Expected k-Coverage in Wireless Sensor Networks

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Abstract

We are concerned with wireless sensor networks where n sensors are independently and uniformly distributed at random in a finite plane. Events that are within a fixed distance from some sensor are assumed to be detectable and the sensor is said to cover that point. In this paper, we have formulated an exact mathematical expression for the expected area that can be covered by at least k out of n sensors. Our results are important in predicting the degree of coverage a sensor network may provide and in determining related parameters (sensory range, number of sensors, etc.) for a desired level of coverage. We demonstrate the utility of our results by presenting a node scheduling scheme that conserves energy while retaining network coverage. Additional simulation results have confirmed the accuracy of our analysis.

Key words: Sensor network, coverage, border effect, node scheduling, uniform distribution

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1 Introduction

Rapid progress in wireless communications and micro-sensing MEMS technology has enabled the deployment of wireless sensor networks. A wireless sensor network consists of a large number of sensor nodes deployed in a region of interest. Each sensor node is capable of collecting, storing, and processing environmental information, and communicating with other sensors. The position of sensor nodes need not be engineered or predetermined [1] for the reason of the enormous number of sensors involved [2] or the need to deploy sensors in inaccessible terrains [1]. Due to technical limitations, each sensor node can detect only events that are within some range from it. A piece of area in the deployment region is said to be *covered* if every point in this area is within the sensory range of some sensor. In this paper, we are concerned with a fundamental property of such network: the area that can be covered by at least k out of n sensors randomly placed in a bounded region. This is referred to as k-coverage [3,4] and the problem of evaluating k-coverage is a form of so-called *coverage problem*.

In the literature, the coverage problem has been formulated in various ways. A related but different formulation is asking how to effectively cover a given region. For example, the *Art Gallery Problem* is to determine the number of guards/cameras and the position of each guard/camera that are necessary to visually cover a polygonal region (the art gallery) [5]. Shakkottai et al. [6] have considered the necessary and sufficient conditions for covering a sensor network with nodes arranged in a grid over a square region. The coverage problem has also been formulated as to determine whether or how well a given set of sensors covers a region [3]. In [7], Meguerdichian et al. defined worst and

best case coverage problems, which are to identify regions of low and high observability, respectively. Geometry techniques such as Voronoi diagram and Delaunay triangulation have been used in solving these problems [7,8]. For other definitions of coverage problem, refer to the survey in [9].

In the aforementioned context, one either needs to determine (as an output) or is given (as an input) the exact position of every sensor. In contrast, it is the distribution of sensor positions rather than exact position of every sensor that is assumed in our problem setting.

The problem of estimating k-coverage is complicated by two factors. First, region covered by each sensor may overlap one another in a stochastic way. Second, a sensor placed near the border of the deployment region will cover less area than sensors placed midway, since not all its disk-shaped sensory region will be within the deployment area. This is referred to as *border effects*. Prior work [10,11] established approximations or asymptotic bounds for 1-coverage problem. In contrast, we have formulated an exact mathematical expression for expected k-coverage in face of border effects. To the best knowledge of the authors, this is the first study that achieves this. A direct application of our result is that given a deployment area and the number of sensors with their sensory range, one can easily point out what level of coverage can be expected. Equivalently, given sensor's sensory range and the expected coverage ratio, one can estimate the number of sensors to be deployed. For a power conserving scheme that allows each sensor to periodically power off its sensory circuitry without coordinations with others, our finding helps in determining the active-to-sleep ratio for a desired network coverage.

The rest of the paper is organized as follows. Problem definition and related

work are presented in the next section. Section 3 analyzes the expected network coverage. Section 4 discusses the applications of our finding, including a node scheduling scheme. Simulation model and numerical results are described in Section 5. Section 6 concludes our work.

2 Problem Definition and Related Work

We assume that each sensor can detect events that are within distance r from it, where r is called *sensory range*. The area of the region that is covered by a sensor is defined to be the sensor's *node coverage*. Let N be a random variable denoting a node's coverage. N is πr^2 if the sensor's sensory region is properly contained in the deployment area. However, when a sensor is placed near the border of the deployment region, N is expected to be less than πr^2 due to border effects. A region is said to be k-covered if every location within it is covered by at least k sensors. Define k-coverage to be the size of the k-covered region after a number of sensors have been randomly placed. We want to express the expected value of k-coverage in terms of $E[N]$.

Traditionally, only 1-coverage is of interest. In [11], Philips et al. analyzed the condition that a given area is 1-covered with high probability by randomly located circles. Their analysis was done under the assumption of Poisson point process [11–15], which assumes a fixed density of nodes λ instead of the exact number of nodes n. With this modeling, whether an elemental area ds contains exactly one node is a binomial distribution with probability $ds\lambda$. For a sufficiently large number of nodes deployed within a sufficiently large system area (but λ remains constant), the node degree can be approximated by a Poisson distribution with mean $\lambda\pi r^2$ [14]. Philips et al. proved that, for any

$\epsilon > 0$, if

$$r = \sqrt{\frac{(1+\epsilon)\ln A}{\pi\lambda}},$$

then $\lim_{A\to\infty} \Pr[\text{the deployment region is 1-covered}] = 1$. Since the obtained results hold on the condition that the system area approaches infinity, where border effects become insignificant, the results are only approximations when applied to reasonable-size deployment region. Furthermore, they were sorely concerned with the condition of fully covering a deployment region; their result cannot be used to estimate the coverage degree of an arbitrary given network scenario.

The expected area that n randomly placed circles may cover in a plane (i.e., 1-coverage) has been analyzed by Hall [10]. He avoided border effects by using the so-called torus convention, which models the deployment region as a torus such that a sensor's sensory region is considered completely within the deployment area. Let A denote the area of the deployment region. Hall has shown that when $n/A \to \lambda$, where $0 < \lambda < \infty$, the ratio of uncovered area in the deployment region approaches $\exp(-\lambda\pi r^2)$ as r increases. Here πr^2 is the node coverage with torus convention.

Although Hall's estimate is only an asymptotic result, we found through experiments that it provides good estimates to a certain degree (details will be presented later). In this paper, we take a different approach and obtain a result that improves the precision of Hall's 1-coverage estimate. The improvement is particularly significant when the network is not fully covered.

We analyze k-coverage based on our estimate of 1-coverage. The degree of coverage is considered a measure of quality of service (QoS) that a sensor

network provides. High QoS is essential for applications that demands high degree of accuracy or reliability. An example is distributed data fusion [16], which is the process of automatic combining or aggregating sensed data from *multiple* sensors.

Network coverage is central to node scheduling schemes that conserve energy by powering off redundant nodes while retaining network coverage. Node scheduling involves the decisions of when and which node can enter power-saving or sleep mode. Based on how these decisions are made, existing approaches can be classified as coordinated or uncoordinated ones. A coordinated coverage-preserving node scheduling scheme presented in [17] demands that each sensor advertises its location information and listens to advertisements from neighbors. After calculating its coverage and its neighbors', a node can determine if it is eligible to turn off its sensory circuitry without reducing overall network coverage. To avoid potential "uncovered hole" due to simultaneous turning off, a back-off protocol is proposed that requires each off-duty eligible sensor to listen to other sensor's status advertisement and, if necessary, announce its own after a random back-off time period expires. The behaviors of other coordinated schemes [18–20] are similar to [17] in that they all require the exchanges of location information and eligibility status.

Cărbunar et al. [21] transform the problem of detecting redundant sensors to that of computing Voronoi diagrams. Node location information is required in their scheme to compute the Voronoi diagram corresponding to the current node deployment. Xing et al. [4] also exploit Voronoi diagram to ensure k-coverage. They have shown that k-coverage is ensured if every critical point (where two sensor's coverage areas intersect or a sensor's coverage area and border line intersect) is covered by at least k sensors. The protocol they pro-

posed needs location information of every sensor as well.

With location information in hand, coordinated node scheduling [17–21,4] can ensure 100% network coverage. However, the requirement of location information may not be practical if energy-hungry GPS (Global Positioning System) device is assumed for this purpose. Moreover, it is questionable whether the energy gained by turning-off sensors could compensate energy loss due to coordination. PEAS [22] is a coordinated node scheduling scheme that demands no location information. Nodes in PEAS periodically alternate between sleep and working modes. When a node wakes up from sleep mode, it can enter sleep mode again if a "probe" message can be received from any working neighbor. PEAS does not guarantee 100% network coverage, yet energy has to be consumed on transmitting and receiving probe messages.

A uncoordinated scheme, on the other hand, demands neither positioning nor communications overhead. However, it is intrinsic that 100% network coverage cannot be guaranteed. In this paper, we present a uncoordinated node scheduling scheme that ensures *expected* network coverage.

3 Network Coverage Estimate

The deployment of n sensors can be modeled as a stochastic process that places sensors one by one according to a uniform distribution over R. For all $1 \leq i \leq n$, let N_i denote the size of the region that is covered by the i-th placed sensor. N_i's are iid random variables with p.d.f. $1/A$ over R, where A

is the size of R. Therefore,

$$E[N_i] = E[N] = \frac{1}{A} \iint_R d(x,y)dydx, \tag{1}$$

where $d(x,y)$ denotes the area covered by a node located at location $(x,y) \in R$. When border effects are not taken into account, $d(x,y) = \pi r^2$ for all $(x,y) \in R$ and $E[N] = \pi r^2$. We shall derive $E[N]$ with the consideration of border effects latter in this section.

Let us start with 1-coverage, based on which the estimate of k-coverage can be obtained. When a node is placed, only a portion of its node coverage gives extra 1-coverage. Let X_i denote the extra 1-coverage area contributed by the i-th placed sensor and C_i be the random variable denoting the size of the 1-covered region collectively offered by i randomly placed nodes. We have $E[C_1] = E[X_1] = E[N]$ and $C_i = C_{i-1} + X_i$ for all i, $2 \leq i \leq n$. In the latter case, $E[C_i] = E[C_{i-1}+X_i]$. Although C_{i-1} and X_i are correlated (a larger C_{i-1} often implies a smaller X_i and vise versa), we still have $E[C_i] = E[C_{i-1}]+E[X_i]$ due to the linearity of expected value (which states that, given m random variables R_i, where $i = 1$ to m, $E[R_1 + R_2 + \cdots + R_m] = E[R_1] + E[R_2] + \cdots + E[R_m]$ regardless whether R_i's are independent to each other [23]). Let $F_i = X_i/N_i$ be the proportion of the extra coverage area contributed by ith placed sensor to its node coverage. It follows that $E[C_i] = E[C_{i-1}] + E[F_iN_i]$.

If border effects are ignored, $E[N_i] = \pi r^2$ by (1), a constant that is independent of F_i, so $E[F_iN_i] = E[F_i] \times E[N_i]$. If border effects must be considered, F_i and N_i are correlated [1]. This can be justified as a smaller N_i implies that the

[1] In fact, it is border effects that makes F_i and N_i dependent. Border effects are also the cause of the dependency between any two links in MANETs [24].

ith node is closer to the boundary, while a larger N_i implies that the node is around the central region. Given C_{i-1}, the value of N_i thus has an effect on the distribution of F_i, though the effect may not be significant. Nevertheless, we propose to approximate $E[F_i N_i]$ by $E[F_i] \times E[N]$, where $E[N]$ is the expected node coverage when border effects are taken into account.

As sensor nodes are uniformly distributed, F_i is expected to be the proportion of the uncovered area to the whole deployment area. Thus we have

$$E[F_i] = \frac{A - E[C_{n-1}]}{A}.$$

It turns out that

$$E[C_i] = E[C_{i-1}] + \left(1 - \frac{E[C_{i-1}]}{A}\right) E[N]. \tag{2}$$

Since $E[C_1] = E[N]$, solving $E[C_n]$ by (2) yields

$$E[C_n] = \left[1 - \left(1 - \frac{E[N]}{A}\right)^n\right] A. \tag{3}$$

$E[N]/A$ is known to be the probability of link occurrence p if the sensory range is viewed as the range of radio communications [24]. Therefore, (3) can also be expressed as $E[C_n] = [1 - (1-p)^n]A$.

Eq. (3) holds for any shape of deployment region as well as for any shape of node's coverage region. It is consistent with the intuition that $\lim_{n\to\infty} E[C_n] = A$ and the experimental observation [7] that after deploying some number of sensors, additional sensors do not improve 1-coverage significantly.

Now we extend the result to general k-coverage cases. For all $0 \leq i \leq n$ and $0 \leq j \leq k$, we define the following random variables:

- C_i^j: the size of the j-covered area after i nodes have been randomly placed. Note that $C_i^0 = A$ and $C_i^1 = C_i$ for all i and $C_i^j = 0$ for all $i < j$.
- X_i^j: the extra area contributed by the i-th placed sensor to the size of j-covered region.
- F_i^j: the proportion of X_i^j to N_i.

By definition, $E[C_i^j] = E[C_{i-1}^j] + E[X_i^j]$ for all $i > j$. We also propose to approximate $E[X_i^j]$ by $E[F_i^j] \times E[N]$. F_i^j is expected to be the proportion of the area that is *exactly* covered by $j-1$ out of $i-1$ sensors to the whole deployment area. Thus we have

$$E[F_i^j] = \frac{E[C_{i-1}^{j-1} - C_{i-1}^j]}{A} = \frac{E[C_{i-1}^{j-1}] - E[C_{i-1}^j]}{A}.$$

It follows that

$$\begin{aligned} E[C_i^j] &= E[C_{i-1}^j] + \left(\frac{E[C_{i-1}^{j-1}] - E[C_{i-1}^j]}{A} \right) E[N] \\ &= (1-p)E[C_{i-1}^j] + pE[C_{i-1}^{j-1}], \end{aligned} \tag{4}$$

where $p = E[N]/A$. Expanding the right-hand side recursively, we obtain

$$E[C_i^j] = \sum_{t=0}^{d} \binom{d}{t} p^{d-t}(1-p)^t E[C_{i-d}^{j-d+t}] \tag{5}$$

for all integer d, $0 \leq d \leq i-j$. It is not efficient to compute $E[C_n^k]$ by applying (5). In fact, an efficient approach to computing $E[C_n^k]$ is by way of dynamic programming [25], where the computation of $E[C_n^k]$ is carried out as a process

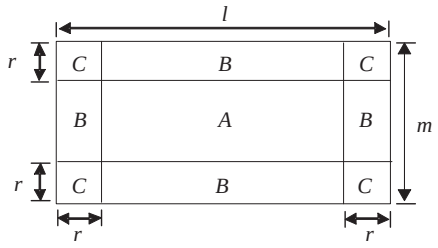

Fig. 1. Regions partitioning an $l \times m$ rectangle R.

of filling a $(n+1) \times (k+1)$ table $c(0 \ldots n, 0 \ldots k)$. Some entries of the table are already known ($c(i,0) = A$ for all i and $c(i,j) = 0$ for all $i < j$); some can be derived by Eq. (3) ($c(i,1) = C_i$ for all i); and the others can be computed by Eq. (4). The time complexity of this approach is $O(nk)$.

Our estimate of network coverage relies on the estimate of node coverage. Let us focus on $l \times m$ rectangular deployment region and disk-shaped sensory region centered at the sensor with sensory range r. We have $A = lm$ and, if border effects are not considered, $E[N] = \pi r^2$. Eq. (3) becomes

$$E[C_n] = \left[1 - \left(1 - \frac{\pi r^2}{lm}\right)^n\right] lm. \tag{6}$$

This is a rough estimation for expected network coverage. In the following, we shall find the value of $E[N]$ in face of border effects with the restriction that $r \leq \min(l,m)/2$. In accordance with the location-dependent nature of coverage, we partition deployment region R into three types of sub-regions, as depicted in Fig. 1.

Let A, B, C represent the events that a sensor node is located in sub-regions A, B, and C, respectively. It follows that

$$E[N] = \Pr[A]\phi_A + \Pr[B]\phi_B + \Pr[C]\phi_C, \tag{7}$$

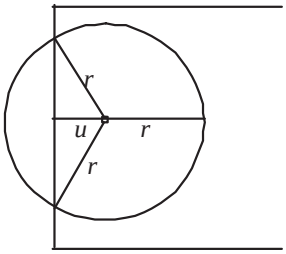

Fig. 2. A sensor node located in region B.

where ϕ_i denotes the expected coverage when the sensor is located in region i. Since sensor's location is determined at random by uniform distribution, we have

$$\Pr[A] = \frac{(l-2r)(m-2r)}{lm}, \Pr[B] = \frac{2r(l+m-4r)}{lm},$$
$$\text{and } \Pr[C] = \frac{4r^2}{lm}. \tag{8}$$

We already know $\phi_A = \pi r^2$. In the following, we are devoted to estimating ϕ_B and ϕ_C.

Computing ϕ_B

Let u denote the distance from a node located in B to the border of R (see Fig. 2). For a given u the overlapped area of the sensor's sensory region and the deployment region is

$$f_B(u) = u\sqrt{r^2-u^2} + \left(\pi - \arccos\left(\frac{u}{r}\right)\right) r^2.$$

Since $0 \le u \le r$, ϕ_B can be computed as

$$\frac{1}{r}\int_0^r f_B(u)du = \frac{1}{r}\left(\int_0^r u\sqrt{r^2-u^2}\,du\right) + \pi r\int_0^r du - r\int_0^r \arccos\left(\frac{u}{r}\right)du.$$

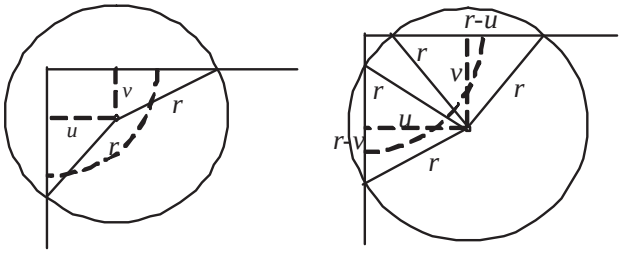

Fig. 3. Two cases of a sensor's location in region C.

It turns out that

$$\phi_B = \left(\pi - \frac{2}{3}\right) r^2. \tag{9}$$

Computing ϕ_C

Let the distances from a node located in C to the two borders of the rectangle be u and v, respectively (refer to Fig. 3). Depending on the location of the sensor node, two cases are possible.

1) The distance to the corner is less than r (Fig. 3a).
2) The distance to the corner is larger than or equal to r (Fig. 3b).

Let ϕ_{C1} and ϕ_{C2} denote the expected coverage in Cases 1 and 2, respectively. We have

$$\phi_C = \Pr[C_1|C]\ \phi_{C1} + \Pr[C_2|C]\ \phi_{C2}, \tag{10}$$

where C_1 and C_2 denote the events that the location of the node belongs to Cases 1 and 2, respectively. Due to uniform distribution of node's location, $\Pr[C_1|C]$ and $\Pr[C_2|C]$ accounts for the proportion of the area where the re-

spect case is concerned. Thus we have

$$\Pr[C_1|C] = \frac{1/4\,\pi r^2}{r^2} = \frac{\pi}{4} \text{ and } \Pr[C_2|C] = 1 - \frac{\pi}{4}. \tag{11}$$

We then compute ϕ_{C1}. Let $f_{C1}(u, v)$ denote the overlapped area of the node's sensory region and the deployment region in Case 1. By geometry we have (refer to Fig. 3a)

$$f_{C1}(u,v) = uv + \frac{u\sqrt{r^2-u^2}}{2} + \frac{v\sqrt{r^2-v^2}}{2} + \left(1 - \frac{\arccos(\frac{u}{r}) + \arccos(\frac{v}{r}) + \frac{\pi}{2}}{2\pi}\right)\pi r^2.$$

The expected area is

$$\phi_{C1} = \frac{1}{\frac{1}{4}\pi r^2} \int_0^r \int_0^{\sqrt{r^2-u^2}} f_{C1}(u,v)\, dv\, du.$$

Due to space limitation, we omit tedious computation details here and simply show the result (for details, refer to [26]).

$$\phi_{C1} = \frac{(\pi^2+1)r^2}{2\pi}. \tag{12}$$

Let $f_{C2}(u, v)$ denote the overlapped area of the node's sensory region and the deployment region in Case 2. We have (refer to Fig. 3b)

$$f_{C2}(u,v) = u\sqrt{r^2-u^2} + v\sqrt{r^2-v^2} + \left(1 - \frac{\arccos(\frac{u}{r}) + \arccos(\frac{v}{r})}{\pi}\right)\pi r^2.$$

Similar technique used in computing ϕ_{C1} can be used here. It turns out that

$$\phi_{C2} = \frac{4r^2(\pi - \frac{4}{3} - \frac{\pi^2}{8})}{4-\pi}. \tag{13}$$

By (10), (11), (12), and (13), we have

$$\phi_C = \left(\pi - \frac{29}{24}\right) r^2. \tag{14}$$

We summarize all derived results by the following two theorems.

Theorem 1 *If a sensor node with sensory range r is uniformly distributed at random in an $l \times m$ rectangular region ($r \leq min(l, m)/2$), its expected coverage is*

$$E[N] = \frac{\frac{1}{2}r^4 - \frac{4}{3}lr^3 - \frac{4}{3}mr^3 + \pi r^2 ml}{ml}.$$

Proof: It can be derived by (7), (8), (9), (14), and the knowledge that $\phi_A = \pi r^2$. □

Theorem 2 *When n sensor nodes each with sensory range r are uniformly distributed at random in an $l \times m$ rectangle ($r \leq min(l, m)/2$), the expected area collectively covered by these sensors is*

$$E[C_n] = \left[1 - \left(1 - \frac{\frac{1}{2}r^4 - \frac{4}{3}lr^3 - \frac{4}{3}mr^3 + \pi r^2 ml}{m^2 l^2}\right)^n\right] lm.$$

Proof: We have $A = lm$ for an $l \times m$ rectangle. By Theorem 1 and (3), we obtain the result. □

4 Discussions

Our theoretical finding is useful in predicting the degree of coverage a sensor network may provide. For example, if 25 sensor nodes with sensory range 100 are uniformly distributed in 1000×1000 rectangle, 51.8% (55.0% by the rough

estimation) of the deployment region is expected to be 1-covered. If we double the number of sensors, the result is increased to 76.8% (79.7% by the rough estimation).

The result can also be used to determine related parameters for a desired network coverage. Define expected network coverage ratio (ENCR) to be $E[C_n^1]/A$. Assuming a fixed sensory range, the following result can be used to determine the minimal number of sensor nodes required for a desired ENCR.

Lemma 3 *Consider a deployment region of size A. Given a fixed sensory range such that the expected node coverage is $E[N]$, the number of sensor nodes needed for ENCR $\geq 1-\epsilon$, where $0 < \epsilon < 1$, is at least*

$$\frac{\ln \epsilon}{\ln\left(1 - \frac{E[N]}{A}\right)}.$$

Proof: We are given the condition

$$1 - \epsilon \leq \left[1 - \left(1 - \frac{E[N]}{A}\right)^n\right] < 1.$$

So we have

$$0 < \left(1 - \frac{E[N]}{A}\right)^n \leq \epsilon < 1,$$

which implies

$$n \ln\left(1 - \frac{E[N]}{A}\right) \leq \ln \epsilon < 0.$$

Since $\ln(1 - E[N]/A) < 0$, we then have

$$n\left[-\ln\left(1 - \frac{E[N]}{A}\right)\right] \geq -\ln \epsilon.$$

It turns out that

$$n \geq \frac{-\ln \epsilon}{-\ln\left(1 - \frac{E[N]}{A}\right)} = \frac{\ln \epsilon}{\ln\left(1 - \frac{E[N]}{A}\right)}.$$

□

By Lemma 3, for more than 99% of the deployment region being 1-covered in the previous example, the number of sensors should be increased to 158 or more.

In case when sensory range r is also tunable, we may adjust both n and r to obtain a desired ENCR. The interesting thing is, whatever n and r are set for a particular ENCR, the expected number of communication links per node (i.e., expected link degree) is bounded.

Theorem 4 *If the radio communication range of every node is the same as the sensory range, the expected link degree is upper-bounded by* $-\ln \epsilon$ *for ENCR* $= 1 - \epsilon$, *where* $0 < \epsilon < 1$.

Proof: Yen and Yu [24] have shown that the expected link degree in a n-node network is $f(n) = (n-1)p$, where p is the probability of link occurrence. Recall that ENCR can be expressed in terms of p as $1 - (1-p)^n$. Letting it be $1 - \epsilon$, where $0 < \epsilon < 1$, we have $p = 1 - \epsilon^{\frac{1}{n}}$ and the expected link degree is $f(n) = (n-1)(1 - \epsilon^{\frac{1}{n}})$. Since $f'(n) = (1 - \epsilon^{\frac{1}{n}}) + (n-1)(n^{-2}\epsilon^{\frac{1}{n}} \ln \epsilon) > 0$ for all $n > 1$, $f(n)$ is monotonically increasing when $n > 1$. To derive the limit of $f(n)$ when n approaches infinity, let $t = 1/n$ and we have

$$\lim_{n\to\infty} (n-1)(1 - \epsilon^{\frac{1}{n}}) = \lim_{t\to 0} \frac{(1-t)(1-\epsilon^t)}{t}.$$

By L'Hôpital's rule,

$$\lim_{t\to 0}\frac{(1-t)(1-\epsilon^t)}{t} = \lim_{t\to 0}\frac{-(1-\epsilon^t)+(1-t)(-\epsilon^t\ln\epsilon)}{1}$$
$$= -\ln\epsilon.$$

Therefore, the expected link degree is upper-bounded by $-\ln\epsilon$. $\square$

Higher link degree usually indicates higher degree of channel contentions and thus poor link performance in case of contention-based medium access control (MAC) protocol. Theorem 4 therefore implies that if contention-based MAC protocol (such as CSMA/CA) is used in sensor networks, the degree of channel contentions can be bounded yet a particular ENCR can be ensured.

The result also has theoretical relevance to other fundamental properties such as network connectivity. Let r_t denote the radio communication range of every sensor. It has been pointed out [15] that, given $r_t = 2r$, a set of communication units are in the same connected component (connected) if the area jointly 1-covered by these units (with sensory range r) is not partitioned. Intuitively, one would not expect separately covered area with a sufficiently high coverage ratio. In fact, it have been proven recently [20,4] that $r_t = 2r$ suffices to ensure network connectedness on the premise of 100% coverage ratio.

The probability of link occurrence becomes $4E[N]/A$ if $r_t = 2r$. Accordingly, the expected link degree is upper-bounded by $-4\ln\epsilon$ for ENCR $= 1-\epsilon$.

We now demonstrate the utility of our result by presenting a uncoordinated node scheduling scheme. It works as follows.

- Each node independently alternates between active and sleep modes. The decision of switching from active to sleep modes or vise versa is purely stochastic. The time periods of active and sleep modes are exponentially distributed random variables with means λ_a and λ_s, respectively.

- The probability of any node being in active mode initially is $p_a = \lambda_a/(\lambda_a + \lambda_s)$.

Suppose that a node has entered active and sleep modes for m times. The total time that the node stays in active and sleep modes are k-Erlang distributions with means $m\lambda_a$ and $m\lambda_s$, respectively. Therefore, the probability that a node is in active mode at any given time is $m\lambda_a/(m\lambda_a + m\lambda_s) = p_a$. Since the states of nodes are not correlated, the number of active nodes at any given time forms a binomial distribution with mean np_a. Therefore, np_a nodes are expected to be active at any time and the expected network coverage can be estimated by substituting np_a for n in Theorem 2. A merit of this approach is that, though the method is stochastic in nature, it is deterministic to set the values of parameters λ_a and λ_s for a desired network coverage. This is not possible without the help of our theoretical finding.

The above node scheduling scheme is similar to that proposed in [19], where all nodes randomly and independently switch operating modes on a time-slot basis. The assumption of time slots implies that all sensors are clock-synchronized, which incurs additional communications overhead. The authors have analyzed the probability of a point being uncovered under the assumption of Poisson point process. Given that a sensor is in active mode with probability p_a (calculated as a long-term average), the probability that a given point is uncovered in a given time slot has been shown to be $\exp(-p_a\lambda\pi r^2)$. This is consistent with Hall's result on the ratio of uncovered area, as the node scheduling effectively drops node density from λ to $p_a\lambda$.

5 Simulation Results

We conducted additional experiments to demonstrate the accuracy of our theoretical findings. A Monte Carlo algorithm [27] is used to calculate the size of k-covered region given a particular sensor deployment. It works as follows. We conducted 10,000 random tests for a given deployment. A point in the target area is randomly chosen in each test and the test successes if this point is covered by at least k sensors. Let p be the total number of tests that success. The k-covered area is $1000^2 \times p/10000$.

The simulation design for 1-coverage is as follows. The number of sensors n is varied 1 to 99 in increments of 2 and sensory range r is varied 1 to 491 in increments of 10. For each combination of n and r, we repeated 100 experiments and took an average on coverage area. In all experiments, sensor nodes are randomly uniformly distributed over a 1000×1000 rectangle.

We measured coverage ratio, the ratio of 1-coverage to the whole system area. Fig. 4(a)-(c) show results estimated by Theorem 2, Eq. (6), and Hall, respectively. Fig. 4(d) shows the results obtained from the experiments. The differences between theoretical estimations and the experimental results are shown in Figs. 5-7, where the difference is defined as value obtained by theoretical estimate minus that of experimental result. Table 1 lists means, standard deviations, maximum values, and minimum values of the differences.

We found that all theoretical predictions overestimate the coverage ratio at most cases. Furthermore, the degree of overestimate is high when the network is not fully covered and approaches zero when 100% coverage ratio is almost ensured. This can be explained as all estimates converge to 100% coverage

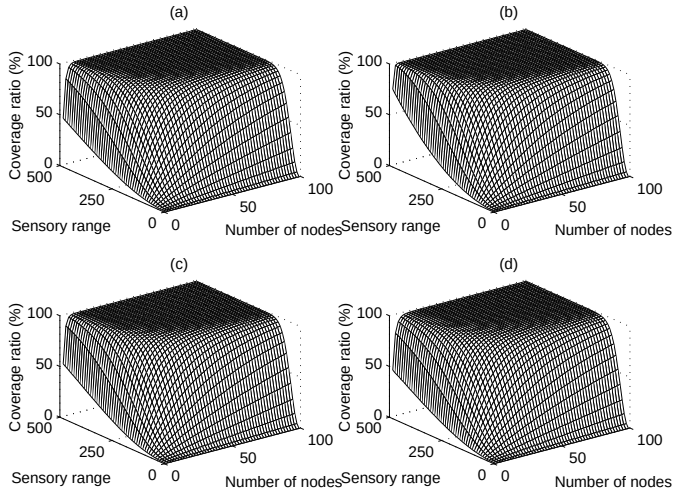

Fig. 4. Network coverage ratios in 1000×1000 rectangle, with n ranging from 1 to 99 and r ranging from 1 to 491. (a) Results estimated by Theorem 2. (b) Results estimated by Eq. (6). (c) Results estimated by Hall [10]. (d) Results obtained from simulations (averaged over 10,000 experiments).

Table 1

Differences of various estimations.

Estimation	Mean	Std. Deviation	Max	Min
Theorem 2	0.3912%	0.5747%	2.9037%	-0.1398%
Eq. (6)	1.6590%	2.9599%	28.6529%	-0.0002%
Hall's	1.1947%	1.6888%	8.0457%	-0.0002%

ratio when the number of nodes or the sensory range goes beyond some value. When numerous sensors are deployed but the sensory range is small enough so that the deployment region is not yet fully covered, the results are similar (Fig. 8). Based on the experimental results, we conclude that Theorem 2 is more accurate and has smaller variance than either Eq. (6) or Hall's estimate, particularly when the network is not completely covered.

In k-coverage experiments, we changed the number of sensors n (ranged from

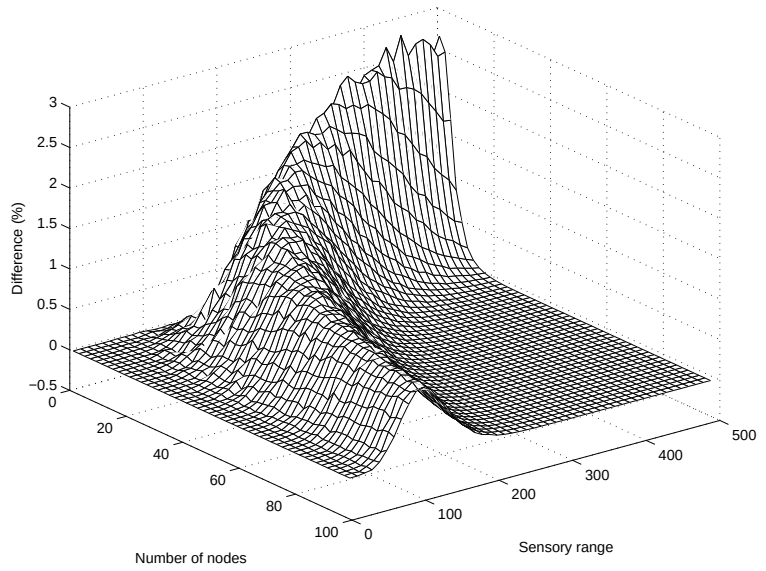

Fig. 5. Differences between Theorem 2's prediction and the experimental results.

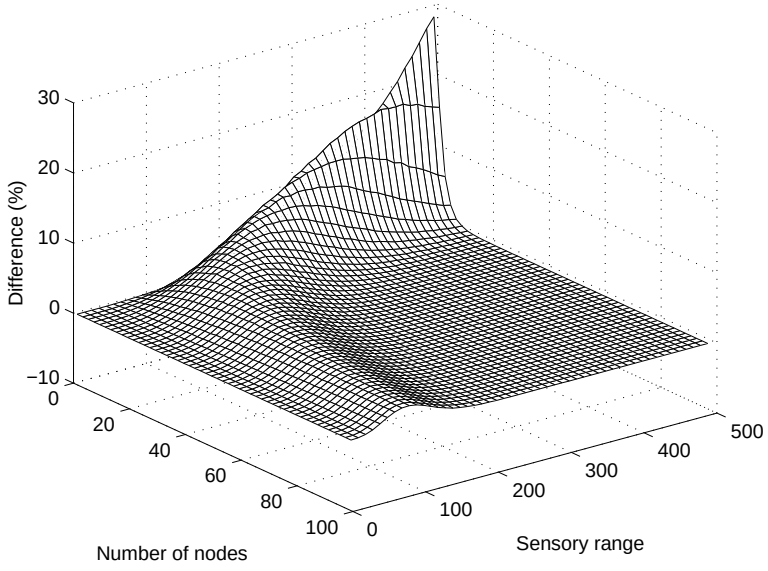

Fig. 6. Differences between Eq. (6)'s prediction and the experimental results.

1 to 199) and measured different k's (from 1 to 10). The sensory range r is fixed to 100 and the deployment region is assumed 1000×1000. For each combination of n and k, we repeated 100 experiments and took an average on the ratio of k-coverage to the whole system area.

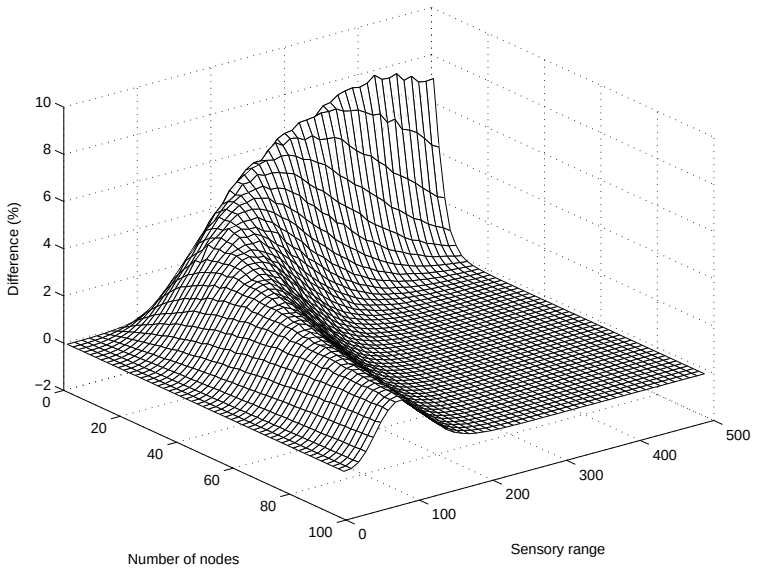

Fig. 7. Differences between Hall's prediction [10] and the experimental results.

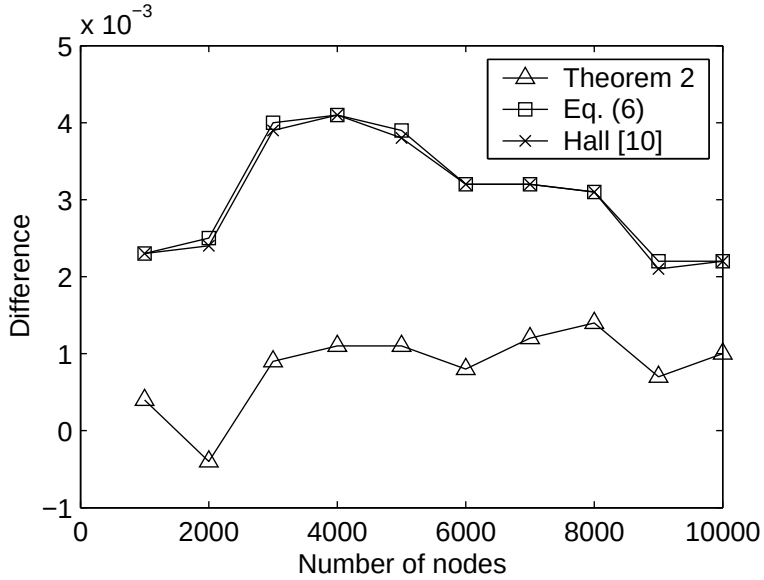

Fig. 8. Differences of all estimates when numerous sensors are deployed. The deployment region is 1000×1000 and the sensory range is 10.

Fig. 9 shows our estimates, while Fig. 10 shows the results obtained from the experiments. The differences between theoretical estimations and the experimental results are shown in Fig. 11. The mean, standard deviation, maximum value, and minimum value of the differences are 2.18×10^{-4}, 0.82×10^{-2},

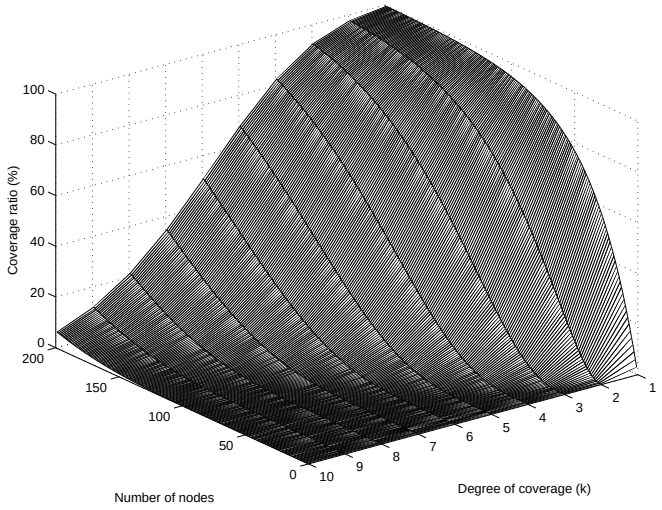

Fig. 9. Estimated k-coverage ratios in 1000×1000 rectangle, with n ranging from 1 to 199 and k ranging from 1 to 10.

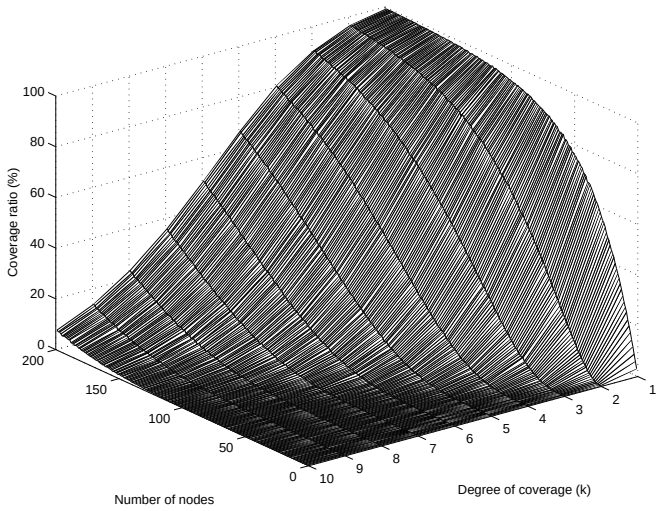

Fig. 10. Measured k-coverage ratios in 1000×1000 rectangle, with n ranging from 1 to 199 and k ranging from 1 to 10.

0.0253, and -0.0190, respectively. The results confirm that our estimate is accurate in general.

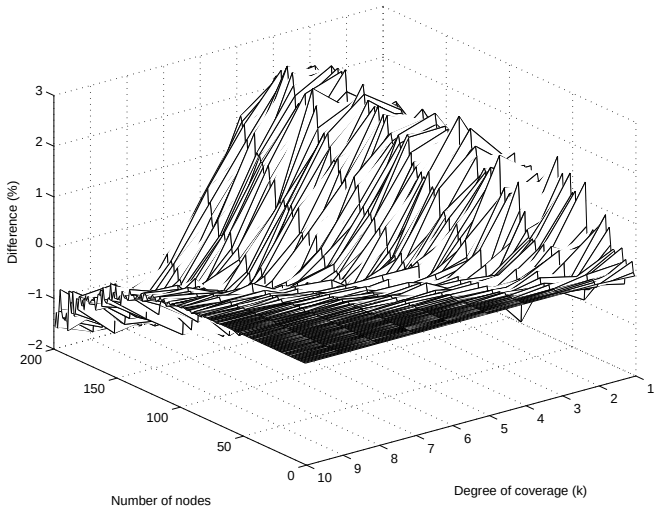

Fig. 11. Differences between theoretical estimations and the measured results.

6 Conclusions

We have analyzed the expected k-coverage offered by a number of randomly placed sensors with the consideration of border effects. We found that, although many combinations of n (the number of sensors) and r (sensory range) can be set for a particular expected 1-coverage ratio, the expected number of communication links per node has a upper bound that depends only on the desired expected 1-coverage ratio, not on any specific values of n and r. Our results have been exploited to design a stochastic node scheduling algorithm that conserves energy yet preserves network coverage. Additionally, simulation results have demonstrated the accuracy of our theoretical findings. We hope that our finding can be a step stone to the ultimate goal of characterizing other related network properties.

Acknowledgements

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