Decentralized Auctioneerless Combinatorial Auctions for Multi-Unit Resource Allocation

Li-Hsing Yen and Guang-Hong Sun
Department of Computer Science, College of Computer Science
National Chiao Tung University, Hsinchu, Taiwan 300, R.O.C.
Email: lhyen@cs.nctu.edu.tw, martian206@gmail.com

***Abstract*—Auction has been used to allocate resources or tasks to processes, machines or other autonomous agents in distributed systems. Among various types of auctions, combinatorial auction (CA) allocates a bundle of items to each agent at once. Finding an optimal auction result for CA that maximizes total winning bid is NP-hard. Many time-efficient approximations to this problem work with a bid ranking function (BRF). However, existing approximations are mostly for single-unit resource and demand an auctioneer. This paper proposes the first auctioneerless open-bid multi-unit CA (MUCA) scheme. It includes a BRF-based winner determination scheme that enables every agent to locally compute a critical bid value for it to win the MUCA and accordingly take its best response to other agent's bid and win declarations. It also allows each winner to locally compute its payment for a critical-value-based pricing scheme. We analyze stabilization, correctness, and consistency properties of the proposed approach. Simulation results confirm that the proposed approach identifies exactly the same set of winners as the centralized counterpart regardless of initial bid setting, but at the cost of lower total winning bid and payment.**

I. Introduction

Auction is a trading process that allows seller to identify potential buyers and the prices the buyers are willing to pay. We may use auctions as resource and task allocation schemes. Unlike conventional approaches that assume zero or fixed price of resource or task, auction-based approaches can allocate resource/task to requesters in a way that reflects actual demand and supply conditions. For this reason, auctions have been used to allocate different types of resources or tasks to a fleet of autonomous, self-interest agents. Existing examples include but not limited to the allocations of wireless spectrum [1], [2], cloud resource [3], [4], [5], [6], servers in mobile edge computing [7], [8], tasks of robots [9], [10], [11], and computation or sensing tasks of mobile devices [12], [13].

Depending on how many types and how many units of the items are to sell, auctions can be classified into four categories as shown in Table I. Single-unit single-item (SUSI) is the simplest form of auction, where only item is to sell and there is only one winner. Some studies considered a single item of multiple supplying units (multi-unit single-item or MUSI) [14], [4], [6], where more than one bidders can be winners. We consider the allocation of multiple types of items via auctions. A typical example is in cloud environment, where we have computation, memory, storage, network, and other types of resources. If there is only one supplying unit for each type of item, the auction is of single-unit multi-item (SUMI), where two or more agents (i.e., bidders) could be winners at the same time provided that no two winners have conflicting interests (i.e., they place bids on a common item). If there can be multiple supplying units for some type of item, the auction is of multi-unit multi-item (MUMI), where two or more agents having conflicting interests can be winners at the same time provided that the set of winning bids is feasible. A set of winning bids is *feasible* if for each type of item, the total amount of units demanded by all winning bids does not exceed the supply.

TABLE I
Classification of auctions.

Type	Related work
Single-Unit Single-Item (SUSI)	[15], [16]
Multi-Unit Single-Item (MUSI)	[14], [17], [4], [6]
Single-Unit Multi-Item (SUMI)	[18], [19], [11], [20], [21]
Multi-Unit Multi-Item (MUMI)	[22], [23], [24], [25], [5]

A. Combinatorial Auction

Identifying a feasible set of winning bids with the highest total bid is *winner determination problem* (WDP). WDP is NP-hard even for SUMI. Some approaches take *sequential single-item auction* [21], [20], [11], where bidders bid for one item at a time. This type of auction can be executed in polynomial time. However, it typically applies to SUMI and is not suitable when agents are *single-minded* [18] (i.e., bidders are only interested in and thus places a bid on a particular bundle of items).

For single-minded agents, *combinatorial auction* (CA) [19], [26] allocates a bundle of items to bidders at once. We refer to CAs for SUMI and MUMI as SUCA and MUCA, respectively. As WDP for CA is NP-hard [19], most existing approaches take approximations. We particularly consider approximations that are based on a *bid ranking function* (BRF), which defines a total order on bid requests. BRF-based approximations are time efficient and suited to single-minded agents, but do not guarantee optimality.

Another point of auction-based resource allocation is to identify bidders' *valuations* on resources. In general, different bidders have different valuations on the same type of resource. For example, computation resource is more valuable to computation-intensive tasks than to communication-intensive tasks. An auction mechanism is *economically efficient* if it

maximizes *social welfare*, i.e., the aggregated valuation from all winning bidders. Economically efficient auction is desirable when it is that the total utility of resource requesters rather than the revenue of the seller that is of concern.

If every bid represents the bidder's true valuation on the bid items, the auction has a nice property called *truthful bidding*. Truthful bidding ensures that the total winning bid is exactly the social welfare that we want to maximize. Truthful bidding is challenging, however, because bidder's valuations on bid items are considered local and private (i.e., not revealed to other bidders and the auctioneer who conducts the auction). Whether bidders are willing to bid truthfully primarily depends on *pricing scheme* that decides the payment of each winner in auction. From the perspective of game theory, a pricing scheme is *incentive compatible* if every bidder's dominant strategy is to bid truthfully.

In a first-price pricing scheme, winner pays exactly her bid. Consequently, rational bidders will not place any bids higher than their valuations on bid items because doing so only incurs negative payoffs. On the other hand, bidders tend to lower their bids so as to increase payoffs by paying less when they turn out to be winners. Therefore, the first-price pricing scheme is not incentive compatible and the auction result is not economically efficient generally. As a remedy, Vickrey proposed second-price scheme [27] for SISU auction, where the winner pays the second highest bid. This payment rule is incentive compatible.

Incentive-compatible pricing scheme together with an optimal winner determination can ensure economical efficiency. The most well-known incentive-compatible pricing scheme for CA is the Vickrey-Clarke-Groves (VCG) mechanism [27], [28], [29]. VCG relies on the optimal solution to the WDP, so it is not computationally feasible. We consider a computationally-efficient heuristic called critical-value-based payment that depends on the definition of the BRF for the associated WDP. It has been proved that if the associated BRF is monotone, a critical-value-based payment is incentive compatible [18]. However, VCG and most BRF-based approaches [18], [30] are for SUCA.

B. Related Work and Motivation

Apart from the types and the number of units of the bid items, auctions can also be classified by how the auction is conducted. Many auctions implicitly assume a single entity, i.e., an auctioneer, to conduct auctions. An auctioneer is needed in a sealed-bid auction, where all participants send their bids to the auctioneer without knowledge of other bids. It is the auctioneer that declares winners and associated payments. Many previous CA approaches fall into the category of sealed-bid auctions and thus all need an auctioneer [31], [18], [30], [23], [32].

The auctioneer is a single point of failure and can be a performance bottleneck. There have been some approaches that attempt to duplicate or partition the load of auctioneer to a set of brokers [33]. Kutanoglu and Wu [34] decomposed the WDP for CA into subproblems each solved by a local agent. However, an auctioneer is still needed to collect and update bidding information for coordinating an iterative auction.

When there are multiple units or types of items to sell, there could be *multiple* auctioneers, for which decentralized approaches also have been proposed. Lewis et al. [17] proposed a decentralized adaptive pricing scheme for sellers to determine the best selling prices in a posted-price model. In [13], buyers send their bids to multiple sellers, which then locally determine the set of winning bid candidates. Each buyer then chooses its final seller. A reverse auction is proposed for the allocation of sensing tasks to smart devices in [12], where buyers (task owners) individually work as auctioneers of their own task auctions. Smart devices as sellers submit their asking prices to buyers, which then assnounce the auction results. An additional step is needed to handle the case when a seller becomes winners of multiple auctions. In all these decentralized approaches, sellers sell the same type of item with different quantities, which renders these approaches decentralized MUSI. Multiple auctioneers have also been proposed for decentralized SUMI auctions [35].

Our study focuses on *decentralized auctioneerless* auction, where bidders themselves coordinately determine the set of winners and payments. This type of auction is preferable in many cases, especially when supplies of and requests for resources exhibit *locality* property. Some examples are listed below.

- Resource is only accessible to "local" users. An example is wireless spectrum resource.
- Users only have interest in locally-accessible resource. An example is virtualized resource provided by edge servers in mobile edge computing environment.

In these cases, potential competitors contending for the same type of resource tend to cluster together. Therefore, a decentralized auction for bidders to coordinate the set of winners is more robust and scalable than an auctioneer-based approach. Furthermore, in applications like robot task allocations, it is more desirable to let robots themselves coordinate their tasks because, compared with a central coordination approach that decides a global task allocation, the decentralized approach has a shorter response time.

For SUSI, Esteva and Padget [15] proposed an auctioneerless approach to WDP based on leader election protocol running on a ring overlay network. Their approach targets at single-item auction so only one winner is possible. For SUMI, sequential single-item auctions are also decentralized and auctioneerless. Each agent independently and incrementally constructs its own bundle of items. In each round of the auction, a single item is to sell to one agent via (possibly reverse) single-item auction. Each agent places its bid on the item based on the reward it might receive from adding the item to its bundle. Sequential single-item auctions have been proposed for the assignment of different tasks to a fleet of robotic agents [21], [9], [20], [11].

In this paper, we propose a decentralized auctioneerless approach to MUCA where bidders autonomously decide whether they themselves are winners and how much they should pay.

TABLE II
SUMMARY OF RELATED WORK

Number of auctioneers	Type	Execution model	Related work
One	SUCA	Centralized	[31], [18], [30]
	MUCA	Centralized	[23], [24], [25], [5], [32]
	SUCA	Distributed	[33], [34]
Multiple	MUSI	Distributed	[17], [12], [13]
	SUMI	Distributed	[35]
None	SUSI	Distributed	[15]
	SUMI	Distributed (sequential single-item auctions)	[21], [9], [11], [20]
	MUCA	Distributed	This work

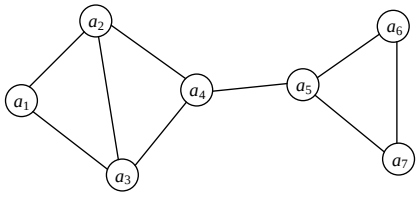


Fig. 1. A conflict graph for seven bidders in a CA

Our approach differs from sequential single-item auctions in the following points. First, these approaches typically apply to SUMI, while ours applies to MUCA. Second, these approaches do not work for single-minded agents. Third, these approaches assume that bidders are intrinsically truthful, while ours does not. Finally, all items must be sold in sequential single-item auctions, but this is not a requirement in our approaches.

Refer to Table II for a summary of related work.

C. Challenges

Without auctioneer, bidders have to exchange bidding information for collaborative winner and payment determinations. Consequently, the decentralization turns a static game (sealed-bid auction) into a dynamic game (open-bid auction). We model it as a dynamic multi-unit CA (MUCA) game with a monotone BRF used for determining the ranks of bid requests. The basic idea is that every agent can locally figure out the minimum bid that can let the agent win the requested items. With this and the agent's valuation, each agent knows and makes its best response in the MUCA game. After the game stabilizes, each winner is able to calculate the minimum bid that still ensures its win and take it as its payment. The auction result can be verified by participants in the auction.

However, the computation and communication overhead may be high if each agent has to exchange information with every other agent. The key to the benefit and the feasibility of the decentralization is that *only* bidders that have conflicting interests need to interact with one another for winner determination. The competitions among bidders can be captured by a *conflict graph*, where bidders are nodes and there is an edge for each pair of competing bidders. Fig. 1 shows an example of conflict graph for seven bidders in a CA. Here, for bidding agent a_5 to determine whether it can win its bid, it only needs to know the bidding information and the status of its competitors (a_4, a_6, and a_7). Agent a_5 is free to declare its win in the CA without information from other bidders (a_1, a_2, and a_3). This suggests a localized, autonomous winner determination mechanism, which is more robust and scalable than a centralized one.

Bidding protocols under this game-theoretic framework face several challenges. First, if bidders can gain extra payoff by having knowledge of other bids before they place their own bids, they may intentionally postpone their decision makings until they receive bidding information of their competitors. The consequence is that the whole system may enter a deadlock state simply because no bidder wants to place its bid first. Second, bidders progress asynchronously due to the lack of a synchronization scheme among them. The non-deterministic interactions among bidders may not converge to a stabilized result without appropriate regulations. Third, even if the protocol reaches a stabilized outcome, the outcome may not be the same as that of the corresponding centralized counterpart using the same BRF.

D. Contribution and Organization

The contributions of this work are summarized as follows.

- We propose a winner determination protocol for MUCA which works with any given BRF that is monotone. This protocol is deadlock free because it allows bidders to revise and update their bid requests whenever they want to react to other bidder's updates (as their best responses). This protocol guarantees stabilization in the face of dynamic bidder interactions. It meets resource constraint and conforms to the BRF-based winner determination rule. For a specific BRF, the proposed decentralized approach can yield the same set of winners as the centralized counterpart, despite that the ranks of bid requests and payments may be different in the two approaches.
- We consider two pricing schemes, first-price payment and critical-value-based payment, and analyze the impact of pricing scheme on agent's bidding strategies in the framework of game theory. We show that, with critical-value-based payment, truthful bidding is every agent's weakly dominant strategy. We also propose a method for each winner to locally determine how much it should pay if critical-value-based payment is in effect.

To the best knowledge of the authors, this is the first decentralized CA approach that possesses these properties. We have conducted extensive simulations to investigate the performance of the proposed approach.

The rest of this paper is organized as follows. Sec. II covers the background and Sec. III presents the game model for our problem. We present the proposed scheme in details in Sec. IV and analyze its properties in Sec. V. Section VI contains the simulation results that confirm the advantage of our scheme. The last section concludes this paper.

II. Problem Definition

We consider a set of n bidding agents (bidders) $A = \{a_1, a_2, \ldots, a_n\}$ and m different types of resources $R = \{r_1, r_2, \ldots, r_m\}$. Let $\mathbf{q} = (q_1, q_2, \ldots, q_m)$ be a supply vector such that $q_i \geq 1$ is the total number of units (or identical instances) of resource type r_i. For SUCA, $q_i = 1$ for all i. Agent a_i submits a *request vector* $\mathbf{s}_i = (s_i^1, s_i^2, \ldots, s_i^m)$, where $s_i^j \leq q_j$ is the number of units of resource type r_j requested by a_i. For SUCA, $\mathbf{s}_i$ reduces to a set (named *bundle*) $S_i \subseteq R$. Theoretically speaking, what a_i requests may deviate from what a_i desires if such a deviation could bring a_i a higher expected payoff. For now we assume that no agent cheats at the request vector. Later we will prove that indeed no agent has the incentive to cheat.

The bid a_i places on $\mathbf{s}_i$ is denoted by $b_i(\mathbf{s}_i)$ or simply b_i, which together with $\mathbf{s}_i$ forms a_i's *bid request* $(\mathbf{s}_i, b_i)$. We assume that an agent only submits one bid request. If an agent may submit multiple requests (i.e., OR bids [36]), we can treat A as a set of requests rather than agents. If an agent is allowed to submit but not to win multiple requests (i.e., XOR bids [36]), we may manually add mutual-exclusive relation between each pair of requests submitted by the same agent[1].

A. Winner Determination Problem

Given a set of bid requests $\mathcal{B} = \{(\mathbf{s}_i, b_i(\mathbf{s}_i))\}_{i=1}^n$, the winner determination problem (WDP) is to find a setting of $X = (x_1, x_2, \ldots, x_n)$, where $x_i \in \{0, 1\}$ for all i, that maximizes the *total winning bid*

$$\sum_{x_i=1} b_i(\mathbf{s}_i) \tag{1}$$

subject to the *resource capacity constraint* defined as

$$\sum_{i=1}^{n} \left(x_i \cdot s_i^k\right) \leq q_k \text{ for all } k = 1, \ldots, m. \tag{2}$$

The WDP for SUCA is an instance of the *maximum weight set packing problem*, which is known to be NP-hard [19]. Some approaches guarantee optimality but may be time-inefficient for some problem instances [36], [37]. Some approaches are time-efficient and achieve optimality by restricting the form or size of bid requests [38]. Some approaches use heuristic or approximation techniques for time efficiency but not optimality. Hoos and Boutilier [39] used stochastic local search algorithm as an approximation to WDP. Zurel and Nisan [40] also proposed an approximation which runs the linear-programming relaxation of the packing problem and then refines the solution by local improvements in the order of bids (hill-climbing). The hill-climbing concept was also adopted by Fukuta and Ito [41] to improve the performance of a simple greedy approach [18]. They also considered the use of simulated annealing technique. Other approximation approaches include dynamic programming [3] and genetic algorithm [42].

[1]One possible way of doing this is through the creation of *dummy goods* [22]. Also note that all OR bids can be converted into equivalent XOR bids [36].

In this paper, we mainly consider approximations that use a BRF to define a total order $\prec$ on $\{(\mathbf{s}_i, b_i)\}_{i=1}^n$ such that $(\mathbf{s}_j, b_j) \prec (\mathbf{s}_i, b_i)$ if $(\mathbf{s}_j, b_j)$ ranks higher than $(\mathbf{s}_i, b_i)$. Algorithm 1 shows the general framework for greedy allocations which examines all bid requests in the order defined by $\prec$ to determine whether each request can be granted.

Algorithm 1 BRF-based Greedy Allocation

```
1: B ← {(s_i, b_i)}_{i=1}^n
2: x_i ← 0 for all i
3: while B ≠ ∅ do
4:     Let (s_k, b_k) be the request that ranks first in B
5:     if q − s_k ≥ 0 then
6:         q ← q − s_k
7:         x_k ← 1
8:     end if
9:     B ← B \ {(s_k, b_k)}
10: end while
```

There have been many BRFs proposed for SUCA. The BRF proposed by Lehmann et al. [18] favors a request that maximizes normalized bid value defined as

$$w_s(S_i, b_i) = \frac{b_i}{|S_i|^\alpha}, \tag{3}$$

where α is a configurable parameter. Mito and Fujita [30] considered several possible BRFs inspired by the heuristics for the maximum weighted independent set (MWIS) problem [43]. Let N_i be the set of all conflicting requests for request (S_i, b_i). One such BRF sets a priority defined as

$$w_n(S_i, b_i) = \frac{b_i}{(|N_i| + 1)^\beta}, \tag{4}$$

where β is a configurable parameter. Another BRF considered by them is

$$w_\phi(S_i, b_i) = \frac{\phi(S_i, b_i)}{(\sum_{(S_j, b_j) \in N_i} b_j + 1)^\beta}, \tag{5}$$

where

$$\phi(S_i, b_i) = \frac{b_i}{(\sum_{(S_j, b_j) \in N_i} |S_i \cap S_j| + 1)^\alpha}. \tag{6}$$

Function $\phi(\cdot)$ alone could also be a BRF.

Not too many approaches have been proposed for MUCA. Leyton-Brown et al. [22] proposed an optimal WDP algorithm. This algorithm uses techniques like branch-and-bound and dynamic programming, which makes it difficult to be decentralized. As an approach to allocating fine-grained spectrum resources, Jia et al. [23] generalized the BRF $w_s(\cdot)$ defined in (3) to MUCA. The proposed BRF is

$$w_m(\mathbf{s}_i, b_i) = \frac{b_i}{(\sum_{k=1}^m s_i^k)^\alpha}. \tag{7}$$

The same BRF has also been used for the allocation of virtual machine instances in clouds [24], [25]. The work in [44] generalized the BRF to consider scarcity of resources with

$\alpha = 0.5$. Mashayekhy et al. [5] considered the following BRF for a bid request in a unit of time.

$$w_d(\mathbf{s}_i, b_i) = \frac{b_i}{\prod_{s_i^k \neq 0} s_i^k}. \tag{8}$$

Some BRFs for SUCA like (4) do not consider the number of resource instances. When being used in MUCA, these BRFs may perform poorly. BRFs like (5) and (6) have not yet been extended to handle multi-unit resources. A possible extension is to replace $|S_i \cap S_j|$ in (6) with some matching term like $\mathbf{s}_i \cdot \mathbf{s}_j$.

B. Pricing Scheme

The VCG mechanism generalizes the second-price scheme to ensure truthful bidding in CAs. VCG demands that each winner a_i in VCG has to pay the social opportunity cost (i.e., the reduction of the total winning bid excluding a_i's) due to the presence of a_i's request. Suppose that we have a set of request pairs $\mathcal{B} = \{(S_i, b_i)\}_{i=1}^n$. Let $\mathcal{B}_{-i}$ denote $\mathcal{B} \setminus \{(S_i, b_i)\}$. Let W and W_{-i} be the sets of winning requests with the highest total bid given $\mathcal{B}$ and $\mathcal{B}_{-i}$, respectively. Each winning request $(S_i, b_i) \in W$ has to pay $p_i(\mathcal{B}) = \sum_{(S_j,b_j)\in W_{-i}} b_j - \sum_{(S_k,b_k)\in W\setminus\{(S_i,b_i)\}} b_k$. VCG payment has been used in [45]. VCG is economically efficient but computationally infeasible because determining W is NP-hard.

For BRF-based winner determination designed for SUCA, Lehmann et al. [18] defined *monotonicity* property for BRF, which states that the BRF gives (S_j, b_j) a rank equal to or higher than that of (S_i, b_i) if $S_j \subseteq S_i$ and $b_j \geq b_i$. BRF $w_s(\cdot)$ has the monotonicity property. BRFs $w_n(\cdot)$, $w_\phi(\cdot)$, and $\phi(\cdot)$ do not ensure monotonicity because it is possible that $S_i' \subset S_i$ but $(S_i', b_i) \not\prec (S_i, b_i)$ as long as the set N_i remains unchanged for both S_i and S_i'.

An allocation of resource to bidders is *exact* if each bidder a_i is allocated either its request $\mathbf{s}_i$ (if a_i wins) or nothing (otherwise) [18]. For a BRF-based winner determination for SUCA with both the exactness and monotonicity properties, it is proved [18] that there is a critical value c_i for each b_i such that a_i gets S_i if $b_i > c_i$ and a_i gets nothing if $b_i < c_i$.

For example, assume that (S_i, b_i) is a winning request and (S_j, b_j) is the request that has the highest rank in the set of requests that do not win because of the presence of (S_i, b_i). For the BRF defined in (3), (S_i, b_i) is a winning request because $w_s(S_i, b_i) > w_s(S_j, b_j)$, which implies that

$$b_i > b_j \frac{|S_i|^\alpha}{|S_j|^\alpha}. \tag{9}$$

On the other hand, (S_i, b_i) would not be a winning request if $w_s(S_i, b_i) < w_s(S_j, b_j)$ or, equivalently, if

$$b_i < b_j \frac{|S_i|^\alpha}{|S_j|^\alpha}. \tag{10}$$

Therefore, $c_i = b_j \times |S_i|^\alpha / |S_j|^\alpha$ is the critical value for b_i.[2]

[2]Though not explicitly stated, critical values should also exist for other monotone BRFs with exact allocations [30].

If a BRF-based winner determination is used for which the criticality property holds, then the following pricing scheme ensures truthful bidding in a sealed-bid CA [18].

$$p_i = \begin{cases} c_i, & \text{if } x_i = 1, \\ 0, & \text{otherwise.} \end{cases} \tag{11}$$

Intuitively, c_i does not depend on b_i, so a_i cannot decrease its payment by unilaterally manipulating b_i. This implies that criticality-based pricing schemes are strategy-proof.

It is not difficult to see that a critical value also exists for each bidder in a MUCA with the same setting. In this paper, we shall extend critical-value-based payment for MUCA.

III. Dynamic MUCA Game

In the proposed framework, bidder independently sets up bid request and then notifies all competitors of that setting. The setting may cause the competitors to make their own moves. Because notifications take arbitrary time and there is no synchronization scheme to coordinate bidder's moves, bidders make moves one after another in a non-deterministic manner. We thus model MUCA as a dynamic game. In contrast, bidders in sealed-bid CAs place their bids without bidding information of any others, rendering it a static (one-shot) game.

We assume a BRF *brf* which maps any bid request to a positive real number. It defines a total order $\preceq$ on $\mathcal{B} = \{(\mathbf{s}_i, b_i)\}_{i=1}^n$ as follows.

Definition 1 (Ranks on Bid Requests): Given two bid requests $(\mathbf{s}_i, b_i)$ and $(\mathbf{s}_j, b_j)$, we have $(\mathbf{s}_i, b_i) \preceq (\mathbf{s}_j, b_j)$ if *brf*$(\mathbf{s}_i, b_i) \geq$ *brf*$(\mathbf{s}_j, b_j)$, and $(\mathbf{s}_i, b_i) \prec (\mathbf{s}_j, b_j)$ if *brf*$(\mathbf{s}_i, b_i) >$ *brf*$(\mathbf{s}_j, b_j)$.

We assume that *brf* is monotone. The monotonicity of BRF defined in [18] is for SUCA. We now generalize the monotonicity property to MUCA as follows.

Definition 2 (Monotonicity for Multi-unit BRF): Let $\mathbf{s}_i = (s_i^1, s_i^2, \ldots, s_i^m)$ be a_i's request vector. Let $\mathcal{S} = \{\mathbf{s}_1, \mathbf{s}_2, \ldots, \mathbf{s}_n\}$. Define binary relation $\leq$ on $\mathcal{S}$ as $\mathbf{s}_i \leq \mathbf{s}_j$ if $s_i^k \leq s_j^k$ for all $k \in \{1, \ldots, m\}$. Define binary relation $<$ on $\mathcal{S}$ as $\mathbf{s}_i < \mathbf{s}_j$ if $\mathbf{s}_i \leq \mathbf{s}_j$ and $\mathbf{s}_i \neq \mathbf{s}_j$. A BRF is monotone if $(\mathbf{s}_j, b_j) \preceq (\mathbf{s}_i, b_i)$ whenever $\mathbf{s}_j \leq \mathbf{s}_i$ and $b_j \geq b_i$.

By this definition, both $w_m(\cdot)$ and $w_d(\cdot)$ defined in Sec. II-A are monotone and can be used in the MUCA game.

To simplify our design and analysis, we assume that for any two bid requests $(\mathbf{s}_i, b_i)$ and $(\mathbf{s}_j, b_j)$, either $(\mathbf{s}_i, b_i) \prec (\mathbf{s}_j, b_j)$ or $(\mathbf{s}_j, b_j) \prec (\mathbf{s}_i, b_i)$. That is, no two bid requests have the same rank. Although BRF like $w_m(\cdot)$ and $w_d(\cdot)$ does not have this property, we can easily make it by introducing some tie-breaking rule for ranks like unique bidder identifiers.

Each agent a_i has a valuation on $\mathbf{s}_i$ denoted by $\nu_i(\mathbf{s}_i)$, which is private. We do not allow for externalities, which means that $\nu_i(\cdot)$ does not depend on any $\nu_j(\cdot)$ with $j \neq i$. Possibly different from $\mathbf{s}_i$, each agent a_i has a *need vector* $\mathbf{d}_i = (d_i^1, d_i^2, \ldots, d_i^m)$, where $d_i^j \leq q_j$ is the units of resource type r_j really needed by a_i. We assume exact allocation, so each bidder a_i is allocated either its request $\mathbf{s}_i$ (if a_i wins)

or nothing (otherwise). Moreover, the assumption of single-minded agents indicates that every agent a_i is interested in $\mathbf{d}_i$ only. Formally,

$$\nu_i(\mathbf{s}_i) = \begin{cases} \nu_i(\mathbf{d}_i), & \text{if } \mathbf{d}_i \leq \mathbf{s}_i, \\ 0, & \text{otherwise.} \end{cases} \tag{12}$$

Therefore, winning $\mathbf{s}_i$ such that $\mathbf{s}_i < \mathbf{d}_i$ gives no value to a_i. On the other hand, the monotonicity property implies that $(\mathbf{d}_i, b_i) \prec (\mathbf{s}_i, b_i)$ for all $\mathbf{d}_i < \mathbf{s}_i$ but $\nu_i(\mathbf{s}_i) = \nu_i(\mathbf{d}_i)$. Therefore, submitting $\mathbf{s}_i$ such that $\mathbf{d}_i < \mathbf{s}_i$ only lowers the probability of winning the auction (due to the monotonicity property) without increasing the value of the win. In other words, agent a_i has no incentive to manipulate $\mathbf{s}_i$ and $\mathbf{s}_i$ is not part of a_i's strategy in the game. Thus it is reasonable to assume that a_i sets up $\mathbf{s}_i$ initially and does not change $\mathbf{s}_i$ during the game.

Each agent's primary strategy is its bid b_i. In open ascending-price auctions and other decentralized auctions [33], [31], [16], agents can only *raise* their bids. We take the same assumption.

Besides b_i, every a_i also needs to declare whether it wins or not currently with b_i. We use x_i to denote a_i's declaration, where $x_i = 1$ if a_i declares a win and $x_i = 0$ otherwise. We use N_i to denote agent a_i's neighboring nodes in the conflict graph, i.e., the set of a_i's competitors. Every agent a_i needs to notify all agents in N_i of x_i. Similarly, a_i needs win declaration x_j of every agent $a_j \in N_i$ for its own win declaration. Win declaration information is needed because of the locality property that we want to exploit in designing decentralized auction protocol. For example, suppose that a_2 in Fig. 1 is a winner only if a_4 is not, which in turn depends on whether a_5 wins. Due to locality, a_2 does not have knowledge of a_5's win and thus cannot locally deduce a_4's win. Therefore, agent a_4 should notify a_2 of its win declaration.

Including x_i in a_i's strategy adds another dimension to agent's strategy space. The value of x_i should be interpreted as a_i's willingness to win and pay. This interpretation allows bidders to withdraw their current bids. In contrast, bidders in any other auction have no freedom to configure x_i's because bidders are implicitly assumed to be always willing to win with their current bids.

It is theoretically possible that a_i declares a win (i.e., $x_i = 1$) or loss ($x_i = 0$) without a matching bid b_i. The correctness of x_i depends on the relationship between b_i and a_i's critical value c_i. As proved by Lehmann et al. [18], if a BRF with both the exactness and monotonicity properties is used for winner determination, there is a critical value c_i for every a_i such that a_i wins if $b_i > c_i$ and does not if $b_i < c_i$. To define the correctness of win declaration in MUCA, we extend the definition of critical value for SUCA in [18] to MUCA as follows.

Definition 3 (Critical Value for MUCA): Given $\mathcal{B}_{-i} = \mathcal{B} \setminus \{(\mathbf{s}_i, b_i)\}$ and $X_{-i} = \{x_j | j \neq i\}$, a_i's critical value c_i is the minimal value that a_i can win by placing a bid $b_i > c_i$ (which is also the maximal value that a_i will definitely lose by placing $b_i < c_i$, if $c_i > 0$) with respect to $\mathcal{B}_{-i}$ and X_{-i}. Formally,

$$\sum_{(\mathbf{s}_j, b_j) \prec (\mathbf{s}_i, b_i)} \left(x_j \cdot s_j^k\right) \leq q_k - s_i^k \text{ for all } s_i^k \neq 0 \tag{13}$$

if $b_i > c_i$. If (13) holds when $b_i \geq 0$, we define $c_i = 0$. Otherwise, we also have

$$\sum_{(\mathbf{s}_j, b_j) \prec (\mathbf{s}_i, b_i)} \left(x_j \cdot s_j^k\right) > q_k - s_i^k \text{ for some } s_i^k \neq 0 \tag{14}$$

when $b_i < c_i$.

Because a_i wins only if $b_i \geq c_i$ and does not only if $b_i \leq c_i$, we have the following definition.

Definition 4 (Correctness of Win Declaration): For a pair (b_i, x_i) declared by any agent a_i, x_i is correct if $x_i = 1$ and $b_i > c_i$ or $x_i = 0$ and $b_i < c_i$. When $b_i = c_i$, which implies that the BRF value of $(\mathbf{s}_i, b_i)$ is the same as that of another bid request $(\mathbf{s}_j, b_j)$, whether x_i is correct depends on the tie-breaking rule used to determine the rank order between $(\mathbf{s}_i, b_i)$ and $(\mathbf{s}_j, b_j)$.

The setting of x_i directly affects a_i's utility. Let p_i be the price that a_i has to pay at the end of the auction. We consider both first-price payment and critical-value-based payment. In the first-price payment, each winner a_i pays its winning bid, i.e., $p_i = b_i$. In the critical-value-based payment, $p_i = c_i$ for each winner a_i. The utility of a_i is defined to be a_i's payoff in the auction, i.e., a_i's valuation on $\mathbf{s}_i$ minus p_i if a_i declares a win, and zero otherwise. Formally, given $\mathcal{B}_{-i} = \mathcal{B} \setminus \{(\mathbf{s}_i, b_i)\}$ and $X = \{x_j\}_{j=1}^n$,

$$u_i(\mathcal{B}_{-i}, X) = x_i \left(\nu_i(\mathbf{s}_i) - p_i\right). \tag{15}$$

The problem with (15) is that agent's utility has nothing to do with the correctness of win declaration. When $\nu_i(\mathbf{s}_i) > p_i$, a_i can get a positive utility by declaring $x_i = 1$ regardless of whether $b_i \geq c_i$. On the other hand, when $\nu_i(\mathbf{s}_i) < p_i$, a_i can get a zero (instead of negative) utility by declaring $x_i = 0$ even if $b_i > c_i$.

To ensure correct win declarations, we propose the following rules for agents that falsify win declarations.

- R1 If $x_i = 0$ but $b_i > c_i$, a_i gets nothing and pays p_i.
- R2 If $x_i = 1$ but $b_i < c_i$, a_i gets $\mathbf{s}_i$ and pay $p_i + \rho$, where $\rho > 0$ is a penalty.

With this treatment, the following theorem shows that falsifying win declaration is not beneficial if any false declaration can always be detected.

Theorem 1: If false win declarations are always detected, no agent has the incentive to falsify win declaration.

Proof: Consider any agent a_i and let $v_i = \nu_i(\mathbf{s}_i)$. One type of false win declaration is $x_i = 0$ but $b_i > c_i$. If $v_i \geq p_i$, then declaring $x_i = 1$ will give a_i a non-negative utility $v_i - p_i$ (instead of $-p_i$ by R1) so a_i has no incentive to declare $x_i = 0$. Therefore, it must be the case that $v_i < p_i$. When $x_i = 0$ is detected false at the end of the auction, a_i has to pay p_i by R1, which is higher than the loss $p_i - v_i$ if a_i declares a win instead. So a_i would rather set x_i to 1. Now consider the other case that $x_i = 1$ but $b_i < c_i$. If $v_i < p_i$, declaring

$x_i = 1$ gives a_i a negative utility (compared with 0 if $x_i = 0$ instead) so a_i has no incentive to do so. Therefore, it must be the case that $v_i \geq p_i$. In that case, raising b_i to some value between c_i and v_i (or c_i if $v_i = c_i$) would give a_i a utility $v_i - p_i$ larger than $v_i - p_i - \rho$ that a_i has to pay by R2. So a_i has no incentive to declare $x_i = 1$ with $b_i < c_i$. ∎

However, we cannot guarantee the detection of false win declarations if some agents collude with one another. If we preclude the possibility of collusion and always detect any single false declaration, then no agent has the incentive to make a false win declaration. Theorem 2 shows the feasibility of detecting any single false win declaration.

Theorem 2: After the MUCA game ends, any single false win declaration can be detected.

Proof: Without loss of generality, let a_i be the only agent with false x_i. Let $A_k = \{a_j | s_j^k \neq 0\}$ be the set of all agents that request r^k. All these agents are competitors so any of them has knowledge of all other's bid requests and win declarations. Consider two possible cases of false declarations:

- $x_i = 1$ is false. If $x_i = 1$ is correct, b_i should be larger than c_i. By Definition 3, $b_i > c_i$ implies that $\sum_{(\mathbf{s}_j, b_j) \prec (\mathbf{s}_i, b_i)} \left(x_j \cdot s_j^k\right) \leq q_k - s_i^k$ for all $s_i^k \neq 0$. All agents in $\cup_{s_i^k \neq 0} A_k$ can collaboratively verify whether the above condition holds. If it does not hold, then the declaration $x_i = 1$ is false.
- $x_i = 0$ is false. If $x_i = 0$ is correct, b_i should be less than c_i. By (14), $b_i < c_i$ implies the existence of some $s_i^k \neq 0$ such that $\sum_{(\mathbf{s}_j, b_j) \prec (\mathbf{s}_i, b_i)} \left(x_j \cdot s_j^k\right) > q_k - s_i^k$. Therefore, any agent in A_k is able to verify whether the above condition holds. If it does not hold, then the declaration $x_i = 0$ is false.

∎

Note that we do not need to perform false win declaration *during* the auction. It suffices to perform the detection once when the auction ends.

IV. MUCA Protocol

Designing a MUCA protocol faces two primary challenges. One is how each bidder locally determines whether the bidder itself is a winner according to the given BRF. It is not trivial because bidders competing for a common resource may be winners at the same time. The other is to make each bidder independently figure out how much it should pay for the auction. This is not trivial for critical-value-based payments. This section addresses these two issues and also discusses agent's bidding strategies with respect to different pricing schemes.

A. Decentralized BRF-based Winner Determination

We now describe the details of the proposed decentralized winner determination scheme. Because pricing does not affect the result of winner determination, the proposed scheme works for both first-price auctions and auctions with critical-value-based payment.

Each agent a_i in the scheme is free to set up b_i and x_i. By (15), any agent a_i's best response (the setting of b_i and x_i) depends on the relationship between $v_i = \nu_i(\mathbf{s}_i)$ and p_i If $p_i > v_i$, then declaring a win (i.e., $x_i = 1$) will give a_i a utility $u_i = v_i - p_i < 0$. Therefore, a_i would rather declare $x_i = 0$. On the other hand, if $p_i < v_i$, then declaring $x_i = 1$ and setting b_i to a value higher than c_i will yield a utility $u_i = v_i - p_i > 0$. This is higher than what a_i can get by declaring $x_i = 0$. Therefore, a_i's best response is

$$BR_i = \begin{cases} x_i = 0, & \text{if } p_i > v_i, \\ x_i = 1 \text{ and } b_i \in (c_i, v_i], & \text{if } p_i < v_i. \end{cases} \tag{16}$$

In (16), a_i does not set b_i a value higher than v_i. The reason is obvious in case of first-price auctions. For critical-value-based payment, this can be justified by the following lemma.

Lemma 1: In case of critical-value-based payment, if an agent a_i can win and get positive utility by setting b_i to some value higher than c_i, the value of b_i should not exceed v_i if a_i does not know when the auction will end.

Proof: The premise indicates that $p_i = c_i < v_i$. If this is the last bid of the auction, setting b_i to some value higher than c_i (say, b_i') and winning the bid gives a_i a utility $u_i = v_i - c_i > 0$. This holds regardless of whether $b_i' > v_i$. That is, picking up $b_i' > v_i$ does not give a_i extra benefit compared with another selection $c_i < b_i' < v_i$. On the other hand, if this is not the last bid of the auction, other agents may raise their bids and thus collectively increase b_i's critical value in the future. It is therefore possible that if a_i picks up $b_i' > v_i$, it may face a critical value c_i' in the future that is greater than v_i and less than b_i'. At that time, declaring $x_i = 0$ is false and will incur a negative utility by R1 while declaring $x_i = 1$ will give a_i a negative utility $u_i = v_i - c_i'$. Therefore, a_i should never set b_i a value higher than v_i. ∎

By Lemma 1, the maximal bid that a_i may place is v_i. Different agents may have different ideas about how to place their initial bids, so we assume that b_i is an arbitrary value in $[\sigma, v_i]$ initially, where σ is the starting bid set by the system. We also assume a minimum bid increment $\epsilon \geq 1$: whenever a_i wants to raise b_i to declare a win, b_i should be at least $c_i + \epsilon$. The initial value of x_i is not important, so it could be either 0 or 1. We assume that each agent a_i broadcasts $(\mathbf{s}_i, b_i, x_i)$ to all other agents in the beginning of the scheme so each agent a_i has knowledge of N_i, $\mathcal{B}$, and $\{x_i\}_{i=1}^n$ initially.

We assume that the logical channel between every agent and any of its neighbor delivers messages without loss and in a first-in-first-out (FIFO) manner. Each agent a_i keeps a local copy of (b_j, x_j) for each $a_j \in N_i$. When a_i receives a new update of (b_j, x_j) from another agent a_j, it executes Algorithm 2 as a response. a_i first updates its knowledge about b_j and x_j (Line 2), and checks whether a_i can win with its current bid by identifying a_i's *key predecessor*.

Definition 5 (Key Predecessor): If a_i can win with its current bid, a_i's key predecessor is a_i itself. Otherwise, a_i's key predecessor is a_k if $(\mathbf{s}_k, b_k)$ is the request that ranks the lowest among all winning requests whose absence alone would make $(\mathbf{s}_i, b_i)$ granted. Intuitively, a_i can make a_k "absent" in

Algorithm 2 Best Response of Agent a_i

1: **On** receiving *update*(b'_j, x'_j) from $a_j \in N_i$
2: $(b_j, x_j) \leftarrow (b'_j, x'_j)$
3: $\mathcal{C} \leftarrow \{(\mathbf{s}_j, b_j) | a_j \in N_i \wedge x_j = 1 \wedge (\mathbf{s}_j, b_j) \prec (\mathbf{s}_i, b_i)\}$
4: $k \leftarrow$ *key_predecessor*$(i, \mathcal{C})$
5: **if** $k = i$ **then**
6: $(b'_i, x'_i) \leftarrow (b_i, 1)$
7: **else** ▷ $(\mathbf{s}_k, b_k) \prec (\mathbf{s}_i, b_i)$
8: $c_i \leftarrow \min_b\{(\mathbf{s}_i, b) \prec (\mathbf{s}_k, b_k)\}$
9: $p_i \leftarrow c_i + \epsilon$ or c_i ▷ get payment; ϵ: minimum allowable increment
10: **if** $p_i < v_i$ **then**
11: $b'_i \leftarrow b \in [c_i + \epsilon, v_i]$; $x'_i \leftarrow 1$
12: **else** ▷ $b_i \leq c_i$ and $\sigma \geq v_i$
13: $(b'_i, x'_i) \leftarrow (b_i, 0)$
14: **end if**
15: **end if**
16: **if** $(b_i, x_i) \neq (b'_i, x'_i)$ **then**
17: $(b_i, x_i) \leftarrow (b'_i, x'_i)$
18: Send *update*(b_i, x_i) to each $a_j \in N_i$
19: **end if**
20: **end**

determining its win by outbidding a_k[3].

If a_i's key predecessor is $a_k \neq a_i$, we have $(\mathbf{s}_k, b_k) \prec (\mathbf{s}_i, b_i)$ and a_k must be a neighboring node of a_i in the conflict graph. Algorithm 3 (Procedure *key_predecessor*) details how to identify the key predecessor for a_i.

Lemma 2: Let j be the value returned by Procedure *key_predecessor*. If $j \neq i$, a_j is a_i's key predecessor.

Proof: The only place for *key_predecessor* to return $j \neq i$ is Line 7. This implies that $x_j = 1$ and $(\mathbf{s}_j, b_j) \prec (\mathbf{s}_i, b_i)$ because $(\mathbf{s}_j, b_j) \in \mathcal{C}$. Because it is a_j that causes *key_predecessor* to return $j \neq i$, we know that

$$\sum_{(\mathbf{s}_l, b_l) \prec (\mathbf{s}_j, b_j)} \left(x_l \cdot s_l^k\right) + x_j \cdot s_j^k + s_i^k > q_k \text{ for some } s_i^k \neq 0. \tag{17}$$

Therefore, a_i can become a winner only if it outbids a_j. On the other hand, since *key_predecessor* does not return l for any bid request $(\mathbf{s}_l, b_l) \in \mathcal{C}$ that outranks $(\mathbf{s}_j, b_j)$, we know that

$$\sum_{(\mathbf{s}_l, b_l) \prec (\mathbf{s}_j, b_j)} \left(x_l \cdot s_l^k\right) + s_i^k \leq q_k \text{ for all } s_i^k \neq 0. \tag{18}$$

Therefore, a_i need not outbid any a_l here to win its bid request. Thus $(\mathbf{s}_j, b_j)$ is the request that ranks the lowest among all winning requests such that $(\mathbf{s}_i, b_i)$ would be granted if $(\mathbf{s}_i, b_i)$ outranked $(\mathbf{s}_j, b_j)$. ∎

If a_i cannot win with its current bid, a_i finds out its critical value c_i, i.e., the minimal value of b_i that allows $(\mathbf{s}_i, b_i)$ to outrank $(\mathbf{s}_k, b_k)$. Assuming $\mathbf{q} = (3, 2, 2, 2, 2)$ and the BRF defined in (7) with $\alpha = 1$, Table III shows an example of key predecessors and critical values. The key predecessor of a_3 is a_2 because $(\mathbf{s}_3, b_3)$ would be granted if it outranked $(\mathbf{s}_2, b_2)$. To let $(\mathbf{s}_3, b_3)$ outrank $(\mathbf{s}_2, b_2)$, b_3 should be greater than

$$c_3 = \frac{b_2(\sum_{k=1}^m s_3^k)^\alpha}{(\sum_{k=1}^m s_2^k)^\alpha} = \frac{70 \times 4}{3} = 93.33. \tag{19}$$

[3] a_i outbids a_k if $(\mathbf{s}_i, b_i)$ outranks $(\mathbf{s}_k, b_k)$.

TABLE IV
AN MUCA WITH $\mathbf{q} = (1, 1)$ AND BRF =$(b_i / \sum_k s_i^k)$

(a) Parameter

Bidder (a_i)	Request vector $(\mathbf{s}_i)$	Valuation (v_i)	Initial (b_i, x_i)
a_1	$(1, 0)$	9	$(2, 1)$
a_2	$(1, 1)$	13	$(8, 1)$
a_3	$(0, 1)$	10	$(1, 0)$

(b) Running Example ($\epsilon = 1$)

Step	Agent	Old (b_i, x_i)	c_i	New (b_i, x_i)
1	a_3	$(1, 0)$	4	$(5, 1)$
2	a_1	$(2, 1)$	4	$(5, 1)$
3	a_2	$(5, 1)$	10	$(11, 1)$
4	a_1	$(5, 1)$	5.5	$(7, 1)$
5	a_2	$(11, 1)$	14	$(11, 0)$

Therefore, a_3's critical value c_3 is 93.33. The key predecessor of a_5 is a_1 because a_5's request can be granted only if a_5 outbids a_1. Neither a_2 nor a_4 is a_5's key predecessor, even though both outbid a_5.

Algorithm 3 Procedure *key_predecessor*$(i, \mathcal{C})$

1: *total_unit*$[k] \leftarrow 0$ for all $k \in \{1, \ldots, m\}$
2: **while** $\mathcal{C} \neq \emptyset$ **do**
3: Let $(\mathbf{s}_j, b_j)$ be the request that has the highest rank in $\mathcal{C}$
4: **for all** $k \in \{1, \ldots, m\}$ **do**
5: *total_unit*$[k] \leftarrow$ *total_unit*$[k] + s_j^k$
6: **if** $s_i^k > 0 \wedge s_i^k +$ *total_unit*$[k] > q_k$ **then**
7: return j
8: **end if**
9: **end for**
10: $\mathcal{C} \leftarrow \mathcal{C} \setminus \{(\mathbf{s}_j, b_j)\}$
11: **end while**
12: return i

Agent a_i can outbid its key predecessor and thus win its request by setting b_i to a value equal to or greater than $c_i + \epsilon$. However, whether it is worthy for a_i to win depends on the relationship between p_i and v_i. If $p_i < v_i$, a_i can win and get positive utility by changing b_i to some value in the range $[c_i + \epsilon, v_i]$ (to be discussed shortly). Otherwise, a_i has no incentive to change b_i because winning the request incurs a negative payoff (as $u_i = v_i - p_i < 0$). If a_i ever changes b_i or x_i, a_i notifies all a_i's competitors of the update.

Table IV shows a running example of the proposed approach, where three agents contend for two types of resources. Two agents turn out to be winners at the end of the auction.

B. Bidding Strategy

Because we design the MUCA protocol to be independent of the pricing scheme, b_i is generally set to be in the range $[c_i + \epsilon, v_i]$ in Line 11 of Algorithm 2. If we set up a particular pricing scheme, agents may have different bidding strategies here.

In the first-price payment, each winner a_i pays its winning bid, i.e., $p_i = b_i$. For this reason, the best response of each agent in the protocol is either to declare $x_i = 0$ or to minimize $b_i = c_i + \epsilon$. We call this bidding strategy *minimal bid increment*.

TABLE III
KEY PREDECESSOR AND CRITICAL VALUE EXAMPLE WITH $\mathbf{q} = (3, 2, 2, 2, 2)$

Bidder (a_i)	Demand vector ($\mathbf{s}_i$)	Bid (b_i)	BRF ($b_i / \sum_k s_i^k$)	x_i	Key predecessor	Critical value (c_i)
a_1	$(1, 0, 1, 0, 0)$	50	25.00	1	a_1	-
a_2	$(0, 0, 0, 2, 1)$	70	23.33	1	a_2	-
a_3	$(0, 1, 0, 1, 2)$	93	23.25	0	a_2	93.33
a_4	$(2, 1, 1, 0, 0)$	90	22.50	1	a_4	-
a_5	$(1, 0, 2, 1, 0)$	63	15.75	0	a_1	100

TABLE V
WINNER'S PAYOFF MATRIX IN CRITICAL-VALUE-BASED-PAYMENT AUCTION

		Competitors	
		Minimal Bid Increment	Truthful Bidding
Agent	Minimal Bid Increment	(median, median)	(low, high)
	Truthful Bidding	(high, low)	(low, low)

In the critical-value-based payment, setting b_i to any value not less than $c_i + \epsilon$ gives a_i the same payment $p_i = c_i$ if a_i wins. To minimize its critical value c_i, a_i needs to minimize the bids of non-winning competitors. Consider two possible bidding strategies: minimal bid increment (placing bid $b_i = c_i + \epsilon$ as a response) and truthful bidding (setting $b_i = v_i$ as a_i's initial bid). The former usually causes incremental increases of competitor's bids. In contrast, announcing an agent's highest possible bid (i.e., its valuation) can make all non-winning competitors quit bidding at the earliest possible time in an ascending-price auction. As a result, the bids of non-winning competitors are minimized. Therefore, truthful bidding is every agent's weakly dominant strategy. Refer to the payoff matrix of winner[4] shown in Table V.

When every agent bids truthfully, the outcome of Algorithm 2 will be equivalent to that of Algorithm 1. However, winner's payoff will also be low. Later we shall show that agent's bidding strategies do not affect the auction outcome (i.e., winners are always winners irrespective of their bidding strategies). Therefore, we do not assume any particular bidding strategy in our protocol design.

C. Payment Determination

After winner determination ends, each winner a_i should independently figure out how much it should pay for the auction. The task is trivial for first-price auctions ($p_i = b_i$). For the critical-value-based payment, the task is to identify a_i's *key successor*.

Definition 6 (Key Successor): Let a_i be a winner. Bidder $a_k \neq a_i$ is a_i's key successor if $(\mathbf{s}_k, b_k)$ is the request that ranks the highest among all non-winning requests that would be granted if $(\mathbf{s}_i, b_i)$ were not present. If there is no such request, a_i's key successor is defined to be a_i itself.

If a_i's key successor is $a_k \neq a_i$, a_i's payment p_i is the minimal value of b_i that makes the rank of $(\mathbf{s}_i, b_i)$ equal to or higher than that of $(\mathbf{s}_k, b_k)$. If a_i's key successor is a_i itself, then $p_i = 0$. This payment is exactly a_i's critical value.

Let us revisit the example shown in Table III. Table VI shows the key successor and payment, respectively, for each winner. The key successor for a_2 is a_3 because $(\mathbf{s}_3, b_3)$ would be granted if $(\mathbf{s}_2, b_2)$ were not present. For $(\mathbf{s}_2, b_2)$ to outrank $(\mathbf{s}_3, b_3)$, b_2 should be greater than

$$c_2 = \frac{b_3(\sum_{k=1}^{m} s_2^k)^\alpha}{(\sum_{k=1}^{m} s_3^k)^\alpha} = \frac{93 \times 3}{4} = 69.75. \tag{20}$$

Therefore, a_2's payment p_2 is 69.75. The key successor for a_1 is a_1 itself because neither a_3's nor a_5's request would be granted if a_1's request were not present. Therefore, a_1's payment is 0.

The relationship between key predecessor and key successor is not symmetric. For example, a_1 is a_5's key predecessor in Table III but a_5 is not a_1's key successor here.

Key predecessor and key successor identifications need different knowledge of bid requests. In the former case, an agent a_i only needs bid requests from all agents in N_i ($\mathcal{C}$ in Algorithm 3). In contrast, a_i in the latter case should have knowledge of all other agent's bid requests ($\mathcal{C} \cup \mathcal{D}$ in Algorithm 4). The reason is that for a_i to determine whether $a_j \in N_i$ is a_i's key predecessor, a_i needs to know whether a_j does not win simply because of a_i, or there is another winner that also prevents a_j from winning the bid. In the latter case, a_j is not a_i's key successor.

Algorithm 4 Procedure *key_successor*$(i, \mathcal{B}, \mathbf{x})$

```
C ← {(s_j, b_j) | (s_j, b_j) ≺ (s_i, b_i)}
total_unit[k] ← 0 for all k ∈ {1, ..., m}
for all (s_j, b_j) ∈ C such that x_j = 1 do
    for all k ∈ {1, ..., m} do
        total_unit[k] ← total_unit[k] + s_j^k
    end for
end for
D ← {(s_j, b_j) | (s_i, b_i) ≺ (s_j, b_j)}
while D ≠ ∅ do
    Let (s_j, b_j) be the request that has the highest rank in D
    if x_j = 1 then
        total_unit[k] = total_unit[k] + s_j^k for all k
    else                                  ▷ a conflicting, non-winning request
        if total_unit[k] + s_j^k ≤ q_k for all k, s_j^k > 0 then
            return j
        end if
    end if
    D ← D \ {(s_j, b_j)}
end while
return i
```

[4]Non-winner's payoff is always zero regardless of the bidding strategy in use.

TABLE VI
KEY SUCCESSOR AND PAYMENT EXAMPLE WITH $\mathbf{q} = (3, 2, 2, 2, 2)$

Bidder (a_i)	Bid (b_i)	Request vector ($\mathbf{s}_i$)	BRF ($b_i / \sum_k s_i^k$)	Result (x_i)	Key successor	Payment (p_i)
a_1	50	$(1, 0, 1, 0, 0)$	25.00	1	a_1	0
a_2	70	$(0, 0, 0, 2, 1)$	23.33	1	a_3	69.75
a_3	93	$(0, 1, 0, 1, 2)$	23.25	0	-	0
a_4	90	$(2, 1, 1, 0, 0)$	22.50	1	a_4	0
a_5	63	$(1, 0, 2, 1, 0)$	15.75	0	-	0

Algorithm 4 details how to identify the key successor for a_i. The loop from Lines 3 to 7 first counts in all resource units that are allocated to winners who outbid a_i. We skip resource allocation to a_i to mimic the absence of a_i in the auction. Then, the loop from Lines 9 to 19 checks all potential key successors a_j in the order of their bid-request ranks. If a_j is a winner, it cannot be a key predecessor so we just count in the amount of resource units allocated to it (Line 12). If a_j is not a winner, a_j is a_i's key successor if its request can be granted with the residual capacity (Line 14).

It is a concern whether a winner a_i is possible to claim a lower payment $c_i' < c_i$. If this happens, a_i's key successor, say, a_j, will find out that $(\mathbf{s}_j, b_j) \prec (\mathbf{s}_i, c_i')$ and become a winner. Therefore, such a false claim is detectable.

V. PROTOCOL ANALYSES

In this section, we analyze whether the proposed protocol stabilizes (i.e., eventually stops), and, if it does, whether the outcome is correct (i.e., meeting the capacity constraint and conforming to the BRF-based winner determination rule) and consistent (with Algorithms 1 in terms of the set of winners). We also show the individual rationality property of the proposed protocol.

A. Stabilization

The protocol may potentially not stabilize because every time a_i changes b_i or x_i, the rank of its bid request and thus the key predecessors of other agents may change. That may cause another agent's reaction and change the set of (declared) winners. For convergence, we consider first how each agent's knowledge about bids and win declarations evolves with time. Let $\mathbf{b}_i^t = (b_1^t, b_2^t, \ldots, b_n^t)$ and $\mathbf{x}_i^t = (x_1^t, x_2^t, \ldots, x_n^t)$ denote a_i's knowledge of all b_j's and x_j's, respectively, after the t-th execution of Line 17 of Algorithm 2 by a_i. Let $\mathbf{b}_i^0$ and $\mathbf{x}_i^0$ be a_i's initial knowledge. We assume $b_j^t = 0$ and $x_j^t = 0$, where $t \geq 0$, for all $a_j \notin N_i$. Because the execution of Line 17 of Algorithm 2 is triggered by a new update on some (b_j, x_j) and changes b_i or x_i, we have $(\mathbf{b}_i^t, \mathbf{x}_i^t) \neq (\mathbf{b}_i^{t+1}, \mathbf{x}_i^{t+1})$ for all $t \geq 0$. A sequence $\xi_i^t = (\mathbf{b}_i^0, \mathbf{x}_i^0), (\mathbf{b}_i^1, \mathbf{x}_i^1), (\mathbf{b}_i^2, \mathbf{x}_i^2), \ldots, (\mathbf{b}_i^t, \mathbf{x}_i^t)$ is a *transition path* of agent a_i that is of length t.

Theorem 3: Any transition path of any agent is finite.

Proof: Consider any agent a_i and one of its transition paths $\xi_i = (\mathbf{b}_i^0, \mathbf{x}_i^0), (\mathbf{b}_i^1, \mathbf{x}_i^1), (\mathbf{b}_i^2, \mathbf{x}_i^2), \ldots$. Since agents can only raise their bids and no agent places a bid higher than its valuation, there exists some integer t_i such that $\mathbf{b}_i^t$ no longer changes when $t \geq t_i$. Although the values of t_i may be different for different a_i's, eventually $\mathbf{b}_i^t$ will stabilize for all a_i's. After that, the rank of bid requests is finalized. Without loss of generality, assume that $(\mathbf{s}_j, b_j) \prec (\mathbf{s}_{j+1}, b_{j+1})$ for all $1 \leq j < n$. We already know that no agent a_j has the incentive to falsify x_j. It follows that the value of x_1 will eventually stabilize. Because of this, the value of x_2 will eventually stabilize, and so on. Therefore, ξ_i must be finite. ■

TABLE VII
TWO CONFORMING OUTCOMES OF THE MUCA SHOWN IN TABLE IV

Bidder (a_i)	Outcome 1 $(\hat{b}_i, \hat{x}_i)$	Outcome 2 $(\hat{b}_i, \hat{x}_i)$
a_1	$(9, 1)$	$(7, 1)$
a_2	$(13, 0)$	$(8, 0)$
a_3	$(10, 1)$	$(3, 1)$

B. Correctness

Because all update messages sent by a_i are delivered in the same order by all agents in N_i, all these agents must have the same knowledge of (b_i, x_i) when the auction ends. Let $(\hat{b}_i, \hat{x}_i)$ be the last value of (b_i, x_i) sent by a_i. When the protocol stabilizes, the outcome (i.e., the collective settings of $\hat{b}_i$'s and $\hat{x}_i$'s) is denoted by $\mathcal{O} = (\hat{\mathbf{b}}, \hat{\mathbf{x}})$, where $\hat{\mathbf{b}} = (\hat{b}_1, \hat{b}_2, \ldots, \hat{b}_n)$ and $\hat{\mathbf{x}} = (\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_n)$. We consider $\mathcal{O}$ *correct* if it meets the capacity constraint and conforms to the BRF-based winner-determination rule defined below.

Definition 7 (BRF-based Winner Determination Rule): Let $\prec$ be a total order defined by a BRF on bid requests $\mathcal{B} = \{(\mathbf{s}_i, b_i\}_{i=1}^n$. An outcome of the CA $\mathcal{O} = (\hat{\mathbf{b}}, \hat{\mathbf{x}})$ conforms to the BRF-based winner determination rule if for every $\hat{x}_i$, where $1 \leq i \leq n$, $\hat{x}_i = 1$ only if

$$s_i^k + \sum_{(\mathbf{s}_j, \hat{b}_j) \prec (\mathbf{s}_i, \hat{b}_i)} \left(\hat{x}_j \cdot s_j^k\right) \leq q_k \text{ for all } s_i^k \neq 0. \qquad (21)$$

It is possible that two or more outcomes conform to the BRF-based winner determination rule. Table VII shows an example with two conforming outcomes. We have $(\mathbf{s}_3, b_3) \prec (\mathbf{s}_1, b_1) \prec (\mathbf{s}_2, b_2)$ in the first outcome and $(\mathbf{s}_1, b_1) \prec (\mathbf{s}_2, b_2) \prec (\mathbf{s}_3, b_3)$ in the second one. Both outcomes conform to the rule, though the bids are different.

With the following two theorems, we show that the proposed protocol always ends up with a correct outcome.

Lemma 3: Let $\mathcal{O} = (\hat{\mathbf{b}}, \hat{\mathbf{x}})$ be an outcome of Algorithm 2. $\mathcal{O}$ conforms to the BRF-based winner determination rule, i.e., for every $\hat{x}_i$, where $1 \leq i \leq n$, $\hat{x}_i = 1$ only if (21) holds.

Proof: Every execution of Algorithm 2 calls procedure *key_predecessor*. For every agent a_i such that $\hat{x}_i = 1$, the

call to *key_predecessor* in a_i's last execution of Algorithm 2 returns either $j \neq i$ or i.

In the former case, a_j is a_i's key predecessor by Lemma 2. The result $\hat{x}_i = 1$ can only be set in Line 11 of Algorithm 2. In the same line, a_i increases b_i to some value $\hat{b}_i$ such that $(\mathbf{s}_i, \hat{b}_i)$ outranks $(\mathbf{s}_l, b_j)$. Because $(\mathbf{s}_i, \hat{b}_i) \prec (\mathbf{s}_l, b_j)$, we have

$$\sum_{(\mathbf{s}_l, b_l) \prec (\mathbf{s}_i, \hat{b}_i)} \left(x_l \cdot s_l^k\right) \leq \sum_{(\mathbf{s}_l, b_l) \prec (\mathbf{s}_i, b_j)} \left(x_l \cdot s_l^k\right). \tag{22}$$

for all $s_i^k \neq 0$. By (18), we have

$$\sum_{(\mathbf{s}_l, b_l) \prec (\mathbf{s}_i, \hat{b}_i)} \left(x_l \cdot s_l^k\right) + s_i^k \leq q_k \text{ for all } s_i^k \neq 0. \tag{23}$$

In the latter case, a_i must pass the *while* loop in Algorithm 3 without finding any $s_i^k \neq 0$ such that

$$\sum_{(\mathbf{s}_j, b_j) \prec (\mathbf{s}_i, \hat{b}_i)} \left(x_j \cdot s_j^k\right) + s_i^k > q_k, \tag{24}$$

which is equivalent to (23).

In either case, any agent $a_l \neq a_i$ may later change b_l or x_l. If any of those changes ever affected a_i's best response, $(\hat{b}_i, \hat{x}_i)$ would not be a_i's final decision. Therefore, (23) still holds when the protocol ends. That is

$$\sum_{(\mathbf{s}_j, \hat{b}_j) \prec (\mathbf{s}_i, \hat{b}_i)} \left(\hat{x}_j \cdot s_j^k\right) + s_i^k \leq q_k \text{ for all } s_i^k \neq 0. \tag{25}$$

■

Lemma 4: Let $\mathcal{O} = (\hat{\mathbf{b}}, \hat{\mathbf{x}})$ be an outcome of Algorithm 2. $\mathcal{O}$ meets the resource capacity constraint specified in (2), i.e.,

$$\sum_{i=1}^{n} \left(\hat{x}_i \cdot s_i^k\right) \leq q_k \text{ for all } k = 1, \ldots, m.$$

Proof: For each k, $1 \leq k \leq m$, let $(\mathbf{s}_i, \hat{b}_i)$ be the bid request that is of the lowest rank in $\mathcal{B}$ such that $\hat{x}_i = 1$ and $s_i^k \neq 0$. This implies for all request $(\mathbf{s}_j, \hat{b}_j) \in \mathcal{B}$ that ranks lower than $(\mathbf{s}_i, \hat{b}_i)$, we have either $s_j^k = 0$ or $\hat{x}_j = 0$. Therefore,

$$\sum_{j=1}^{n} \left(\hat{x}_j \cdot s_j^k\right) = s_i^k + \sum_{(\mathbf{s}_j, \hat{b}_j) \prec (\mathbf{s}_i, \hat{b}_i)} \left(\hat{x}_j \cdot s_j^k\right).$$

Because $\hat{x}_i = 1$, we know that (21) holds by Lemma 3. We thus have the proof. ■

Theorem 4: The outcome of Algorithm 2 is correct.

Proof: It directly follows from Lemmas 3 and 4. ■

C. Consistency

Even though Algorithm 2 converges and the outcome is correct, the outcome may deviate from that obtained by Algorithm 1 with the same BRF. Let $\hat{\mathbf{b}} = (\hat{b}_1, \hat{b}_2, \ldots, \hat{b}_n)$ and $\hat{\mathbf{b}}' = (\hat{b}'_1, \hat{b}'_2, \ldots, \hat{b}'_n)$ be two vectors that represent the final bids found by Algorithms 2 and 1, respectively. Because $0 \leq \hat{b}_i \leq v_i$ for all $\hat{b}_i \in \hat{\mathbf{b}}$ and $\hat{b}'_i = v_i$ for all $\hat{b}'_i \in \hat{\mathbf{b}}'$, we have $\hat{\mathbf{b}} \leq \hat{\mathbf{b}}'$. Consequently, the ranks of bid requests in these two algorithms can be different. In fact, Outcomes 1 and 2 in Table VII are exactly the results found by Algorithms 1 and 2, respectively. The ranks of bid requests are obviously different.

Despite of the difference in ranks, we shall prove that Algorithm 2 is *consistent* in the sense that it identifies the same set of winners as Algorithm 1 using the same BRF.

Theorem 5: Let $\hat{\mathbf{b}} = (\hat{b}_1, \hat{b}_2, \ldots, \hat{b}_n)$ and $\hat{\mathbf{b}}' = (\hat{b}'_1, \hat{b}'_2, \ldots, \hat{b}'_n)$ be two vectors that represent the final bids found by Algorithms 2 and 1, respectively. Let $\hat{\mathbf{x}} = (\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_n)$ and $\hat{\mathbf{x}}' = (\hat{x}'_1, \hat{x}'_2, \ldots, \hat{x}'_n)$ be two vectors that represent the final win declarations of Algorithms 2 and 1, respectively. If these two algorithms use the same BRF, then $\hat{\mathbf{x}} = \hat{\mathbf{x}}'$.

Proof: Without loss of generality, we assume that $(\mathbf{s}_i, \hat{b}'_i = v_i)$ is ranked i-th in Algorithm 1. However, the corresponding bid request $(\mathbf{s}_i, \hat{b}_i)$ is not necessarily ranked i-th in Algorithm 2. Let $\mathbf{c} = (c_1, c_2, \ldots, c_n)$ and $\mathbf{c}' = (c'_1, c'_2, \ldots, c'_n)$ be two vectors that represent the critical values for $(\hat{\mathbf{b}}, \hat{\mathbf{x}})$ and $(\hat{\mathbf{b}}', \hat{\mathbf{x}}')$, respectively.

By way of contradiction, assume that $\hat{\mathbf{x}} \neq \hat{\mathbf{x}}'$. Let k be the smallest number such that $\hat{x}_k \neq \hat{x}'_k$. There are two possible cases.

- $(\hat{x}_k, \hat{x}'_k) = (0, 1)$. A necessary condition for $\hat{x}'_k = 1$ is

  $$\sum_{j=1}^{k} (\hat{x}'_j \cdot s_j^l) \leq q_l \text{ for all } l \text{ such that } s_k^l \neq 0. \tag{26}$$

  Another necessary condition for $\hat{x}'_k = 1$ is $c'_k < v_k$. Because $\hat{\mathbf{b}} \leq \hat{\mathbf{b}}'$, we have $c_k \leq c'_k$. Together with the condition that $c'_k < v_k$, Algorithms 2 is free to set $\hat{b}_k$ to some value $b \in (c_k, v_k]$ and thus wins the auction. Therefore, it is impossible that $\hat{x}_k = 0$.
- $(\hat{x}_k, \hat{x}'_k) = (1, 0)$. The result $\hat{x}'_k = 0$ implies that

  $$s_k^l + \sum_{j=1}^{k-1} (\hat{x}'_j \cdot s_j^l) > q_l \text{ for some } l \text{ such that } s_k^l \neq 0. \tag{27}$$

  Let $\bar{l}$ be one such l. That is,

  $$s_k^{\bar{l}} + \sum_{j=1}^{k-1} (\hat{x}'_j \cdot s_j^{\bar{l}}) > q_{\bar{l}}. \tag{28}$$

  If $k = 1$, which implies $s_1^{\bar{l}} > q_{\bar{l}}$, then $\hat{x}_1$ must be 0 as well by Lemma 4. Therefore, k must be greater than 1. Let $\mathcal{A}'_k = \{a_j | j < k, s_j^{\bar{l}} \neq 0, \hat{x}'_j = 1\}$ be the set of agents that also request $r_{\bar{l}}$ with bid requests outranking a_k's and getting granted by Algorithm 1. Let $\mathcal{B}_k = \{(\mathbf{s}_j, \hat{b}_j) | a_j \in \mathcal{A}'_k\}$ be the set of bid requests submitted by all the agents in $\mathcal{A}'_k$ when running Algorithm 2. If $(\mathbf{s}_k, \hat{b}_k)$ does not outrank any $(\mathbf{s}_j, \hat{b}_j) \in \mathcal{B}_k$ in the end of Algorithm 2, then

  $$s_k^{\bar{l}} + \sum_{(\mathbf{s}_j, \hat{b}_j) \prec (\mathbf{s}_k, \hat{b}_k)} (\hat{x}_j \cdot s_j^{\bar{l}}) \geq s_k^{\bar{l}} + \sum_{j=1}^{k-1} (\hat{x}'_j \cdot s_j^{\bar{l}}) \tag{29}$$

  because $\hat{x}_j = \hat{x}'_j$ for all $j < k$. By (28) and (29), we have

  $$s_k^{\bar{l}} + \sum_{(\mathbf{s}_j, \hat{b}_j) \prec (\mathbf{s}_k, \hat{b}_k)} (\hat{x}_j \cdot s_j^{\bar{l}}) > q_{\bar{l}}, \tag{30}$$

which implies that $\hat{x}_k$ cannot be 1. Therefore, $(\mathbf{s}_k, \hat{b}_k)$ must outrank some $(\mathbf{s}_j, \hat{b}_j) \in \mathcal{B}_k$ to declare $\hat{x}_k = 1$. Let $(\mathbf{s}_p, \hat{b}_p)$ be the bid request that ranks the lowest in $\mathcal{B}_k$. It have two properties. First, $\hat{x}_p = 1$ because $a_p \in \mathcal{A}'_k$ and $\hat{x}_j = \hat{x}'_j$ for all j, $1 \leq j < k$. By Lemma 3, we then have

$$s_p^{\bar{l}} + \sum_{(\mathbf{s}_j, \hat{b}_j) \prec (\mathbf{s}_p, \hat{b}_p)} (\hat{x}_j \cdot s_j^{\bar{l}}) \leq q_{\bar{l}}. \tag{31}$$

Second, the set $\{(\mathbf{s}_j, \hat{b}_j) | (\mathbf{s}_j, \hat{b}_j) \prec (\mathbf{s}_p, \hat{b}_p)\}$ includes $(\mathbf{s}_k, \hat{b}_k)$ (because $(\mathbf{s}_k, \hat{b}_k)$ must outrank $(\mathbf{s}_p, \hat{b}_p)$) as well as all bid requests in the set $\mathcal{B}_k \backslash \{(\mathbf{s}_p, \hat{b}_p)\}$. It may include other bid requests. Therefore,

$$\sum_{(\mathbf{s}_j, \hat{b}_j) \prec (\mathbf{s}_p, \hat{b}_p)} (\hat{x}_j \cdot s_j^{\bar{l}}) \geq \sum_{1 \leq j \leq k, j \neq p} (\hat{x}_j \cdot s_j^{\bar{l}}). \tag{32}$$

By (31), we have

$$s_p^{\bar{l}} + \sum_{1 \leq j \leq k, j \neq p} (\hat{x}_j \cdot s_j^{\bar{l}}) \leq q_{\bar{l}}. \tag{33}$$

Because $\hat{x}_k = 1$, $\hat{x}'_p = 1$, and $\hat{x}'_j = \hat{x}_j$ for all $1 \leq j < k$, (33) can be rewritten as

$$s_k^{\bar{l}} + \sum_{1 \leq j < k} (\hat{x}'_j \cdot s_j^{l}) \leq q_{\bar{l}}. \tag{34}$$

This contradicts with (28).

Therefore, it is impossible that $\hat{\mathbf{x}} \neq \hat{\mathbf{x}}'$. ∎

D. Individual Rationality

An auction scheme provides *individual rationality* if no participant receives a negative utility. By (15), any agent a_i not declaring a win ($x_i = 0$) gets zero utility. If agent a_i declares a win ($x_i = 1$), its utility is $\nu_i(\mathbf{s}_i) - p_i$ by (15). By (16), the declaration of $x_i = 1$ implies $p_i < v_i$, where $v_i = \nu_i(\mathbf{s}_i)$, since otherwise x_i must be 0. Therefore, any agent a_i declaring $x_i = 1$ has a utility $\nu_i(\mathbf{s}_i) - p_i > 0$. Since no agent receives a negative utility, the proposed protocol ensures individual rationality.

VI. Numerical Results

We conducted simulations to evaluate the performance of the proposed scheme in terms of total winning bid, total payment, and the time to convergence. The first two performace metrics are compared with those obtained by two centralized BRF-based greedy allocations. One was proposed by Mito and Fujita [30] with BRF $w_n(\cdot)$ defined in (4), where the value of β is set to 0.5. The other was proposed by Jia et al. [23] with BRF $w_m(\cdot)$ defined in (7), where the value of α is set to 1.

We assumed n bidders and m types of items. For supply side, the number of identical instances for any resource type r_j, q_j, was distributed over the set of integers in the range $[1, q_{\max}]$, where $q_{\max}$ is a fixed number. For demand side, the probability that any agent a_i requests any resource type r_j was a tunable parameter p_s. If a_i did request r_j, s_i^j was distributed over the set of integers in the range $[1, q_j]$. Let $v_{i,j}$ be a_i's valuation on one instance of resource type r_j. The set $\{v_{i,j}\}_{i=1}^n$ are i.i.d's which follows a Gaussian distribution truncated at 0 and 50 with mean μ_j, where μ_j is a random variable uniformly distributed over the range $[10, 20]$. Agent a_i's valuation on $\mathbf{s}_i$ was set to

$$\nu_i(\mathbf{s}_i) = \sum_{j=1}^{m} (s_i^j \times v_{i,j}). \tag{35}$$

As mentioned, the proposed approach does not demand particular initial value of each x_i. We thus tested three possible settings for the initial values of x_i's: all 1's, all 0's, and randomly selected 1's and 0's.

We assumed critical-value-based payments but did not assume any particular bidding strategy. More specifically, the new bid in Line 11 of Algorithm 2 was randomly selected from the range $[c_i + \epsilon, v_i]$. The value of ϵ was set to a small number (0.001) to maximize the range of the new bid selection.

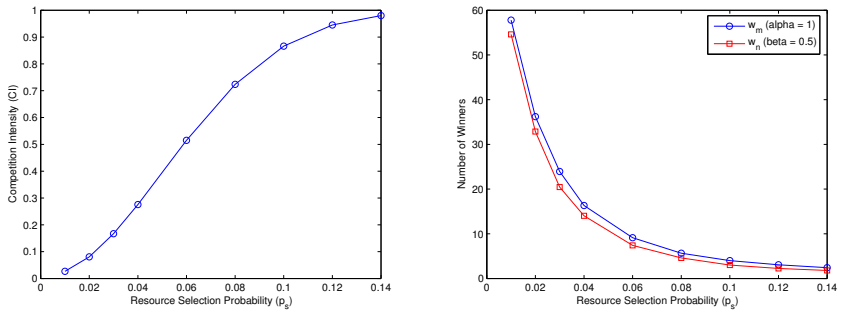
For each possible setting, we generated 100 test data and performed 10 trials for each data. Each result is an average over these 1000 trials.

A factor that affects the performance metrics is *competition intensity* (CI), the ratio of the total number of edges in the conflict graph to the maximum (i.e., $n(n-1)/2$). We fixed m, n, and $q_{\max}$, and varied the value of p_s from 0.01 to 0.14 to adjust CI. Fig. 2a shows how CI changes with increasing p_s.

When CI increased, the numbers of winners identified with both BRFs decreased, as Fig. 2b shows. Here w_m slightly outperformed w_n due to the consideration of the number of instances in its function definition. For a specific BRF, the centralized greedy approach and the proposed decentralized approach yielded exactly the same set of winners.

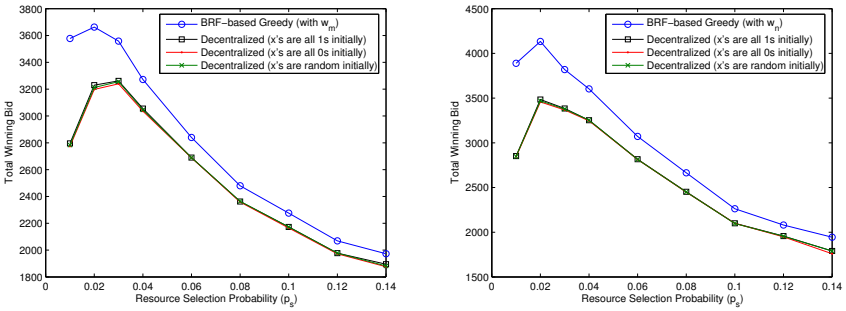
Figure 3 shows how total winning bid changes with respect to p_s. For both BRFs, the centralized approach outperformed the decentralized counterpart, and the performance of the decentralized approach was not affected by the initial setting of x_i's. Although there were more winners with $p_s = 0.01$ than with $p_s \geq 0.02$ in both BRFs, the total winning bid with $p_s = 0.01$ was not always higher than that with $p_s \geq 0.02$. The reason is that although there were more winners with $p_s = 0.01$ than with $p_s \geq 0.02$, each winner with $p_s = 0.01$ generally requested fewer resource instances than that with $p_s \geq 0.02$. Because each agent's bid was roughly in proportional to the number of requested instances, we obtained the highest total winning bid with $p_s = 0.02$ or $p_s = 0.03$. When p_s increased further, the total winning bid decreased because the number of winners decreased significantly as Fig. 2b indicates.

Figure 4 shows how the total payment changes with increasing CI. Here all approaches exhibit behaviors similar to the result of total winning bid. However, the total payment is always lower than the total winning bid under any circumstance. The performance gap between the centralized and the proposed decentralized approach becomes smaller when CI is larger. This trend is also similar to that exhibited in Fig. 3.



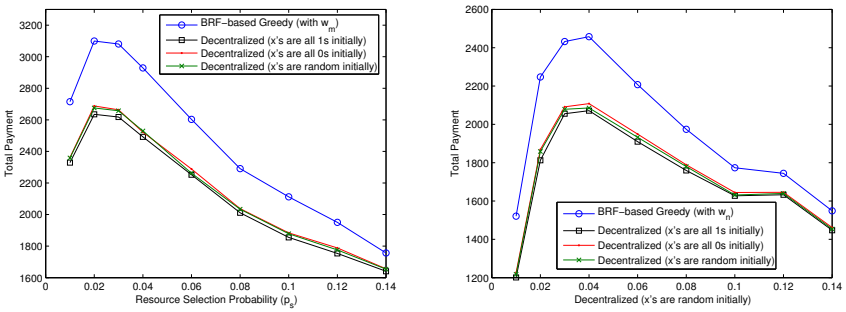
(a) (b)

Fig. 2. Results with $q_{\max} = 5$, $n = 100$, and $m = 200$. (a) Competition intensity (CI) versus p_s. (b) Number of winners versus p_s.



(a) (b)

Fig. 3. Average total winning bid with (a) w_m ($\alpha = 1$) and (b) w_n ($\beta = 0.5$). ($q_{\max} = 5$, $n = 100$, and $m = 200$)



(a) (b)

Fig. 4. Average payment with (a) w_m ($\alpha = 1$) and (b) w_n ($\beta = 0.5$). ($q_{\max} = 5$, $n = 100$, and $m = 200$)

We studied the convergence time of the decentralized approach by measuring the total number of moves taken by all bidders before reaching the final result. We did not count the initial setting and broadcast of bid requests; only changes of bid requests (sending of updates) count. Fig. 5 shows the results, which clearly depend on the initial settings of x_i's. Generally speaking, the all-1s initial setting demanded fewer moves than the all-0s initial setting when there were more winners (i.e., when p_s is small). On the other hand, the all-0s initial setting outperformed the all-0s initial setting when there were few winners (i.e., when p_s is large). The random setting generally lies between these two extremes, and would be the best choice if we do not know the CI value beforehand.

Regardless of the initial setting of x_i's, on average each agent took fewer than two moves before stability.

VII. Conclusions

We have proposed decentralized winner and payment determination protocols based on a given BRF for MUCA. It allows bidders to locally determine their bid and willingness to win by identifying their key predecessors. By exchanging that information with other competitors, other bidders can take moves so as to reach a consensus. For critical-value-based payment, winners determine their payments by identifying their respective key successors. We have proved that the proposed approach eventually stabilizes and is correct in the sense that the result meets the capacity constraint and conforms to the BRF-based winner-determination rule. We also proved that the proposed approach is consistent with

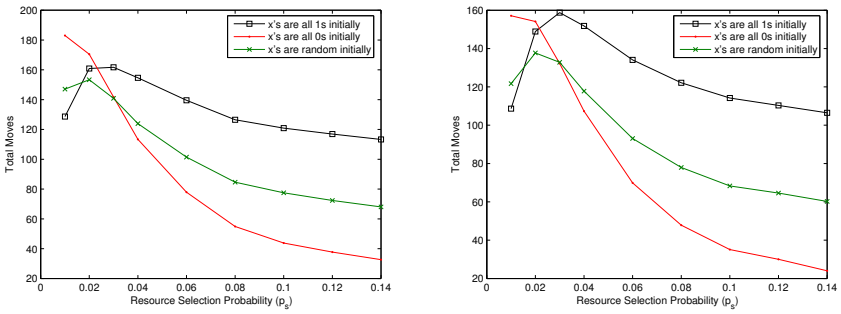


Fig. 5. Average number of moves with (a) w_m ($\alpha = 1$) and (b) w_n ($\beta = 0.5$). ($q_{\max} = 5$, $n = 100$, and $m = 200$)

the centralized counterpart using the same BRF in the sense that both approaches identify the same set of winners. The proposed approach also ensures truthful bidding and individual rationality. Simulation results confirms the correctness and consistency of the proposed approach at the cost of lower total winning bid and payment.

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