Link-Preserving Channel Assignment Game for Wireless Mesh Networks

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Abstract—To deliver user traffic in a wireless mesh network, mesh stations equipped with multiple wireless interfaces communicate with one another utilizing multiple orthogonal channels. Channel assignment in such an environment is to assign one channel to each interface to minimize co-channel interference among wireless links while preserving link connectivity. The interference and connectivity objectives are generally conflicting. This paper proposes a game-theoretic approach that jointly considers the two objectives with a unified payoff function. We prove that the proposed approach is an exact potential game, which guarantees stability in a finite time. We also prove the link-preserving property of the approach. Simulation results show that the proposed approach generally outperforms counterparts in terms of network interference when a moderate number of channels are available.

Index Terms—channel assignment; wireless mesh network; interference; connectivity; game theory

I. INTRODUCTION

To provide wireless access service on a large geographical area, we need to deploy a number of access points connected via a wired or wireless infrastructure. The latter case corresponds to a wireless mesh network (WMN). Elemental devices in a WMN called mesh stations [1] connect with each other through backhaul links to exchange control information and forward user traffic. To increase bandwidth capacity and enhance reliability, mesh stations are usually equipped with multiple wireless interfaces and allocated multiple orthogonal channels. Channel assignment is to allocate one channel to each interface so as to minimize co-channel interference (i.e., to maximize bandwidth capacity) while maximizing link connectivity (i.e., maximizing reliability).

We consider a decentralized channel assignment, where each mesh station autonomously assigns one channel to each of its interfaces. To successfully establish a link between two mesh stations, the selection of channels should meet two constraints. First, *common-channel constraint* demands that the two stations, one on each side of the link, should have at least one interface tuned to a common channel. A assignment that meets this constraint on every link is said to be *linkpreserving*. Second, *interference constraint* demands that the interference experienced on the channel should be sufficiently low. If channels are statically assigned to interfaces, the assignment is also subject to *interface constraint*, i.e., the total number of channels assigned to a station cannot exceed Bo-Rong Ye

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Fig. 1. A channel assignment example

the number of interfaces on that station.¹ With the interface constraint, meeting the other two constraints are conflicting in nature. Fig. 1 shows a channel assignment example where each station has two wireless interfaces. Given the assignment results of all the neighbors of station p_1 as shown in the figure, p_1 may choose channel set $\{2, 4\}$ to meet the common-channel constraint for all of its three links. However, this choice also incurs a high degree of interference. On the other hand, if p_1 chooses channel set $\{1, 3\}$ instead, the link between p_1 and p_4 will be broken as not meeting the common-channel constraint. However, this choice minimizes the degree of interference.

This study seeks a link-preserving channel assignment scheme that minimizes overall network interference. Existing approaches toward this goal are diverse. Some approaches superficially treated the link-preserving requirement [2]–[5]. It is also possible to ignore the link-preserving requirement initially when minimizing co-channel interference and then revise the results to meet this requirement at a latter stage [6]. Another trend of design is to restrict the range of selectable channels so as to meet the common-channel constraint by the Pigeonhole principle [7], [8].

This paper proposes a game-theoretic approach to the channel assignment problem in WMNs. The approach models mesh stations as players and channels as strategies. The payoff of a player with a particular channel configuration is defined to be the gain of connectivity minus the cost of co-channel interference between neighboring links. We prove that this game model is an exact potential game [9], which guarantees

¹We assume that the number of available channels is at least as many as the number of interfaces, so the number of available channels is not another constraint.

stability regardless game dynamics. We also prove that, if link connectivity is preserved initially, it will be preserved as well whenever the game ends. We conducted simulations to investigate the performance of the proposed approach. Compared with other link-preserving schemes, the proposed approach yielded the lowest interference when a moderate number of channels are available.

The remainder of this paper is organized as follows: Background information and related work are presented in Section II. In the following section, we present a game model for the channel assignment problem and prove the stability and correctness of the proposed approach. Section IV studies the performance of the proposed approach through simulations. The simulation results are compared with those of existing solutions. Section V concludes this paper.

II. BACKGROUND AND RELATED WORK

An intuitive way to meeting the common-channel constraint is to assign channels to links rather than individual interfaces. However, this approach may violate the interface constraint and thus need extra efforts to reduce channel usages. Subramanian et al. [6] proposed a two-phase channel assignment algorithm. In the first phase, an algorithm with Tabu search [10] assigns channels to links to minimize interference while disregarding the interface constraint. The algorithm then resolves all interface constraint violations by condensing channel usages in the second phase.

Assigning one channel to each interface trivially meets the interface constraint. The challenge is then to meet the common channel constraint. There are several alternatives. A naive method is to uniformly assign Channel 1 to the first interface of each mesh station, Channel 2 to the second interface of each mesh station, and so forth [2]. Another solution is to uniformally fix the channel of the first interface to a default one, and perform interference-minimization channel assignments only for the other interfaces [3]-[5]. These two approaches generally incur severe co-channel interference on the common or default channel. The third approach is to confine the set of selectable channels for each mesh station so that the common channel constraint is ensured by the Pigeonhole principle [7], [8]. Connected Low Interference Channel Assignment (CLICA) [11] is a greedy approach that assigns channels in a node-by-node manner. When assigning channels to links of a node, CLICA ensures connectivity by prioritizing the assignments of links that are likely to fail the common-channel constraint due to uncoordinated channels assignments among nodes.

Channel assignment schemes that meet the commonchannel and interface constraints can be evaluated by the degree of interference incurred. There are two fundamental models for the impact of interference [12]. The physical model considers the aggregated intensity of interference on the same channel from all other transmitters at the receiver side. On the other hand, the protocol model only concerns whether the same channel is used by some transmitter within a limited interference range. What really matters in the protocol model is that the interference relationship between links is binary and symmetric. Here we assume that the interference range is equal to the transmission range. With this definition, two links l_u and l_v are *adjacent* to each other (denoted by $l_u \leftrightarrow l_v$) iff they are incident on a common node, and two adjacent links interfere with each other if they are assigned the same channel. Following [13] and [6], we define *network interference* to be the number of adjacent links that are assigned a common channel.

Definition 1 (Network interference): For a particular channel assignment, let $c(l_i)$ be the function that returns the channel assigned to a link l_i . The overall network interference I with respect to $c(\cdot)$ is

$$I = \sum_{l_u \leftrightarrow l_v} [c(l_u) = c(l_v)].$$

There have been many game-theoretic approach to channel assignment problems. Many approaches [5], [14]-[16] aim at minimizing co-channel interference and do not address the link connectivity issue. In some other approaches [17]-[20], connectivity is not a problem simply because channels are allocated to links rather than individual interfaces. Duarte et al. [21] formulated channel assignments with the consideration of link connectivity as a cooperative game where players have common interest. Their work assumes non-orthogonal channels and thus considers adjacent-channel interference in addition to co-channel interference. Yen and Dai [8] assumed the physical interference model and proposed a non-cooperative game for interference-minimization channel assignments. Their work ensures link connectivity by limiting the range of channel selections. To summarize, to the best knowledge of the authors, our work here is the first noncooperative game approach that ensures link connectivity by defining a utility function that incorporates the connectivity requirement with the impact of co-channel interference.

III. THE PROPOSED APPROACH

We name the proposed approach link-preserving interference-minimization (LPIM) game. This section presents details about LPIM.

A. The LPIM Game

We model a backhaul network by a undirected *connectivity* graph G = (P, E), where $P = \{p_1, p_2, \ldots, p_n\}$ is the set of mesh stations and E is the set of mesh station pairs such that $(p_i, p_j) \in E$ iff (p_i, p_j) is a potential link between mesh stations p_i and p_j , i.e., p_i and p_j are within the communication range of each other. We assume that $C = \{k_1, k_2, \ldots, k_m\}$ is the set of orthogonal channels to be allocated. Each mesh station has at most r_{\max} interfaces. The actual number of interfaces used by station p_i is $r_i = \min(r_{\max}, |N_i|)$, where $N_i = \{p_j | (p_i, p_j) \in E\}$ is the set of p_i 's neighbors in the connectivity graph.

Mesh stations are players in the LPIM game. The strategy of player p_i is a vector $s_i = (c_1^i, c_2^i, \ldots, c_m^i)$, where $c_j^i = 1$ or 0 indicating whether p_i assigns channel k_j to one of its interfaces. The strategy s_i is subject to $\sum_{1 \le j \le m} c_j^i = r_i$. All valid s_i 's comprise p_i 's strategy set S_i . The strategy space of the game is defined by $\Sigma = \prod_{1 \le i \le n} S_i$. A strategy profile $S \in \Sigma$ represents a channel configuration of the network. We sometimes express S as (s_i, s_{-i}) , where s_{-i} indicates a tuple of all player's strategies other than p_i 's.

Given a strategy profile, our objective is to define a utility function for each player that incorporates both the gains of connectivities and the impacts of co-channel interference. The challenge is to minimize overall co-channel interference while still preserving the connectivity of every potential link. To capture the gains of connectivities, we define

$$L_i(S) = \sum_{p_j \in N_i} C_i(s_i, s_j), \tag{1}$$

where

$$C_i(s_i, s_j) = \begin{cases} -|N_i| & \text{if } s_i \cdot s_j = 0\\ 0 & \text{otherwise.} \end{cases}$$
(2)

Note that the dot product of s_i and s_j is the number of common channels assigned by both p_i and p_j . It equals zero only if p_i and p_j assign no common channel. On the other hand, this value also reflects the degree of co-channel interference caused by p_i with respect to S. Thus we define the impact as

$$I_i(S) = -\sum_{p_j \in N_i} (s_i \cdot s_j).$$
(3)

Combining the gains of connectivity and the impacts of interference, we have

$$t_i(S) = \beta L_i(S) + I_i(S), \tag{4}$$

where $\beta > r_{\text{max}}$ is a constant to ensure that connectivity is always important than interference. Finally, the utility of p_i is defined to incorporate the gains and impacts of p_i itself and all its neighbors.

$$u_i(S) = t_i(S) + \sum_{p_j \in N_i} t_j(S).$$
 (5)

It is not difficult to see that the game is an exact potential game [9] with exact potential function defined as $\phi(S) = \sum_i t_i(S)$.

Theorem 1: $\phi(S) = \sum_{i} t_i(S)$ is an exact potential function for the LPIM game with utility function defined as (5).

Proof: Consider any player p_i that changes its strategy. Let the strategy profile before and after this change be S and \bar{S} , respectively. The difference of $\phi(\cdot)$ after and before the change is

$$\phi(S) - \phi(S) = t_i(S) - t_i(S) + \sum_{p_j \in N_i} (t_j(\bar{S}) - t_j(S)) + \sum_{p_j \in P \setminus N_i} (t_j(\bar{S}) - t_j(S)).$$
(6)

Because $t_j(\cdot)$'s for all $p_j \in P \setminus N_i$ are not affected by p_i 's move, we have

$$\phi(\bar{S}) - \phi(S) = t_i(\bar{S}) - t_i(S) + \sum_{p_j \in N_i} (t_j(\bar{S}) - t_j(S)), \quad (7)$$

which is exactly the utility gain of p_i .

Theorem 1 indicates that the LPIM game always stabilizes when players change their strategies following the so-called *best-response rule*. This rule states that player p_i selects strategy s_i^* only if

$$s_i^* = \operatorname*{arg\,max}_{s_i \in S_i} u_i(s_i, s_{-i}). \tag{8}$$

Any game play sequence following the best-response rule is a *best-reply path* [22]. For exact potential games, a best-reply path always ends at a Nash equilibrium.

Definition 2 (Nash equilibrium): Given a game $\Gamma = [P; \{S_i\}_{i=1}^n; \{u_i\}_{i=1}^n]$, a strategy profile $S = (s_1, s_2, \ldots, s_n)$ is a Nash equilibrium if $\forall i \in \{1, 2, \ldots, n\}$: $\forall s_i^* \in S_i :: u_i(s_i, s_{-i}) \geq u_i(s_i^*, s_{-i})$.

In fact, an exact potential game also ends at a Nash equilibrium if every player p_i follows the *better-response rule*, i.e., changing its strategy from s_i to s_i^* only if

$$u_i(s_i^*, s_{-i}) > u_i(s_i, s_{-i}).$$
 (9)

We also need to ensure the connectivity of every potential link.

Theorem 2: If the connectivity of every potential link is ensured initially, i.e., $s_i \cdot s_j \neq 0$ for every $(p_i, p_j) \in E$, then the connectivity is still preserved when the LPIM game ends.

Proof: We prove this by showing that no player has the incentive to trade connectivity for the improvement of interference. For any player p_i , the highest interference value occurs when $s_i = s_j$ for all $p_j \in N_i$. In that case,

$$I_i(S) = -\sum_{p_j \in N_i} \min(r_i, r_j) \ge -r_{\max}|N_i|.$$
 (10)

On the other hand, if p_i shares no common channel with some $p_i \in N_i$, we have

$$L_i(S) \le -|N_i|. \tag{11}$$

Therefore,

$$\beta L_i(S) \le -\beta |N_i| < -r_{\max}|N_i|. \tag{12}$$

By (10) and (12), the highest possible improvement of interference (from $-r_{\max}|N_i|$ to 0) brought by p_i 's strategy change cannot compensate the loss of connectivity of any single potential link (p_i, p_j) (which is at least $-r_{\max}|N_i|$). This holds for p_i and all $p_j \in N_i$. Therefore, p_i will not break the connectivity of any potential link during game play.

Figure 2 shows the topology of a five-station mesh network. Common Channel Assignment (CCA) [2] is assumed to make initial channel configuration. For this Table I shows a bestreply path. Note that the path is not unique (For example, p_1 could choose $\{3, 5, 6\}$ instead of $\{2, 5, 7\}$ in the fourth step). Figure 3 shows the final channel configuration after the LPIM game ends.



Fig. 2. Initial channel configuration of a five-station mesh network

TABLE I A possible best-reply path

Step	s_1	s_2	s_3	s_4	s_5
0	$\{1, 2, 3\}$	$\{1, 2, 3\}$	$\{1, 2, 3\}$	$\{1, 2, 3\}$	$\{1, 2, 3\}$
1	$\{1, 2, 3\}$	$\{1, 2, 3\}$	$\{1, 4, 5\}$	$\{1, 2, 3\}$	$\{1, 2, 3\}$
2	$\{1, 2, 3\}$	$\{1, 2, 3\}$	$\{1, 4, 5\}$	$\{2, 4, 6\}$	$\{1, 2, 3\}$
3	$\{1, 2, 3\}$	$\{1, 2, 3\}$	$\{1, 4, 5\}$	$\overline{\{2,4,6\}}$	$\{3, 5, 6\}$
4	$\underline{\{2, 5, 7\}}$	$\{1, 2, 3\}$	$\{1, 4, 5\}$	$\{2, 4, 6\}$	$\overline{\{3,5,6\}}$

B. One Channel Per Link

The LPIM game ensures that when the game ends, stations at the two ends of any link share *at least one* common channel. It is possible that these two stations share two or more common channels, effectively creating multiple links between these two stations. The same situations also occur to other channel assignment schemes [7], [8], [11].

In terms of connectivity, multiple common channels between two neighboring stations do no harm. However, the concurrent use of multiple channels can increase interference. Therefore, we want to to designate only one channel to use for the minimization of network interference. A straightforward way to designating one channel to each link is to randomly select one common channel among all. Yen and Dai [8] proposed a simple heuristic that picks up a common channel that minimizes the amount of conflicts with the current channel assignments of adjacent links. This is also the approach adopted by LPIM.



Fig. 3. Channel configuration of the five-station mesh network after LPIM ends



Fig. 4. Network interference versus the number of channels $(n = 50, r_{\text{max}} = 3)$

IV. SIMULATION RESULTS

We conducted simulations on unit disk graphs [23] for performance evaluation. We randomly placed n mesh nodes in a 1000 m × 1000 m area, where the communication range of each mesh node is assumed 200 m. A link exists between two nodes if and only if these two nodes are within the communication rage of each other. We precluded topologies with isolated nodes. Each node had $r_{\text{max}} = 3$ interfaces. Total m channels were used.

LPIM can start with any channel configuration that meets the common-channel constraint. We found through experiments that the result of LPIM is not really sensitive to the initial configuration. For simplification, we used CCA [2] to perform initial channel assignment. Afterwards, players were randomly selected to make decisions. When making a decision, the player followed the better-response rule. The game ended when no player could increase its utility unilaterally.

Two representative approaches, CLICA [11] and Tabu [6], were tested and compared with LPIM. To investigate the design effectiveness of LPIM, we also tested a variant of LPIM called LPIM(PP) that considers only the impact of interference and adheres to the Pigeonhole principle when selecting channels. More explicitly, the utility function of each player in LPIM(PP) is defined as

$$u_i(S) = -\sum_{p_j \in N_i} (s_i \cdot s_j) \tag{13}$$

with the condition that $c_k^i = 0$ for all p_i and $k > \min_{p_j \in N_i} \{r_i + r_j - 1\}$. LPIM(PP) is also an exact potential game (the proof is analogous to [24]).

Since all these approaches are link-preserving, we measured averaged network interference for performance comparison. Each result was averaged over 1000 trials.

We first studied how different approaches decrease network interference with increased number of available channels. Figures 4 and 5 show the results for a 50-node and 70-node mesh networks, respectively, with $r_{\text{max}} = 3$. When only three channels are available, Tabu performed the best, thanks to its interference-minimization design in the first phase. Its performance gradually improved as more channels were available.



Fig. 5. Network interference versus the number of channels ($n = 70, r_{\text{max}} = 3$)



Fig. 6. Network interference versus the number of mesh stations (m = 7, $r_{\text{max}} = 3$)

The performance of CLICA is close to that of Tabu when considerable channels are available. LPIM(PP) performed nearly the same as LPIM when no more than five channels were available. The reason is that $\min_{p_j \in N_i} \{r_i + r_j - 1\}$ was generally five for every p_i in LPIM(PP), so all channels were assignable with the Pigeonhole principle when the number of available channels does not exceed five. When more than five channels are available, LPIM(PP) did not improve its performance further as at most five channels were assignable by the Pigeonhole principle. In contrast, LPIM successfully decreased network interference with a moderate number of channels. Oddly, the network interference with LPIM started increasing with more channels. The reasons are still unknown and under investigation.

We also studied how the network interference changes with increasing number of mesh stations. Figures 6 and 7 show the results for seven and nine channels, respectively, with $r_{\rm max} = 3$. Observe that the network interference increased exponentially with the number of mesh stations. Nevertheless, the relative rank of each scheme remains the same.



Fig. 7. Network interference versus the number of mesh stations (m = 9, $r_{\rm max} = 3$)

V. CONCLUSIONS

We have proposed LPIM, a non-cooperative game design for the channel assignment problem in WMNs. To the best knowledge of the authors, LPIM is the first non-cooperative game approach that ensures link connectivity by defining a utility function that incorporates the connectivity requirement with the impact of co-channel interference. The LPIM game eventually enters a Nash equilibrium regardless of initial channel configuration. We have proved that, as long as the link-preserving property is ensured initially, the property will also be preserved at the end of the game. The performance of the proposed approach in terms of network interference was studied through simulations. The simulation results indicate that the proposed game-theoretic approach generally outperforms the other existing approaches when a moderate number of channels are available.

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