Link-Preserving Channel Assignment Game for Wireless Mesh Networks

Li-Hsing Yen and Bo-Rong Ye

***Abstract*—To deliver user traffic in a wireless mesh network, mesh stations equipped with multiple interfaces communicate with one another utilizing multiple orthogonal channels. Channel assignment is to assign one channel to each interface to minimize co-channel interference among wireless links while preserving link connectivity. The interference and connectivity objectives are generally conflicting. This paper first analyzes the probability of link connectivity when channels are randomly assigned to interfaces. We then propose a game-theoretic approach that jointly considers the two objectives with a unified payoff function. We prove that the proposed approach is an exact potential game, which guarantees stability in a finite time. We also prove the link-preserving property of the approach. Simulation results show that the proposed approach generally outperforms counterparts in terms of network interference when a moderate number of channels are available. For fairness of link interference, both the proposed approach and its variant outperform the counterparts.**

***Index Terms*—channel assignment; wireless mesh network; interference; connectivity; game theory**

I. Introduction

To provide wireless access service on a large geographical area, we need to deploy a number of access points connected via a wired or wireless infrastructure. The latter case corresponds to a wireless mesh network (WMN). Elemental devices in a WMN called mesh stations (Hiertz et al. 2010) connect with each other through wireless backhaul links to exchange control information and forward user traffic. To increase bandwidth capacity and enhance reliability, mesh stations are usually equipped with multiple wireless interfaces and allocated multiple orthogonal channels. Channel assignment is to allocate one channel to each interface so as to minimize co-channel interference (i.e., to maximize bandwidth capacity) while maximizing link connectivity (i.e., maximizing reliability).

We consider a decentralized channel assignment, where each mesh station autonomously assigns one channel to each of its interfaces. To successfully establish a link between two mesh stations, the selection of channels should meet two constraints. First, *common-channel constraint* demands that the two stations, one on each side of the link, should have at least one interface tuned to a common channel. An assignment that meets this constraint on every link is said to be *link-preserving*. Second, *interference constraint* demands that the interference experienced on the channel should be sufficiently low. If channels are statically assigned to interfaces, the assignment is also subject to *interface constraint*, i.e., the total number of channels assigned to a station cannot exceed the number of interfaces on that station.[1] With a limited number of interfaces and the interface constraint in mind, meeting the other two constraints are conflicting in nature. Fig. 1 shows a channel assignment example where each station has two wireless interfaces. Given the assignment results of all the neighbors of station p_1 as shown in the figure, p_1 may choose channel set $\{2, 4\}$ to meet the common-channel constraint for all of its three links. However, this choice also incurs a high degree of interference[2]. On the other hand, if p_1 chooses channel set $\{1, 3\}$ instead, the link between p_1 and p_4 will be broken as not meeting the common-channel constraint. However, this choice minimizes the degree of interference.

This study seeks a link-preserving channel assignment scheme that minimizes overall network interference. Existing approaches toward this goal are diverse. Some approaches superficially treated the link-preserving requirement (Jain et al. 2005, Ko et al. 2007, Skalli et al. 2007). It is also possible to ignore the link-preserving requirement initially when minimizing co-channel interference and then revise the results to meet this requirement at a later

[1] We assume that the number of available channels is at least as many as the number of interfaces, so the number of available channels is not another constraint.

[2] We shall show how to measure interference shortly.

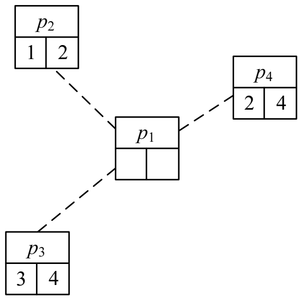

Fig. 1. A channel assignment example

stage (Subramanian et al. 2008). Another trend of design is to restrict the range of assignable channels so as to meet the common-channel constraint by the Pigeonhole principle (Yen & Huang 2015).

This paper first analyzes the probability of meeting the common-channel constraint for any link when channels are randomly assigned to interfaces. This is to demonstrate the need for an algorithmic design. We then propose a game-theoretic approach to the channel assignment problem in WMNs. It is natural to model the channel assignment problem as a game because players in the game, either mesh stations, interfaces, or links, have conflicting interests concerning co-channel interference. This is also the case in cognitive radio (CR) networks, where CR users perform spectrum sensing to avoid interference with other users among which no link is to establish (Southwell et al. 2014, Xu et al. 2012). In WMNs, however, we also need to meet the common-channel constraint, which has not yet been seriously considered in any non-cooperative game model for channel assignment. Many game models superficially treated or simply disregard the link connectivity issue (Chen et al. 2013, Song et al. 2008, Xiao et al. 2008, Yuan et al. 2010). Some game models (Chen & Zhong 2009, Gao & Wang 2008, Vallam et al. 2011, Yang et al. 2012) consider links as players so that the common-channel property is not a constraint but rather an implicit assumption. These games are subject to the interface constraint. Our prior work (Yen & Dai 2015) considered link connectivity as a requirement external to the game and dealt with it by limiting the range of assignable channels. To the best knowledge of the authors, the only game model that considers both co-channel interference and link connectivity is the work proposed by Duarte et al. (Duarte et al. 2012). This work models the channel assignment problem as a *cooperative* rather than a non-cooperative game.

The proposed *non-cooperative game* models mesh stations as players and channels as their strategies. The payoff of a player with a particular channel configuration is defined to be the gain of connectivity minus the cost of co-channel interference between neighboring links. We prove that this game model is an exact potential game (Monderer & Shapley 1996), which guarantees stability regardless game dynamics. We also prove that, if link connectivity is preserved initially, it will be preserved as well whenever the game ends. We conducted simulations to investigate the performance of the proposed approach. Compared with other link-preserving schemes, the proposed approach yielded the lowest network interference when a moderate number of channels are available. In terms of fairness of link interference, a variant of the proposed approach yields the best result.

The remainder of this paper is organized as follows: Background information and related work are presented in Section II. Section III analyzes the probability of meeting the common-channel constraint for any link if channels are randomly assigned. In the following section, we present a game model for the channel assignment problem and prove the stability and correctness of the proposed approach. Section V studies the performance of the proposed approach through simulations. The simulation results are compared with those of existing solutions. Section VI concludes this paper.

II. Background and Related Work

An intuitive way to meeting the common-channel constraint is to assign channels to links rather than individual interfaces, assuming that stations at both ends of each link should allocate an interface for the assignments. However, this approach may violate the interface constraint and thus need extra efforts to reduce channel usages. For example, Fig. 2 shows a channel assignment for all the links associated with station p_1. Though this result may minimize co-channel interference, p_2 has only two interfaces, not enough to accommodate three different channels demanded by the result.

Subramanian et al. (Subramanian et al. 2008) proposed a two-phase channel assignment algorithm. In the first phase, an algorithm with Tabu search (Hertz

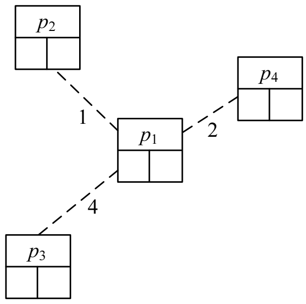

Fig. 2. Assigning channels to links

& de Werra 1987) assigns channels to links to minimize interference while disregarding the interface constraint. The algorithm then resolves all interface constraint violations by condensing channel usages in the second phase. Other researchers improved this two-phase approach by using simulated annealing (Chen & Chen 2015).

Connected Low Interference Channel Assignment (CLICA) (Marina et al. 2010) is a greedy approach that assigns channels in a station-by-station manner. When assigning channels to all links of a station, CLICA ensures connectivity by prioritizing the assignments of links that are likely to fail the common-channel constraint due to uncoordinated channels assignments among stations.

Assigning one channel to each interface trivially meets the interface constraint. The challenge is then to meet the common channel constraint. If channels are randomly assigned to interfaces, link-preserving is surely not guaranteed. We provide a probability analysis on link connectivity in Sec. III. However, if we confine the set of assignable channels for each mesh station, random channel assignment may meet the common channel constraint by the Pigeonhole principle (Yen & Dai 2015, Yen & Huang 2015).

There are several deterministic approaches to the common-channel constraint. A naive method is to uniformly assign Channel 1 to the first interface of each mesh station, Channel 2 to the second interface of each mesh station, and so forth (Jain et al. 2005). Another solution is to uniformly fix the channel of the first interface to a default one, and perform interference-minimization channel assignments only for the other interfaces (Chen et al. 2013, Ko et al. 2007, Skalli et al. 2007). These two approaches generally incur severe co-channel interference on the common or default channel.

Channel assignment schemes that meet the common-channel and interface constraints can be evaluated by the degree of interference incurred. There are two fundamental models for the impact of interference (Gupta & Kumar 2000). The physical model considers the aggregated intensity of interference on the same channel from all other transmitters at the receiver side. On the other hand, the protocol model only concerns whether the same channel is used by some transmitter within a limited interference range. What really matters in the protocol model is that the interference relationship between links is binary and symmetric. Here a link exists between two stations if these two stations are within the transmission range of each other. If we assume that the interference range is equal to the transmission range, the *adjacency* or *potential interference* relation on two links can be defined as follows.

Definition 1 (Link adjacency): For two links $l = (u, v)$ and $l' = (u', v')$, l and l' are *adjacent* to each other (denoted by $l \leftrightarrow l'$) iff (u, u'), (u, v'), (v, u'), or (v, v') is a link.

With this definition, two adjacent links do interfere with each other if they are assigned the same channel. Following (Kodialam & Nandagopal 2003) and (Subramanian et al. 2008), we define *network interference* to be the number of adjacent links that are assigned a common channel.

Definition 2 (Network interference): For a particular channel assignment, let $c(l_i)$ be the function that returns the channel assigned to a link l_i. The overall network interference I with respect to $c(\cdot)$ is

$$I = \sum_{l_u \leftrightarrow l_v} [c(l_u) = c(l_v)].$$

There have been many game-theoretic approaches to channel assignment problems. Many approaches (Chen et al. 2013, Song et al. 2008, Xiao et al. 2008, Yuan et al. 2010) aim at minimizing co-channel interference and do not address the link connectivity issue. In some other approaches (Chen & Zhong 2009, Gao & Wang 2008, Vallam et al. 2011, Yang et al. 2012), connectivity is not a problem simply because channels are allocated to links rather than individual interfaces. Duarte et al. (Duarte et al. 2012) formulated channel assignments with the consideration of link connectivity as a cooperative game where players have common interest. Their work assumes non-orthogonal channels and thus considers adjacent-channel interference in addition to co-channel interference. Yen and Dai (Yen & Dai 2015) assumed the physical interference model and proposed a non-cooperative game for interference-minimization channel assignments. Their work ensures link connectivity by limiting the range of channel selections. To summarize, to the best knowledge of the authors, our work here is the first non-cooperative game approach that ensures link connectivity by defining a utility function that incorporates the connectivity requirement with the impact of co-channel interference.

III. Probability Analysis on Link Connectivity

Connectivity or link-preserving becomes a probabilistic event if the range of channels to be allocated to interfaces of a station is not narrowed by the Pigeonhole principle. Assume that there are n stations numbered from 1 to n. Consider a link $l(i, j)$ between two stations i and j. Suppose that stations i and j have r_i and r_j interfaces, respectively. If i and j randomly and independently allocate channels from a given set of k channels (without repetitions) to their interfaces, the probability that stations i and j share at least one common channel is

$$p_{i,j} = \begin{cases} 1 & \text{if } r_i + r_j > k, \\ 1 - \dfrac{\binom{k-r_i}{r_j}}{\binom{k}{r_j}} & \text{otherwise.} \end{cases} \tag{1}$$

This is essentially the link probability concerning the common-channel constraint for the link between i and j with random channel assignments. Figure 3 shows the common-channel probabilities for the link between i and j with $k = 12$ channels.

A station is effectively *isolated* if it shares no common channel with any of its neighboring stations. Let N_i be the set of i's neighboring stations. The probability that station i becomes isolated using random channel assignments is

$$\prod_{j \in N_i} (1 - p_{i,j}) = \prod_{j \in N_i} \frac{\binom{k-r_i}{r_j}}{\binom{k}{r_j}}. \tag{2}$$

Suppose that every station has uniformly r interfaces and let $d_i = |N_i|$. If $2r > k$, then all the d_i links for each i meet the common-channel constraint by the Pigeonhole principle. Otherwise,

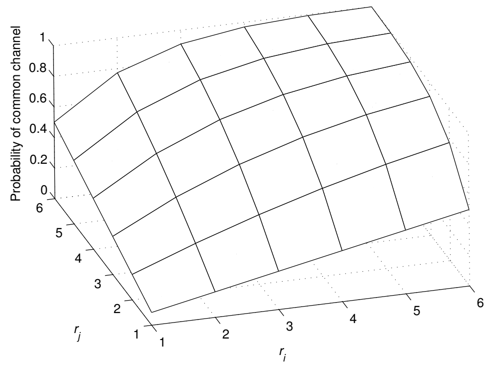

Fig. 3. Probability of common channel between i and j with $k = 12$ channels.

the probability that c out of d_i links ($0 \le c \le d_i$) meet the common-channel constraint is

$$p_c(d_i, c) = \binom{d_i}{c} \left(1 - \frac{\binom{k-r}{r}}{\binom{k}{r}}\right)^c \left(\frac{\binom{k-r}{r}}{\binom{k}{r}}\right)^{d_i - c}. \tag{3}$$

Let E_i, $1 \le i \le n$, be a random variable representing the number of links incident to station i that meet the common-channel constraint after random channel assignments. It follows that

$$\Pr[E_i = 0] = p_c(d_i, 0) = \left(\frac{\binom{k-r}{r}}{\binom{k}{r}}\right)^{d_i}. \tag{4}$$

The probability that there is some isolated station in the network is

$$\begin{aligned} \Pr[\vee_{i=1}^{n} (E_i = 0)] &\ge \max_{1 \le i \le n} \Pr[E_i = 0] \\ &= \left(\frac{\binom{k-r}{r}}{\binom{k}{r}}\right)^{d_{\min}}, \end{aligned} \tag{5}$$

where $d_{\min} = \min_{1 \le i \le n}\{d_i\}$. Therefore, the probability of no isolated station after channel assignments is upper-bounded by

$$1 - \left(\frac{\binom{k-r}{r}}{\binom{k}{r}}\right)^{d_{\min}}. \tag{6}$$

Figure 4 shows the upper bounds of this probability with $k = 12$ channels.

Concerning the link-preserving requirement, the probability that all of station i's links meet the common-channel constraint is

$$p_c(d_i, d_i) = \left(1 - \frac{\binom{k-r}{r}}{\binom{k}{r}}\right)^{d_i}. \tag{7}$$

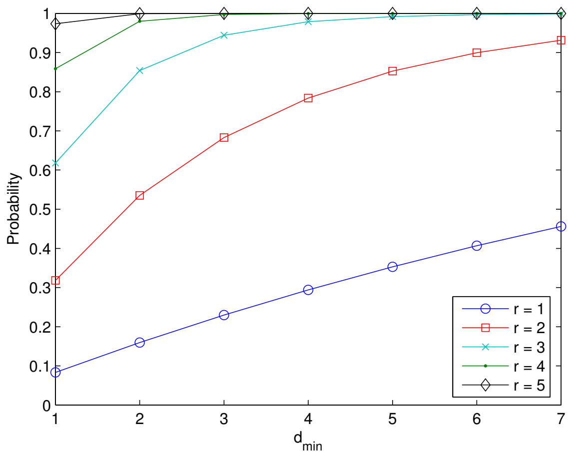

Fig. 4. Upper bounds of the no-isolated-station probability with $k = 12$ channels.

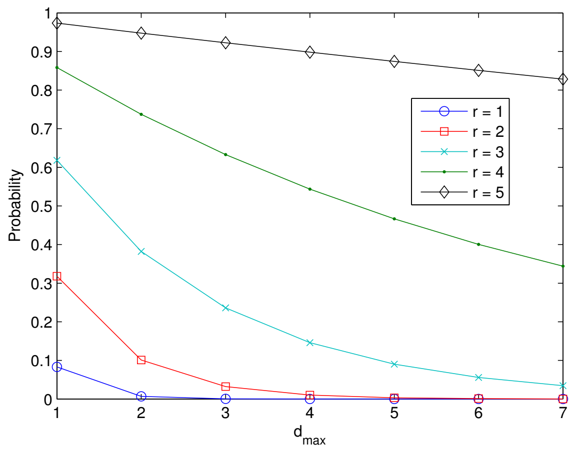

Fig. 5. Lower bounds of link-preserving probability with $k = 12$ channels.

The probability of meeting the common-channel constraint for every link after channel assignments is

$$\begin{aligned}\Pr[\wedge_{i=1}^{n}(E_i = d_i)] &\leq \min_{1\leq i\leq n} \Pr[E_i = d_i] \\ &= \left(1 - \frac{\binom{k-r}{r}}{\binom{k}{r}}\right)^{d_{\max}}, \end{aligned} \tag{8}$$

where $d_{\max} = \max_{1\leq i\leq n}\{d_i\}$. The probability of link-preserving is therefore upper-bounded by

$$\left(1 - \frac{\binom{k-r}{r}}{\binom{k}{r}}\right)^{d_{\max}}. \tag{9}$$

Figure 5 depicts the lower bounds of link-preserving probability with $k = 12$ channels.

IV. The Proposed Approach

We name the proposed approach link-preserving interference-minimization (LPIM) game. This section presents all the details about LPIM.

A. The LPIM Game

We model a backhaul network by a undirected *connectivity graph* $G = (P, E)$, where $P = \{p_1, p_2, \ldots, p_n\}$ is the set of mesh stations and E is the set of mesh station pairs such that $(p_i, p_j) \in E$ iff (p_i, p_j) is a link between mesh stations p_i and p_j, i.e., p_i and p_j are within the transmission range of each other. We assume that $C = \{k_1, k_2, \ldots, k_m\}$ is the set of orthogonal channels to be allocated. Each mesh station has at most $r_{\max}$ interfaces. The actual number of interfaces used by station p_i is $r_i = \min(r_{\max}, |N_i|)$, where $N_i = \{p_j|(p_i, p_j) \in E\}$ is the set of p_i's neighbors in the connectivity graph.

Mesh stations are players in the LPIM game. The strategy of player p_i is a vector $s_i = (c_1^i, c_2^i, \ldots, c_m^i)$, where $c_j^i = 1$ or 0 indicating whether p_i assigns channel k_j to one of its interfaces. The strategy s_i is subject to $\sum_{1\leq j\leq m} c_j^i = r_i$. All valid s_i's comprise p_i's strategy set S_i. The strategy space of the game is defined by $\Sigma = \Pi_{1\leq i\leq n} S_i$. A *strategy profile* $S \in \Sigma$ represents a channel configuration of the network. We sometimes express S as (s_i, s_{-i}), where s_{-i} indicates a tuple of all player's strategies other than p_i's.

Given a strategy profile, our objective is to define a utility function for each player that incorporates both the gains of connectivities and the impacts of co-channel interference. The challenge is to minimize overall co-channel interference while still preserving the connectivity of every link. To capture the gains of connectivities, we define

$$L_i(S) = \sum_{p_j \in N_i} C_i(s_i, s_j), \tag{10}$$

where

$$C_i(s_i, s_j) = \begin{cases} -|N_i| & \text{if } s_i \cdot s_j = 0 \\ 0 & \text{otherwise.} \end{cases} \tag{11}$$

Note that the dot product of s_i and s_j is the number of common channels assigned by both p_i and p_j. It equals zero only if p_i and p_j assign no common channel. On the other hand, this value also reflects the degree of co-channel interference caused by p_i with respect to S. Thus we define the impact as

$$I_i(S) = -\sum_{p_j \in N_i} (s_i \cdot s_j). \tag{12}$$

Combining the gains of connectivity and the impacts of interference, we have

$$t_i(S) = \beta L_i(S) + I_i(S), \tag{13}$$

where $\beta > r_{\max}$ is a constant to ensure that connectivity is always important than interference. Finally, the utility of p_i is defined to incorporate the gains and impacts of p_i itself and all its neighbors.

$$u_i(S) = t_i(S) + \sum_{p_j \in N_i} t_j(S). \tag{14}$$

It is not difficult to see that the game is an exact potential game (Monderer & Shapley 1996) with exact potential function defined as $\phi(S) = \sum_i t_i(S)$.

Theorem 1: $\phi(S) = \sum_i t_i(S)$ is an exact potential function for the LPIM game with utility function defined as (14).

Proof: Consider any player p_i that changes its strategy. Let the strategy profile before and after this change be S and $\bar{S}$, respectively. The difference of $\phi(\cdot)$ after and before the change is

$$\begin{aligned}\phi(\bar{S}) - \phi(S) = {} & t_i(\bar{S}) - t_i(S) \\ & + \sum_{p_j \in N_i} (t_j(\bar{S}) - t_j(S)) \\ & + \sum_{p_j \in P \setminus N_i} (t_j(\bar{S}) - t_j(S)).\end{aligned} \tag{15}$$

Because $t_j(\cdot)$'s for all $p_j \in P \setminus N_i$ are not affected by p_i's move, we have

$$\phi(\bar{S}) - \phi(S) = t_i(\bar{S}) - t_i(S) + \sum_{p_j \in N_i} (t_j(\bar{S}) - t_j(S)), \tag{16}$$

which is exactly the utility gain of p_i. ■

Theorem 1 indicates that the LPIM game always stabilizes when players change their strategies following the so-called *best-response rule*. This rule states that player p_i selects strategy s_i^* only if

$$s_i^* = \operatorname*{argmax}_{s_i \in S_i} u_i(s_i, s_{-i}). \tag{17}$$

Any game play sequence following the best-response rule is a *best-reply path* (Milchtaich 1996). For exact potential games, a best-reply path always ends at a Nash equilibrium.

Definition 3 (Nash equilibrium): Given a game $\Gamma = [P; \{S_i\}_{i=1}^n; \{u_i\}_{i=1}^n]$, a strategy profile $S = (s_1, s_2, \ldots, s_n)$ is a Nash equilibrium if $\forall i \in \{1, 2, \ldots, n\} : \forall s_i^* \in S_i :: u_i(s_i, s_{-i}) \geq u_i(s_i^*, s_{-i})$.

In fact, an exact potential game also ends at a Nash equilibrium if every player p_i follows the *better-response rule*, i.e., changing its strategy from s_i to s_i^* only if

$$u_i(s_i^*, s_{-i}) > u_i(s_i, s_{-i}). \tag{18}$$

We also need to ensure the connectivity of every link.

Theorem 2: If the connectivity of every link is ensured initially, i.e., $s_i \cdot s_j \neq 0$ for every $(p_i, p_j) \in E$, then the connectivity is still preserved when the LPIM game ends.

Proof: We prove this by showing that no player has the incentive to trade connectivity for the improvement of interference. For any player p_i, the highest interference value occurs when $s_i = s_j$ for all $p_j \in N_i$. In that case,

$$I_i(S) = -\sum_{p_j \in N_i} \min(r_i, r_j) \geq -r_{\max}|N_i|. \tag{19}$$

On the other hand, if p_i shares no common channel with some $p_j \in N_i$, we have

$$L_i(S) \leq -|N_i|. \tag{20}$$

Therefore,

$$\beta L_i(S) \leq -\beta|N_i| < -r_{\max}|N_i|. \tag{21}$$

By (19) and (21), the highest possible improvement of interference (from $-r_{\max}|N_i|$ to 0) brought by p_i's strategy change cannot compensate the loss of connectivity of any single link (p_i, p_j) (which is at least $-r_{\max}|N_i|$). This holds for p_i and all $p_j \in N_i$. Therefore, p_i will not break the connectivity of any link during game play. ■

Figure 6 shows the topology of a five-station mesh network with seven channels available for allocation. Common Channel Assignment (CCA) (Jain et al. 2005) is assumed to make initial channel configuration. For this Table I shows a best-reply path. Note that the path is not unique (For example, p_1 could choose $\{3, 5, 6\}$ instead of $\{2, 5, 7\}$ in the fourth step). Figure 7 shows the final channel configuration after the LPIM game ends.

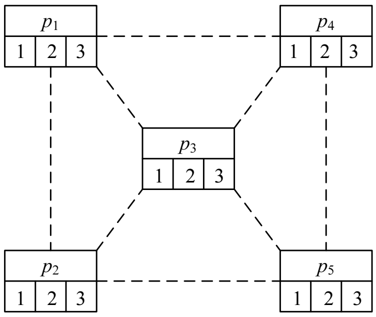

Fig. 6. Initial channel configuration of a five-station mesh network

TABLE I
A POSSIBLE BEST-REPLY PATH

Step	s_1	s_2	s_3	s_4	s_5
0	$\{1,2,3\}$	$\{1,2,3\}$	$\{1,2,3\}$	$\{1,2,3\}$	$\{1,2,3\}$
1	$\{1,2,3\}$	$\{1,2,3\}$	$\underline{\{1,4,5\}}$	$\{1,2,3\}$	$\{1,2,3\}$
2	$\{1,2,3\}$	$\{1,2,3\}$	$\{1,4,5\}$	$\underline{\{2,4,6\}}$	$\{1,2,3\}$
3	$\{1,2,3\}$	$\{1,2,3\}$	$\{1,4,5\}$	$\{2,4,6\}$	$\underline{\{3,5,6\}}$
4	$\underline{\{2,5,7\}}$	$\{1,2,3\}$	$\{1,4,5\}$	$\{2,4,6\}$	$\{3,5,6\}$

B. *One Channel Per Link*

The LPIM game ensures that when the game ends, stations at the two ends of any link share *at least one* common channel. It is possible that these two stations share two or more common channels, effectively creating multiple links between these two stations. The same situations also occur to other channel assignment schemes (Marina et al. 2010, Yen & Dai 2015, Yen & Huang 2015).

Multiple common channels between two neighboring stations in fact provide additional connectivity. However, the concurrent use of multiple channels causes self-interference. Therefore, we want to designate only one channel to use for the minimization of network interference. A straightforward way to designating one channel to each link is to randomly select one common channel among all. Yen and Dai (Yen & Dai 2015) proposed a simple heuristic that picks up a common channel that minimizes the amount of conflicts with the current channel assignments of adjacent links. This is also the approach adopted by LPIM.

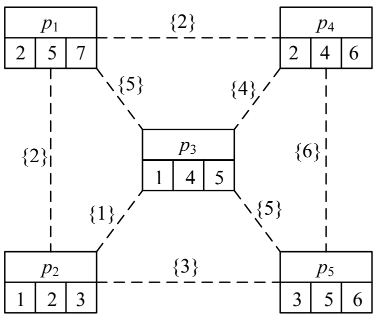

Fig. 7. Channel configuration of the five-station mesh network after LPIM ends

V. SIMULATION RESULTS

We conducted simulations on unit disk graphs (Clark et al. 1990) for performance evaluation. We randomly placed n mesh stations in a 1000 m $\times$ 1000 m area, where the communication range of each mesh station is assumed 200 m. A link exists between two stations if and only if these two stations are within the communication rage of each other. We precluded topologies with isolated stations. Each station had $r_{\text{max}} = 3$ interfaces. Total m channels were used.

LPIM can start with any channel configuration that meets the common-channel constraint. We found through experiments that the result of LPIM was not really sensitive to the initial configuration. For simplification, we used CCA (Jain et al. 2005) to perform initial channel assignment. Afterwards, players were randomly selected to make decisions. When making a decision, the player followed the better-response rule. The game ended when no player could increase its utility unilaterally.

Two representative approaches, CLICA (Marina et al. 2010) and Tabu (Subramanian et al. 2008), were tested and compared with LPIM. To investigate the design effectiveness of LPIM, we also tested a variant of LPIM called LPIM(PP) that considers only the impact of interference and adheres to the Pigeonhole principle when selecting channels. More explicitly, the utility function of each player p_i in LPIM(PP) is defined as

$$u_i(S) = -\sum_{p_j \in N_i} (s_i \cdot s_j) \tag{22}$$

with the condition that $c_k^i = 0$ for all p_i and $k > \min_{p_j \in N_i}\{r_i + r_j - 1\}$. LPIM(PP) is also an exact potential game (the proof is analogous to (Bilò et al. 2011)).

Since all these approaches are link-preserving, we measured averaged network interference for performance comparison. We are also interested in the distribution of interference experienced by all links. To quantify the degree of fairness, we measure

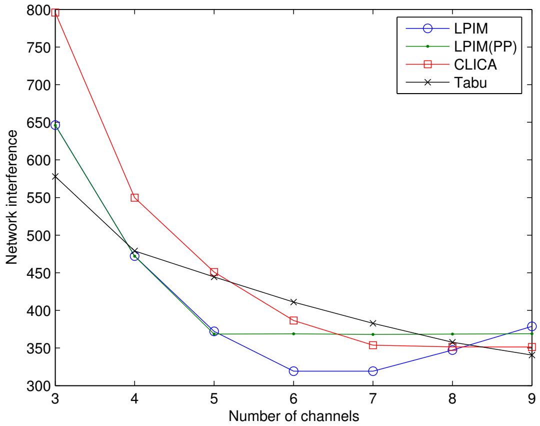

Fig. 8. Network interference versus the number of channels ($n = 50$, $r_{\max} = 3$)

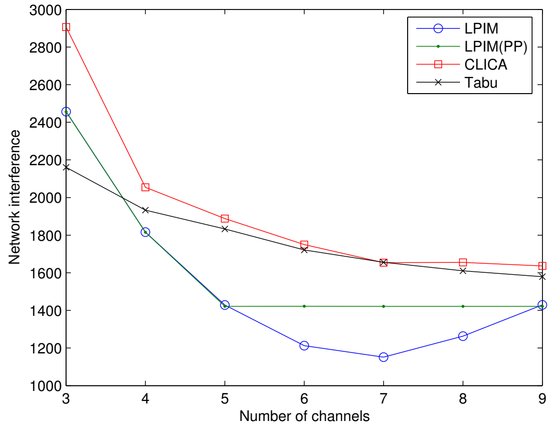

Fig. 9. Network interference versus the number of channels ($n = 70$, $r_{\max} = 3$)

fairness index β (Chiu & Jain 1989) for a channel configuration S as

$$\beta = \frac{(\sum I_i(S))^2}{n \times \sum I_i(S)^2}. \tag{23}$$

The value of β becomes 1 when all links get the same degree of interference, and it approaches $1/n$ in case of extremely unfair interference distribution. Each result was averaged over 1000 trials.

A. Network Interference

We first studied how different approaches decrease network interference with increased number of available channels. Figures 8 and 9 show the results for a 50-node and 70-node mesh networks, respectively, with $r_{\max} = 3$. When only three channels are available, Tabu performed the best, thanks to its interference-minimization design in the first phase. Its performance gradually improved as more channels were available. The performance of CLICA is close to that of Tabu when considerable channels are available. LPIM(PP) performed nearly the same as LPIM when no more than five channels were available. The reason is that $\min_{p_j \in N_i}\{r_i + r_j - 1\}$ was generally five for every p_i in LPIM(PP), so all channels were assignable with the Pigeonhole principle when the number of available channels does not exceed five. When more than five channels are available, LPIM(PP) did not improve its performance further as at most five channels were assignable by the Pigeonhole principle. In contrast, LPIM successfully decreased network interference with a moderate number of channels. Oddly, the network interference with LPIM started increasing with more channels. The reasons are still unknown and under investigation.

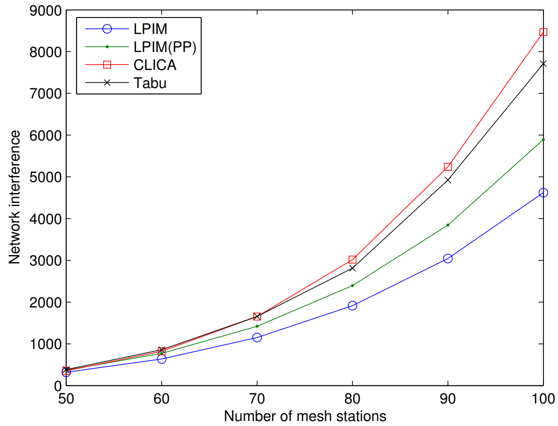

Fig. 10. Network interference versus the number of mesh stations ($m = 7$, $r_{\max} = 3$)

We also studied how the network interference changes with increasing number of mesh stations. Figures 10 and 11 show the results for seven and nine channels, respectively, with $r_{\max} = 3$. Observe that the network interference increased exponentially with the number of mesh stations. Nevertheless, the relative rank of each scheme remains the same.

B. Fairness

Figures 12 and 13 show how the fairness index varies with increasing number of channels in 50-station and 70-station networks, respectively. Observe that LPIM and LPIM(PP) both yielded

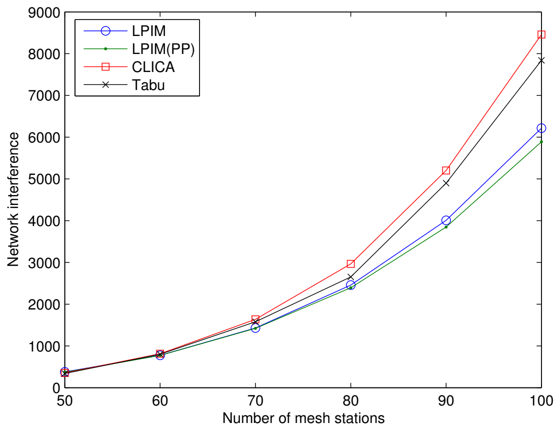

Fig. 11. Network interference versus the number of mesh stations ($m = 9$, $r_{\max} = 3$)

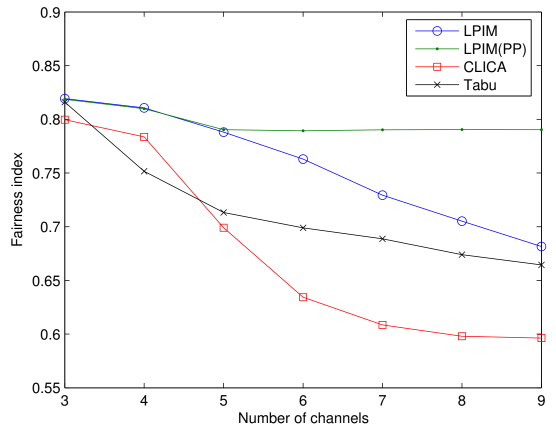

Fig. 12. Fairness index versus the number of channels ($n = 50$, $r_{\max} = 3$)

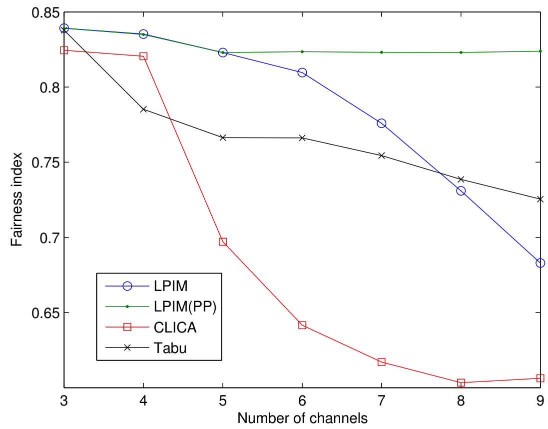

Fig. 13. Fairness index versus the number of channels ($n = 70$, $r_{\max} = 3$)

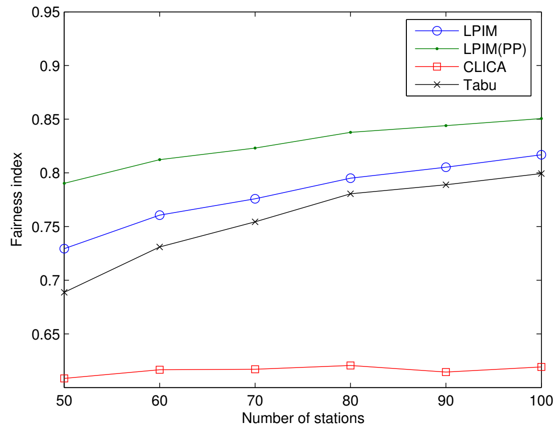

Fig. 14. Fairness index versus the number of stations ($m = 7$, $r_{\max} = 3$)

the highest results when no more than five channels were available. When six or more channels were available, LPIM(PP) maintained its superiority while LPIM gradually degraded its performance. The performance of Tabu is generally next to those of LPIM and LPIM(PP) but better than that of CLICA. The only exception is when four channels were available, for which CLICA outperformed Tabu.

Figure 14 shows the relationship between fairness index and the number of stations. Here the number of channels was fixed to seven. The results show that except for CLICA, increasing the number of stations generally improved fairness. Among all, LPIM(PP) performed the best, followed by LPIM and then Tabu. CLICA yielded the worst results.

VI. Conclusions

We have analyzed the probability of link connectivity for random channel assignments. We have proposed LPIM, a non-cooperative game design for the channel assignment problem in WMNs. To the best knowledge of the authors, LPIM is the first non-cooperative game approach that ensures link connectivity by defining a utility function that incorporates the connectivity requirement with the impact of co-channel interference. The LPIM game eventually enters a Nash equilibrium regardless of initial channel configuration. We have proved that, as long as the link-preserving property is ensured initially, the property will also be preserved at the end of the game. The performance of the proposed approach in terms of network interference was studied through simulations. The simulation results indicate that the proposed game-theoretic approach generally outperforms the other existing approaches when a moderate number of channels are available. Concerning the distribution of link interference, both the proposed approach and its variant outperform the counterparts in terms of fairness.

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