Exercise 1.1

12. Hint: computing dy/dt.

18. Possible interval of definitions: $(\pi/2, 5\pi/2)$, $(5\pi/2, 9\pi/2)$, …, etc.

30. Hint: check the continuity of $y(x)$ at $x = 0$.

45. Hint: check your calculus book. Each question has a family of solutions.

58. Solution: $x^2y'' - 2xy' + 2y = 0$.

60. Solution (a) $y = 5$ (b) $(-\infty, 5)$ and $(5, \infty)$.

Exercise 1.2

7. Solution: $x = -\cos t + 8 \sin t$.

22. Solution: any region where $y \neq -1$.

30. Solution:

(b) Solving $y(0) = \tan C = 0$, we have $C = 0$ and $y = \tan x$. Since tan x is discontinuous at $x = \pm\pi/2$, the solution is not defined on $(-2, 2)$ because it contains $\pm\pi/2$.
(c) The largest interval of definition on which the solution can exist is $(-\pi/2, \pi/2)$.

46. Solution: $y = 2x^3 - x^2 - 5x + 8$.

49. If the solution is tangent to the x-axis at $(x_0, 0)$, then $y' = 0$ when $x = x_0$ and $y = 0$. Substituting these values into $y' + 2y = 3x_0 - 6$ we get $0 + 0 = 3x_0 - 6$ or $x_0 = 2$.