: $x \sin y + y \cos x - \frac{1}{2}y^2 = c$.

14. Solution: $x + y + xy - 3 \ln |xy| = c$.

26. Hint: use the integration table for $1/(1 + y^2)$. Solution: $xy^2 - y\cos x - \tan^{-1} y = -1 - \pi/4$.

35. Solution: $\frac{10}{3}e^{3x} - 2ye^{3x} + x = c$.

40. No. For the interval where the integrating factor $\mu = 0$, the second equation may have a solution $y(x)$ that is not a solution of the first equation.

43. Hint: $dF(x, y) = F_x(x, y)dx + F_y(x, y)dy$. Try to compute $d\left(\sqrt{x^2 + y^2}\right)$.

Solution: $\sqrt{x^2 + y^2} = x + c$.