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A Two-Stage Game for Allocating Channels and Radios to Links in Wireless Backhaul Networks

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Abstract Radio interfaces and channels are two sorts of resources in a multi-channel multi-radio wireless mesh network. Efficient allocation of radio resources to mesh nodes should be done under the constraints of reducing co-channel interference yet with increased network connectivity. However, these two constraints conflict in nature as far as allocating radios (i.e., transceivers) and channels to links is concerned. In consideration of physical-layer interference, this paper proposes two non-cooperative games that play in sequence for radio resource allocation. The first game assigns channels to radios while the second distributes the resulting radio-channel pairs to links. The proposed games are shown to always reach a Nash equilibrium regardless of initial configurations, and together guarantee network connectivity while minimizing co-channel interference of each individual radio. We have conducted simulations to analyze game behaviors and carried out performance comparisons. The results indicate that game convergence time depends on the behavior of the first game. The proposed approach leads to more operative links than counterpart schemes when only two radios are available at each node, but loses its advantage over centralized, greedy methods when more radios are available.

Keywords radio resource · game theory · wireless mesh network

1 Introduction

A wireless mesh network (WMN) interconnects radio nodes in a mesh topology, providing frame delivery services to stations equipped with radio interfaces. A WMN comprises a wireless access and a wireless backhaul network. The wireless access network consists of dozens of mesh access points (MAPs)

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that cooperatively provide wireless access services to mesh clients in a large geographical area. The backhaul network links MAPs, allowing for multiple gateways to the wired backbone and multiple frame forwarding paths between each pair of MAPs. We assume that the wireless access network uses a different technology or spectrum from that used in the wireless backhaul network (e.g., IEEE 802.11g and 802.11a) such that communication in the access network cannot interfere with that in the backhaul network.

Basic devices in the backhaul network for traffic forwarding are mesh points (MPs). One MP may establish several wireless links called *designated links*, each to its neighboring MP. This study investigates radio resource (i.e., channels and dedicated transceivers) allocations for designated links in the backhaul network. All channels under consideration are non-overlapping (ruling out interference from adjacent channels), so only co-channel interference is of concern. Note that some channel allocation schemes addressing interference from adjacent channels can be found in [1,2]. The basic requirement in our study is that two wireless devices should have dedicated transceivers that tune to the same channel before they can communicate. However, when any other devices in the proximity generate signal on the same channel, the receiving end of the current communication is likely to experience co-channel interference. From physical layer perspective, co-channel interference degrades the quality of received signal, causing high bit error rate. If viewed from the link layer or above, co-channel interference brings about transmission collision and bandwidth contention, thus degrading goodputs. IEEE 802.11a as well as other wireless technology specifies several non-overlapping channels for use. Utilizing these channels efficiently can prevent or alleviate performance degradation by co-channel interference.

There has been research on efficient utilization of multiple channels. Some studies such as [3,4] proposed dynamic channel switching to allow the use of multiple channels by a single transceiver (also called radio in this paper). Channel switching works by dividing link-layer transmission time into fixed-length time slots and scheduling transmission and reception slots to reduce possible co-channel interference among nearby transceivers. Dynamic channel switching, however, requires network-wide tight synchronization among all the involved nodes at a non-trivial cost. Furthermore, switching channels incurs delay. The resulting delay is significant for some applications, particularly when channels taken by all the nodes in a multi-hop routing path diverge greatly.

If devices are equipped with multiple transceivers, each operating on a dedicated channel, then a device can communicate simultaneously with other devices yet free from switching channels. However, such arrangements should be done in an appropriate way to make all the designated links operative. We consider a link *operative* if both ends of the link have radios operating on the same channel (*common channel constraint*) and experiencing sufficiently low interference (*interference constraint*) [5]. These two constraints are often conflicting; meeting the common channel constraint for each designated link may inevitably cause severe co-channel interference to other links. This holds particularly in a dense network, where a limited number of radios and channels are to be allocated to a larger number of designated links in close proximity. On the other hand, leaving out some designated links in radio resource allocation (breaking the common channel constraint) can decrease interference with the others. How to maximize the number of operative links subject to the two constraints is an optimization problem. This study respects the common channel constraint and considers co-channel interference a performance metric to minimize rather than a requirement to meet in allocating radio resources to links.

Previous studies on the optimization problem mostly concern the effects of interference on the link layer or above. The protocol model [6], the most commonly adopted interference model, asserts binary interference relation on transceivers based on the notion of *interference range*. More specifically, transceiver u interferes with transceiver v (and v thus becomes inoperative) if v is within the interference range of u. On the other hand, the physical model [6] considers the intensity of interference experienced by transceivers, which is quantified with an exponentially decreasing function of distance to the interferer. In the physical model, transceiver u becomes inoperative if the aggregated interference intensity from all other transceivers exceeds some threshold. The physical model is considered more general than the protocol model. Accordingly this text adopts the physical model and uses the signal-to-interference ratio (SIR) to assess the operability of links.

As opposed to current schemes operating primarily based on heuristics, this study proposes a game-theoretic approach. Game theory provides a mathematical framework for strategic decision making in a competition where players have conflicting benefits or goals. For the last decade, game theory has been applied to deal with resource/duty sharing problems in wireless network environments. Founded on game theory, our development consists of two stages. The first stage allocates channels to radios with the objective of maximizing the SIR value each radio experiences. This is modeled as a non-cooperative game where radios act as players whose strategies are available channels. The game will produce a set of radio-channel pairs. By limiting the set of strategies available to each player, the produced result guarantees that the common channel constraint is met for every link. After the first stage, each link may have more than one candidate radio-channel pair to select. Subsequently the second stage formulates another non-cooperative game for distributing radio-channel pairs to links, with an aim to minimize the number of conflicting channels between neighboring links. We shall prove the stability of both games, showing that each game always ends up with a Nash equilibrium regardless of its initial configuration. We also conduct simulations to examine which policy (best response or better response) each player should follow in choosing its strategy to maximize the number of operative links, and how such a policy affects game convergence time. We compare the proposed approach with existing methods (centralized, greedy approaches) in terms of the number of operative links as well.

The remainder of this paper is organized as follows. Background and related work are presented in Section 2. Next we present the proposed game-theoretic approach to radio resource allocation in backhaul networks. In Section 4, simulation results of subject schemes are discussed and compared. Lastly Section 5 concludes this article.

2 Preliminaries

2.1 Background

A backhaul link is to connect two nodes. Therefore, radio resource allocation is essential to arrange a radio and assign a channel for each designated link at both ends of the link. The arrangements and assignments can be achieved in various ways. Traditional heuristics focus on one resource type at a time and can be categorized into three broad kinds. *Link-centric* schemes allocate radios/channels to links in some order [7–9, 5]. *Radio-centric* schemes assign channels and serving links to individual radios [10]. *Node-centric* schemes perform allocation in a node-by-node manner. When processing a node, these schemes assign channels to all radios of the node or to all links incident on the node [1, 11, 12].

Some channel allocation methods are traffic-aware [9, 11, 13–15], associating link or node traffic with a weight to determine which link or node is to be assigned a channel next. An important issue arises where traffic conditions are constantly changing, making it difficult to acquire accurate yet representative traffic dynamics. These approaches also incur extra overhead if allocation is computed anew whenever traffic condition changes. In view of such overhead, we do not take link traffic information into account.

The radio resource allocation problem implicitly assumes that available radios at a node are fewer than prospective links incident on the node in number (otherwise, the problem is trivial). The assumption places challenge on link connectivity. Fig. 1 shows an example where either of nodes A and B has three links to build with only two radios. Given that channels already assigned to radios of A and B are all distinct, the link connecting A and B is no longer possible to meet the common channel constraint. Skipping this link degrades the connectivity of the network and increases the average length of routing paths because of detours. In the worst case, the whole network may even become disconnected. The common channel assignment (CCA) assigns channel 1 to radio 1, channel 2 to radio 2, and so on at each node [16]. CCA ensures network connectivity, but does not fully utilize radio resources to minimize co-channel interference, resulting in the same degree of interference as what nodes would experience in a single-radio environment. As a more widely-adopted solution, every node reserves a radio that operates on a *default channel* [11, 1] in light of the common channel constraint. Other radios can be assigned to channels besides the default channel for maximized radio resource utilization.

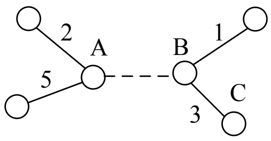

Fig. 1 Connectivity problem with link-centric schemes

Subramanian et al. [7] proposed a two-phase link-centric scheme where the first phase assigns channels to links to minimize interference but disregards the constraint of available radios. Therefore, link (A, B) in the above example can be assigned channel 4 (for example) to minimize potential interference among these links. The second phase processes all nodes that have inadequate radios, adopting a channel-merging procedure that forces adjacent links to reuse the same channel. This procedure may cause chain reactions to other nodes. In Fig. 1, the procedure may reassign channel 2 or 5 to link (A, B) so that A can operate with only two radios. However, one of B's links then must undergo another channel-merging procedure, which may cause further channel reassignment to one of B's other neighbors, say, node C if any.

One way to let several adjacent links share a common radio without inter-link interference is to have the radio operated on multiple channels in a time-multiplexing manner. This approach, called dynamic assignment, demands that when transmitting a packet, both the sending and the receiving radios tune to the same channel at the same time. This can be done through a prearranged schedule or by an on-line coordination, at expense of additional delay for switching channels. When channels taken by all nodes in a multi-hop routing path diverge, the resulting delay may significantly increase end-to-end message delay.

In hybrid assignments, radios are partitioned into two sets: one using a fixed assignment and the other a dynamic assignment. The fixed assignment can tune a dedicated radio to a default channel to ensure basic connectivity between neighboring nodes [10]. Kyasanur and Vaidya [17] proposed that each node uses a dedicated radio that tunes to a particular channel to receive data. Other radios are to send data only and can dynamically change channels. When sending a packet, the sender must use one of these radios and tune to the receiver's receiving channel.

Interference between radios or links can be captured by the protocol model or the physical model [6]. In the protocol model, transmission from nodes u to v is successful only if v is not within the interference range of some node that is also transmitting at the same time. The physical distance between the receiver and the possible interferer is not the only determinant for the interference relation. Some research took the hop count of the shortest path connecting two nodes [1]. In the physical model, the degree of interference experienced by a receiver is a real value collectively determined by all radios in the network that operate on the same channel at the same time. For a receiver

v, the intensity of interference caused by one of these radios w depends on the physical distance between v and w, nothing to do with interference range.

Because the common-channel constraint and the interference constraint can be conflicting, it is not always possible to make every designated link operative. The connectivity constraint is a weaker requirement, demanding that the whole network remain connected in the presence of some inoperative links [10, 11, 1]. The scheme by Rajakumar et al. [8] assumes the physical model and assigns a channel to a link provided that the resulting interference is below a threshold. After channel assignment, all designated links that are not yet assigned channels are to be replaced by free-space optical links. Rajakumar et al. used a genetic algorithm to minimize the number of required optical links, though not handling the case where channels assigned to a node outnumber the node's radio interfaces.

Heuristics-based schemes differ largely in terms of goals to achieve. Current proposals aim at a context of minimizing local interference of individual nodes [1], minimizing overall network interference [7, 15], minimizing the maximal link interference [12], maximizing the number of operative links [8, 5], and maximizing network throughput [13, 11]. Among others, this study aims at maximizing the number of operative links with game-theoretic underpinnings.

2.2 Game-Theoretic Approaches

Table 1 summarizes recent game-theoretic research on radio resource allocation, in following lines. Song et al. [18] tackled the problem of adjusting both operating frequency (i.e., channel number) and transmission power of each access point in order to maximize the signal-to-interference-plus-noise ratio (SINR) experienced at each receiver. The problem was modeled as a cooperative game and a non-cooperative game, respectively. However, Song et al. assumed a single radio at each access point and did not consider the connectivity requirement. Yuan et al. [19] also assumed a single radio at each node and proposed a means to maximize the total capacity of wireless access networks. Channel assignments in both studies were intended for access links rather than for backhaul links.

Duarte et al. [2] assumed overlapping channels under the protocol interference model, and formulated the channel allocation problem as a cooperative game where players have common interest. The common-interest assumption naturally leads to the existence of Nash equilibria, in which two ways to reach a Nash equilibrium were devised. Network connectivity was considered in the design of the game utility function.

Gao and Wang [20] considered a disjoint set of communication sessions where each sender and relay node equipped with multiple radios allocates one transmission channel for each of its radios to maximize the data rate of its involved sessions. The problem was approached by a cooperative game in multi-hop networks. The authors treated links individually but did not address network connectivity.

Name	Radio	Channel type	Game type	Connectivity	Collision domain	Interference model
CTMG [18]	Single	Non-overlapping	Cooperative	Not addressed	Single	Physical
NTMG [18]	Single	Non-overlapping	Non-cooperative	Not addressed	Single	Physical
CMG [19]	Single	Non-overlapping	Cooperative	Not addressed	Multiple	Protocol
CoCAG [2]	Multiple	Overlapping	Cooperative	Considered in the game	Multiple	Protocol
Gao & Wang [20]	Multiple	Non-overlapping	Cooperative	Not a problem because channels are allocated to links; network-wide connectivity is not considered	Single	
STG [21]	Multiple	Non-overlapping	Non-cooperative	Not addressed	Multiple	Protocol
Vallam et al. [22]	Multiple	Non-overlapping	Non-cooperative	Same as Gao & Wang [20]	Multiple	Protocol
ChAlloc [23]	Multiple	Non-overlapping	Non-cooperative	Same as Gao & Wang [20]	Multiple	Protocol
Perfectly-fair [24]	Multiple	Non-overlapping	Non-cooperative	Same as Gao & Wang [20]	Single	
MCAG [25]	Multiple	Non-overlapping	Non-cooperative	Not addressed	Single	Protocol
This work	Multiple	Non-overlapping	Non-cooperative	Ensured by limiting the number of assignable channels	Single	Physical

In a joint routing and channel assignment game proposed by Xiao et al. [21], channels were allocated to given source-sink node pairs (i.e. routing paths) on condition that co-channel interference is not allowed between routing paths. Interference among nodes within a path, however, degrades throughput. Neither did that study consider network connectivity.

Chen and Zhong [24] treated the whole wireless network as a single collision domain. They assumed that all radios operating on the same channel evenly share the bandwidth of a single channel, and modeled channel assignment as a non-cooperative game. As game players, nodes seek to maximize the amount of obtainable bandwidth. A special solution was derived that was proven to be a Nash equilibrium yet perfectly fair. Similar to [20], Chen and Zhong's scheme and other avenues [22,23] operated by allocating channels to link pairs, without regard to network-wide connectivity.

Assuming channels with unequal bandwidth and a common channel among all the nodes, Chen et al. [25] formulated channel allocation as a non-cooperative game where nodes acting as players have different satisfaction levels on the achieved bandwidth. They showed the existence of Nash equilibria and attempted to find one that maximizes the sum of all player's utilities. Neither did that study consider network connectivity.

To recap previous work, some considered only single radio [18,19] or assumed overlapping channels [2]. Some were based on cooperative game model [18,20,19,2]. These previous schemes distinguish themselves from our study in either problem settings or game modeling. Most importantly, almost all previous schemes did not consider network connectivity. The most comparable is Duarte et al's scheme that, under the assumption of overlapping channels, considered network connectivity in the utility function [2] and modeled the problem as a cooperative game.

3 The Proposed Approach

Our resource allocation is approached by two non-cooperative games running in stages. The first game assigns channels to radios. The second game then assigns the resulting radio-channel pairs to links.

3.1 Stage One: Allocating Channels to Radios

We assume that n nodes numbered from 1 to n are deployed in a backhaul network. Let r_i be the number of radio interfaces available to node i. The first-stage game models radios as players, so there are total of $m = \sum_i r_i$ players in the game. Let $P = \{p_1, p_2, \ldots, p_m\}$ be the set of players. We use $p_i \bowtie p_j$ to denote the relation that p_i and p_j are two radios located in the same node.

Suppose that all non-overlapping channels are numbered from 1 to k. To guarantee the common channel constraint, the proposed game follows the rule presented in [5] to limit the set of channels that can be assigned to each node.

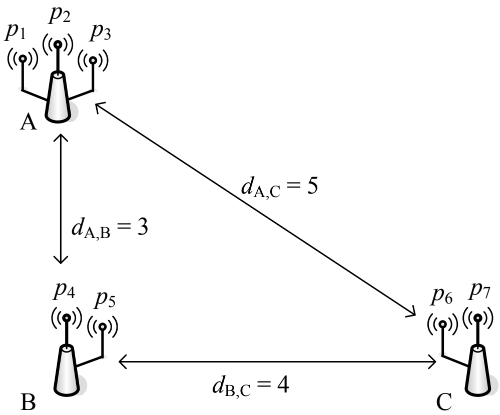

Fig. 2 A network with three nodes.

More specifically, letting N_i be the set of node i's neighboring nodes, the set of channels that can be allocated to radios of node i is $\{1, 2, \ldots, u\}$, where

$$u = \min(k, \min_{j \in N_i}\{r_i + r_j - 1\}). \tag{1}$$

The common channel constraint is ensured for every link by the Pigeonhole Principle. If all nodes have equally r radios, (1) reduces to $u = \min(k, 2r - 1)$.

Let S_i denote player p_i's *strategy set*, the set of all channels available to p_i subject to (1). A *strategy profile* is an m-tuple $C = (c_1, c_2, \ldots, c_m)$, where $c_i \in S_i$ represents player p_i's choice. We may sometimes express C as (c_i, C_{-i}). Given a strategy profile C, we define the utility of p_i associated with C as

$$u_i(C) = u_i(c_i, C_{-i}) = -\sum_{j \neq i} f(c_i, c_j), \tag{2}$$

where $f(c_i, c_j)$ is a function that returns the cost of choosing strategy c_i (by player p_i) with respect to strategy c_j (by another player $p_j \neq p_i$). The definition of $f(c_i, c_j)$ is as follows.

$$f(c_i, c_j) = \begin{cases} 1/d_{i,j}^{\alpha} & \text{if } c_i = c_j \text{ and } p_i \not\bowtie p_j \\ \beta & \text{if } c_i = c_j \text{ and } p_i \bowtie p_j \\ 0 & \text{if } c_i \neq c_j, \end{cases} \tag{3}$$

where $d_{i,j}$ is the physical distance between p_i and p_j; α and β are constants. The value of $f(c_i, c_j)$ reflects the degree of interference experienced by p_i or p_j when p_i chooses channel c_i while p_j chooses channel c_j. If $c_i = c_j$ and p_i and p_j belong to different nodes, the degree of interference is proportional to $1/d_{i,j}^{\alpha}$, where α ranging from 2 to 4 stands for the path loss exponent. If $c_i = c_j$ and p_i and p_j are located in the same node, $f(c_i, c_j)$ returns a large cost $\beta \gg 1/d^{\alpha}$ to represent severe self-interference [2] that occurs between any two radios within a node. If $c_i \neq c_j$, then there is no cost at all.

Our definition of utility function takes physical interference into account, where distance between radios matters. Consider a simple backhaul network with three nodes shown in Fig. 2. Suppose that p_4 and p_6 already select c_4 and c_6, respectively, and no any other node selects either channel. If p_1 has to make a selection between c_4 and c_6, it will choose c_6 because it is farther from p_6 than p_4. In many game-theoretic approaches that adopt the protocol interference model, the payoff of selecting one channel is set to the data rate provided by that channel under the assumption that channel capacity is equally shared among all interfering radios [22,2,25]. With this setting, there is no difference between c_4 and c_6 for p_1.

Our utility definition also considers interference intensity. Suppose that, in addition to p_6, in Fig. 2 p_7 also selects c_6[1]. Player p_1 still selects c_6 since the aggregated cost of selecting c_6 is still lower than that of selecting c_4. In contrast, p_1 would rather choose c_4 in many other approaches [22,2,25] because it is the number of interfering players rather than the intensity of interference that counts in these approaches.

The proposed channel allocation game can be represented as $\Gamma = [P; \{S_i\}_{i=1}^m; \{u_i\}_{i=1}^m]$. This is a non-cooperative game, meaning that players do not cooperate with each other to seek system's benefit. In fact, all players are selfish. This game is also a dynamic game, as players take turns to make their decisions, knowing what decisions have already been made. Players are also myopic, meaning that a player will change its strategy whenever that change increases its utility. Formally, we can define two types of response function for players. The *better response* function for player p_i is

$$r_i(c_i, C_{-i}) = \{c_j \in S_i | u_i(c_j, C_{-i}) > u_i(c_i, C_{-i})\}, \tag{4}$$

which represents a subset of S_i that can yield a higher utility value than p_i's current strategy c_i provided that all other player's strategies remain unchanged. The *best response function* is defined as

$$\begin{aligned} b_i(c_i, C_{-i}) = &\{c_j \in r_i(c_i, C_{-i}) | \forall c'_j \in r_i(c_i, C_{-i}) : \\ &u_i(c_j, C_{-i}) > u_i(c'_j, C_{-i})\}. \end{aligned} \tag{5}$$

A centralized stochastic procedure that mimics the proposed game takes the following steps to work.

1. Assign every player a strategy uniformly or randomly determined.
2. For each p_i, compute $\Sigma_i = r_i(c_i, C_{-i})$ (in case of better-response) or $\Sigma_i = b_i(c_i, C_{-i})$ (in case of best-response).
3. If $\Sigma_i = \emptyset$ for all p_i, then the procedure stops. Otherwise, pick some p_i such that $\Sigma_i \neq \emptyset$ and change c_i to a strategy randomly chosen from Σ_i.
4. Go to the second step.

Suppose that more than four channels are available to nodes in Fig. 2. According to (1), the highest channel numbers that can be allocated to radios

[1] This could happen only during a game play; it cannot be the final result of the game.

Table 2 A possible game evolving sequence

Step	Before	After
1	(1,2,3,1,2,1,$\underline{2}$)	(1,2,3,1,2,1,$\underline{3}$)
2	($\underline{1}$,2,3,1,2,1,3)	($\underline{4}$,2,3,1,2,1,3)

of nodes A, B, C are 4, 3, and 3, respectively. Table 2 shows a possible game evolving sequence (i.e., transitions of strategy profiles) if CCA is initially used to allocate channels to radios. When the game ends up with strategy profile $(4, 2, 3, 1, 2, 1, 3)$, no player has the incentive to further change its strategy. That is, the game enters a Nash equilibrium.

In what follows we shall prove the stability of the game. First of all, the utility of player p_i after it changes strategy to c_i' is

$$u_i(c_i', C_{-i}) = -\sum_{j \neq i} f(c_i', c_j). \tag{6}$$

For other players $p_j \neq p_i$, its utility after p_i changes strategy to c_i' is as follows.

Lemma 1 *Let $C = (c_i, C_{-i})$. For each player $p_j \neq p_i$, its utility if p_i changes strategy from c_i to c_i' is*

$$u_j(c_i', C_{-i}) = u_j(C) + f(c_j, c_i) - f(c_j, c_i'). \tag{7}$$

Proof Before p_i changes strategy, the utility of $p_j \neq p_i$ is

$$u_j(c_i, C_{-i}) = -\sum_{k \neq i,j} f(c_j, c_k) - f(c_j, c_i). \tag{8}$$

After p_i changes strategy, the utility of $p_j \neq p_i$ becomes

$$u_j(c_i', C_{-i}) = -\sum_{k \neq i,j} f(c_j, c_k) - f(c_j, c_i'). \tag{9}$$

Subtracting (8) from (9) yields the change of p_j's utility due to p_i's change of strategy:

$$u_j(c_i', C_{-i}) - u_j(c_i, C_{-i}) = -f(c_j, c_i') + f(c_j, c_i). \tag{10}$$

Therefore,

$$u_j(c_i', C_{-i}) = u_j(c_i, C_{-i}) + f(c_j, c_i) - f(c_j, c_i'). \tag{11}$$

□

Let $U = \sum_j u_j(c_i, C_{-i})$ and $U' = \sum_j u_j(c_i', C_{-i})$ be the sums of all player's utilities before and after p_i changes strategy from c_i to c_i', respectively. We can prove the stability of this game by showing that $U' > U$. That is, every time a player changes its strategy, the sum of all player's utility is increased. Since we cannot unlimitedly increase the sum, the game eventually ends up with a solution in which no player can further increase its utility unilaterally (i.e., a Nash equilibrium).

Theorem 1 *The proposed channel allocation game will end up with a Nash equilibrium regardless of its initial configuration.*

Proof We can express U' as $U' = u_i(c'_i, C_{-i}) + \sum_{j \neq i} u_j(c'_i, C_{-i})$. By Lemma 1, we have

$$\begin{aligned} U' &= u_i(c'_i, C_{-i}) + \sum_{j \neq i} [u_j(C) + f(c_j, c_i) - f(c_j, c'_i)] \\ &= u_i(c'_i, C_{-i}) + \sum_{j \neq i} u_j(C) + \sum_{j \neq i} f(c_j, c_i) - \sum_{j \neq i} f(c_j, c'_i) \\ &= u_i(c'_i, C_{-i}) + U - u_i(c_i, C_{-i}) - u_i(c_i, C_{-i}) + u_i(c'_i, C_{-i}) \\ &= U + 2(u_i(c'_i, C_{-i}) - u_i(c_i, C_{-i})) \end{aligned} \tag{12}$$

Since p_i changes strategy from c_i to c'_i only if $u_i(c'_i, C_{-i}) > u_i(c_i, C_{-i})$, (12) implies that $U' > U$. Because the total utility cannot be increased unlimitedly, it is ensured that the game eventually reaches a Nash equilibrium. □

Theorem 1 in fact shows that U is a potential function, which makes the proposed game a potential game [26]. Potential games possess the *finite improvement property*, which means that a sufficiently long sequence of better responses can lead the game into a Nash equilibrium regardless of the initial configuration of the game.

3.2 Stage Two: Assigning Radio-Channel Pairs to Links

After the first-stage game allocates one channel to every radio, the second stage assigns the resulting radio-channel pairs to links. The design of the first-stage game ensures that each link has at least one radio-channel to select. However, there may be several candidate radio-channel pairs for a link, each having a different level of interference with other links because several links may need to share one radio-channel pair. Therefore, although connectivity is not affected in the second stage, assignments in the second stage do affect the resulting interference. Note that because radios are all identical, it is only necessary to determine channels for links in this assignment task.

Consider the scenario shown in Fig. 3, where channels have been allocated to radios. Since nodes A and C are allocated two common channels (channels 3 and 5), either channel can be assigned to link (A, C). However, channel 3 seems to have lower interference than channel 5 as fewer radios are allocated channel 3. On the other hand, there are four links incident on node C, but C has only three radio-channel pairs. Therefore, at least one channel must be shared between two links.

In our preliminary study [27], a greedy approach was developed for the allocation of radio-channel pairs to links. The greedy approach takes the following steps.

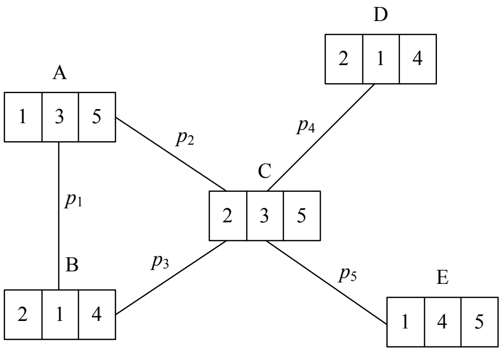

Fig. 3 A scenario illustrating a channel allocation result.

1. For all links that have only one candidate channel, assign the only channel to the link. Taking Fig. 4 as an example, this step will assign channel 1 to link (A, B).
2. After all links with k candidate channels ($k \geq 1$) have been assigned channels, process all links that have $k + 1$ candidate channels in an arbitrary order. For each link, assign it a candidate channel that conflicts with the least number of its neighboring links, and arbitrarily break ties. For example, link (A, C) in Fig. 3 has two candidate channels (channels 3 and 5) while all other links have only one. When all other links have been assigned channels, this step assigns channel 3 to link (A, C) because channel 3 is least assigned to neighboring links than channel 5.
3. Stop the procedure when all links have been assigned channels.

The above procedure has been proved efficient [27]. However, it is a centralized approach which demands global information and does not scale well. As a decentralized approach, this paper formulates the problem of assigning radio-channel pairs to links as another non-cooperative game. Now players in the game are links, while a player's strategy set is the collection of all radio-channel pairs that meet the common channel constraint. Detailed game formulation follows.

1. Assume that total m links are sorted in some order (for example, the lexicographic order) as $\{(u_1, v_1), (u_2, v_2), \ldots, (u_m, v_m)\}$. The player set is $P = \{p_1, p_2, \ldots, p_m\}$, where $p_i = (u_i, v_i)$ for all i, $1 \leq i \leq m$. For example, five players (links) can be defined in Fig. 3. So $P = \{(A, B), (A, C), (B, C), (C, D), (C, E)\}$ in case of the lexicographic order.
2. The strategy set of player p_i is $S_i = K_{u_i} \cap K_{v_i}$, where K_x is the set of channels that have been allocated to radios of node x in the first stage. In Fig. 3, all players but p_2 have only one strategy. Here $S_2 = \{3, 5\}$.
3. The utility function of p_i is defined as the negative of the number of neighboring players that choose the same strategy as p_i. Here two players (links) are neighbors if one end node of any link is adjacent to any end node of the other link. Let N_x be the set of nodes adjacent to node x. The neigh-

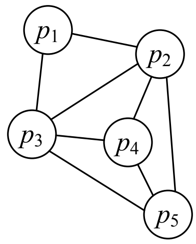

Fig. 4 Neighbor graph for links in Fig. 3

borhood relationship between players can be captured by an undirected *neighbor graph* $G = (P, E)$, where $(p_i, p_j) \in E$ iff $u_j \in N_{u_i} \vee u_j \in N_{v_i} \vee v_j \in N_{u_i} \vee v_j \in N_{v_i}$. Fig. 4 shows the neighbor graph for links in Fig. 3. Define $N(p_i) = \{p_j | (p_i, p_j) \in E\}$. Given a strategy profile $C = (c_1, c_2, \ldots, c_m)$, where $c_i \in S_i$ for all i, the utility function of p_i is formally defined as

$$u_i(c_i, C_{-i}) = -|\{p_j | p_j \in N(p_i) \wedge c_i = c_j\}|. \tag{13}$$

For example, $u_2(C)$ will be 0 given $C = (1, 3, 2, 2, 5)$. On the other hand, if $c_2 = 5$ instead of 3, $u_2(C)$ will be -1 since one of its neighboring players, i.e, p_5, also chooses channel 5.

In this game, player's payoff (utility) received from a strategy depends on the congestion level of that strategy (i.e., the total number of players that choose the strategy). Therefore, this game belongs to congestion games [28]. Furthermore, the utility of any player only hinges on the choices of its neighboring players. This makes this game a graphical game [29]. These two properties imply that the proposed game is a graphical congestion game [30]. More precisely, this game is a graphical linear congestion game since the utility received by a player from a strategy is a linear function of the number of the player's neighbors that choose the same strategy. Bilò et al. [30] prove that every graphical linear congestion game defined over an undirected social graph is an exact potential game [26], which is a special class of potential games that also possesses the finite improvement property. Therefore, either better response or best response can guarantee the stability of this game.

4 Simulation Results

Simulations were carried out to investigate how many operative links can be yielded by a given approach. We measured the *operational link ratio* (OLR)—ratio of operative links to total designated links. For each transmitter-receiver pair of a certain distance apart, we used the log-distance path loss model [31] to calculate the received signal strength (RSS). Only path loss was considered in signal strength measurements. Shadowing effect and fading were left out because these factors are environment-dependent, time-varying, and difficult to be incorporated into the game model. Table 3 lists all related parameters.

Table 3 Simulation Parameters

Parameter	Value
Path loss model	Log-distance
Transmit power	15 dBm
Reference distance	1 m
Path loss at reference point	35 dB
Path loss exponent	3.0
Background noise	−95 dBm [32]

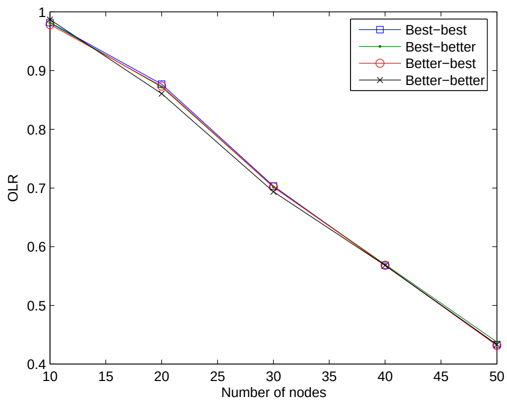

Fig. 5 OLRs of different game behaviors versus the number of nodes with $r = 2$. (r_t = 125 m)

After channel assignment is done, RSS information together with background noise settings is then used to assess the SINR of each link. A link was considered operative only if its SINR was greater than 1 dB. The simulations involved 100 scenarios that served as test cases, from which the average was taken as the result. In each scenario, a number of nodes were randomly placed in a 1000×1000 m^2 area. We varied the total number n of nodes, the number r of radios per node, and the transmission range r_t.

4.1 Game Behavior

We first examine how best response and better response affect the performance of our proposed approach. Since our approach is composed of two games, we have four possible combinations. Figs. 5 to 7 show how OLR changes with the number of deployed nodes in three settings ($r = 2$, $r = 4$, and $r = 6$). Figs. 8 to 10 show how OLR changes with the number of radios per node in three cases of transmission range ($r_t = 125$ m, $r_t = 250$ m, and $r_t = 500$ m). Observe that the game behavior (best response or better response) in either stage does not vary OLR significantly.

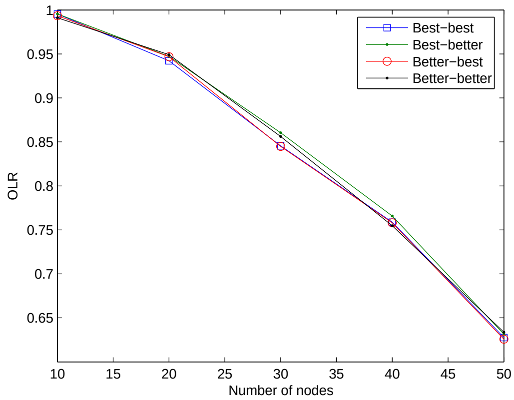

Fig. 6 OLRs of different game behaviors versus the number of nodes with $r = 4$. (r_t = 125 m)

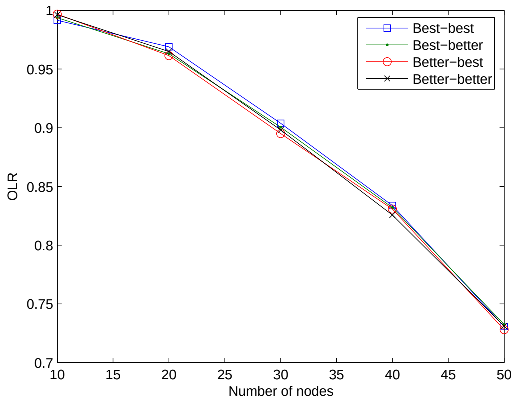

Fig. 7 OLRs of different game behaviors versus the number of nodes with $r = 6$. (r_t = 125 m)

Although game behavior did not affect OLR values, it might affect game convergence time. We therefore measured the total number of strategy transitions divided by the number of radios (which is exactly the number of players in the first-stage game) in every setting. Figs. 11 to 13 show results from various r and n. When $r = 2$, different game behaviors make no much difference. When $r = 4$ or $r = 6$, it is the behavior of the first-stage game that determines the result, and best response has shorter convergence time than better response. The superiority of best response over better response in terms of game convergence time is still observed when we varied r_t (Figs. 14 to 16). In those experiments, the average number of strategy transitions per radio generally increases with the number of radios. This increasing trend declines at $r = 7$ because we have only 12 assignable channels. In summary, concerning game

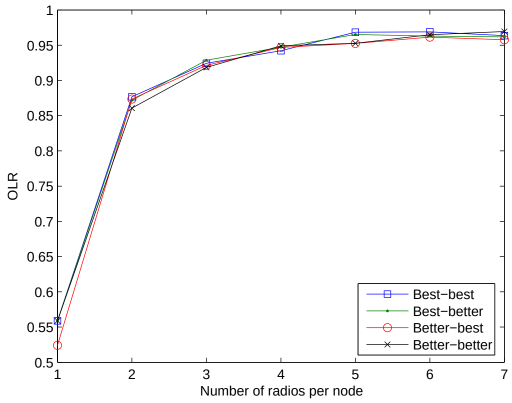

Fig. 8 OLRs of different game behaviors versus the number of radios with transmission range set to 125 m. ($n = 20$)

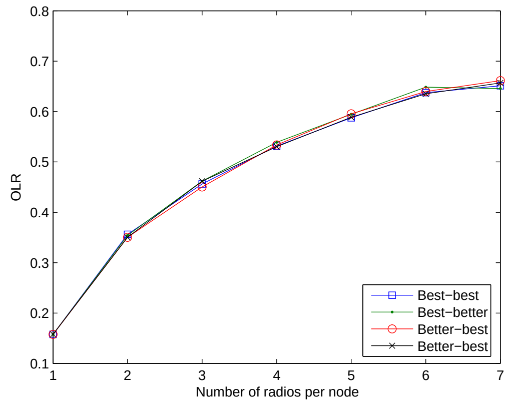

Fig. 9 OLRs of different game behaviors versus the number of radios with transmission range set to 250 m. ($n = 20$)

convergence time, the behavior of the first-stage game does matter. The behavior of the second-stage game seems irrelevant to game convergence time. For this reason, we pick best-best (i.e., games in both stages take best response) as our representative in the following comparisons with other approaches.

4.2 Comparisons with Other Approaches

Let us proceed to compare the performance of the proposed approach with that of several well-known methods. Under discussion are recent methods, including a heuristic method termed *link-preserving* [5], a cooperative channel assignment game (CoCAG) [2], and a non-cooperative game termed *perfectly-*

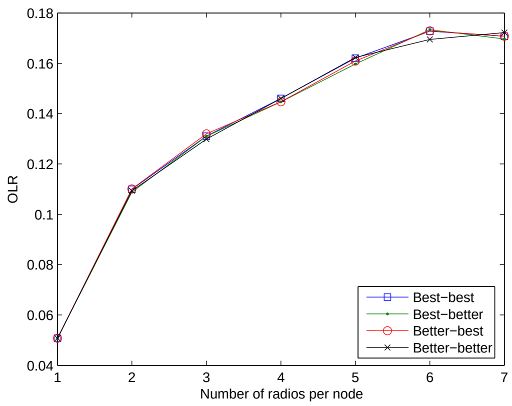

Fig. 10 OLRs of different game behaviors versus the number of radios with transmission range set to 500 m. ($n = 20$)

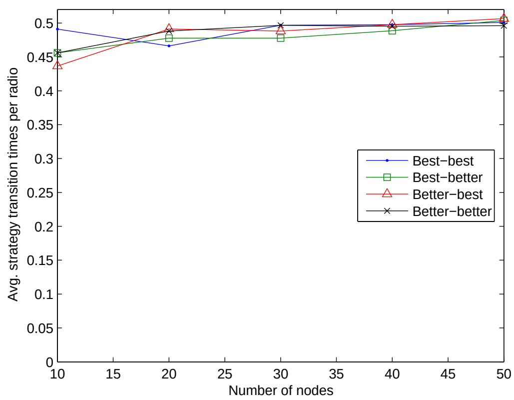

Fig. 11 Average number of strategy transition times per radio before Nash equilibrium with $r = 2$.

fair [24]. Our prior work characterizing a game-theoretic scheme in the first stage and a greedy approach in the second stage [27] was tested as well. That work is referred to as *best-greedy* here in that the first-stage game therein takes best-response strategies.

The first set of simulations assumed a fixed number of radios in each node to reflect how OLR changes with the number of deployed nodes. Figs. 17 to 19 show results from $r = 2$, $r = 4$, and $r = 6$, respectively. In all cases, the introduction of more nodes increases interference and thus decreases the resulting OLR values. On the other hand, more radios decrease interference and thus lead to increased OLR values. When each node has only two radios (Fig. 17), the proposed approach exhibits similar performance to best-greedy and outperforms all others. However, when each node has four (Fig. 18) or

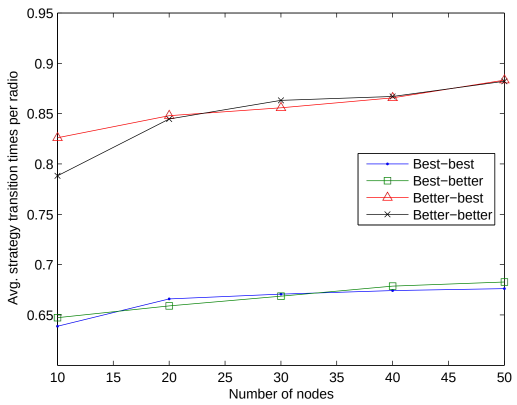

Fig. 12 Average number of strategy transition times per radio before Nash equilibrium with $r = 4$.

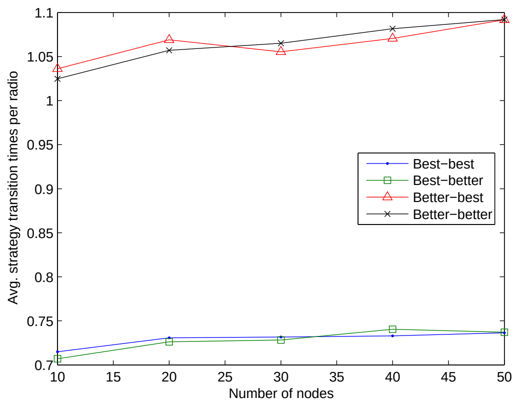

Fig. 13 Average number of strategy transition times per radio before Nash equilibrium with $r = 6$.

six (Fig. 19) radios, link-preserving performs the best, followed by best-greedy and then the proposed approach. This is justifiable, as link-preserving uses a centralized greedy algorithm, best-greedy uses a centralized greedy algorithm in the second stage, whereas our approach is fully distributed.

The next set of simulations were run under a fixed r_t to determine the relationship between r and OLR. When $r_t = 125$ m (Fig. 20), link density was low and OLR could grow higher than 0.9 as long as adequate radios were provided. When we increased r_t to 250 m and thus created more designated links, no method achieved an OLR higher than 0.9 (Fig. 21). The upper bound dropped to 0.24 when we further increased r_t to 500 m (Fig. 22). The proposed approach and best-greedy had comparable performance when only two or three

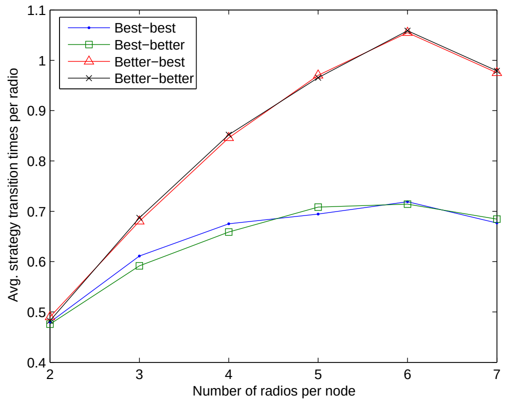

Fig. 14 Average number of strategy transition times per radio before Nash equilibrium with transmission range set to 125 m.

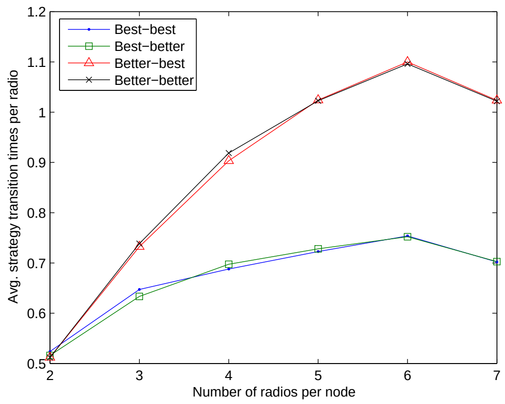

Fig. 15 Average number of strategy transition times per radio before Nash equilibrium with transmission range set to 250 m.

radios were available at each node, and comprise the leading group. When more radios were available, the link-preserving method performed the best.

Foregoing simulation results indicate that the proposed game-theoretic approach outperforms counterpart schemes in the number of operative links when only two radios are available at each node. When more radios are available, centralized, greedy approaches perform better. A compromised performance is observed in a hybrid approach where the second-stage game is replaced with a greedy approach.

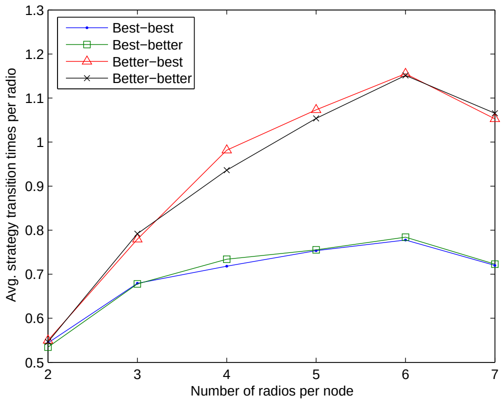

Fig. 16 Average number of strategy transition times per radio before Nash equilibrium with transmission range set to 500 m.

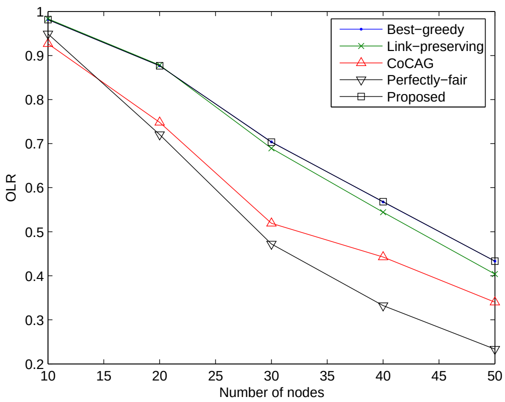

Fig. 17 OLR versus the number of nodes with $r = 2$.

5 Conclusions

In closing, this paper proposes a two-stage radio resource allocation scheme for multi-channel, multi-radio wireless backhaul networks. The first stage, modeled as a non-cooperative game, assigns channels to radios. The second stage, modeled as another non-cooperative game, distributes the resulting radio-channel pairs to links. This scheme guarantees the common channel constraint and is proved stable as a whole, i.e., games always lead to Nash equilibria regardless of initial configurations. We have conducted simulations to demonstrate the effectiveness of best- and better-response behaviors and compare the performance of subject schemes. Simulation results show that game behavior does not affect the number of operative links, but game convergence

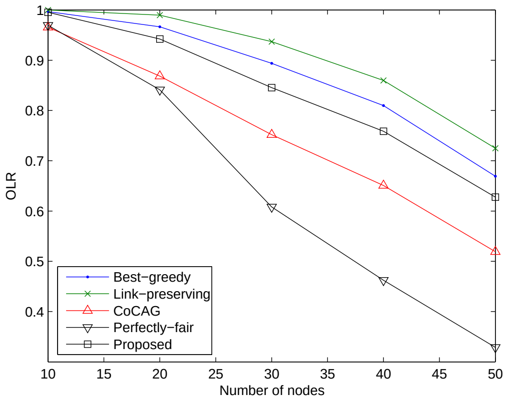

Fig. 18 OLR versus the number of nodes with $r = 4$.

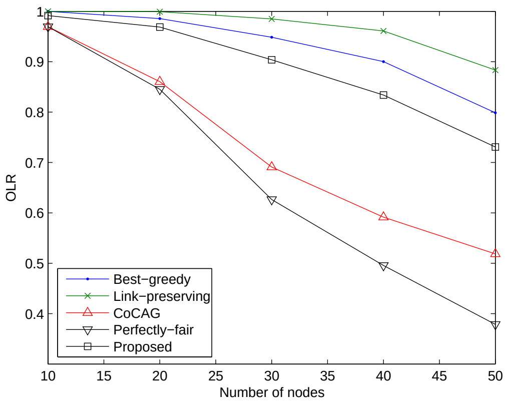

Fig. 19 OLR versus the number of nodes with $r = 6$.

time depends on the behavior of the first-stage game. Performance results also indicate that our approach generally yields more operative links than counterparts when only two radios are available at each node. However, when more radios are available, centralized, greedy approaches perform better.

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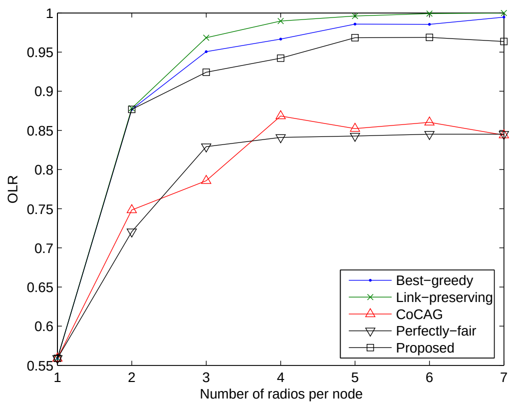

Fig. 20 OLR versus the number of radios with transmission range set to 125 m.

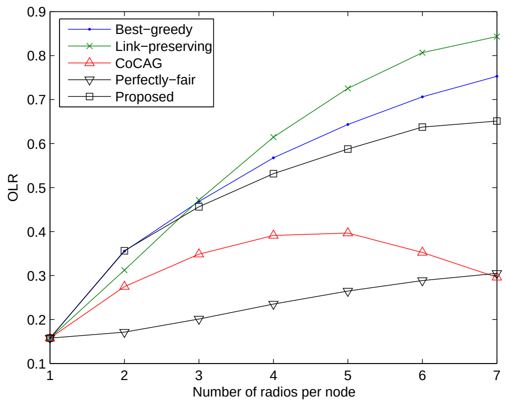

Fig. 21 OLR versus the number of radios with transmission range set to 250 m.

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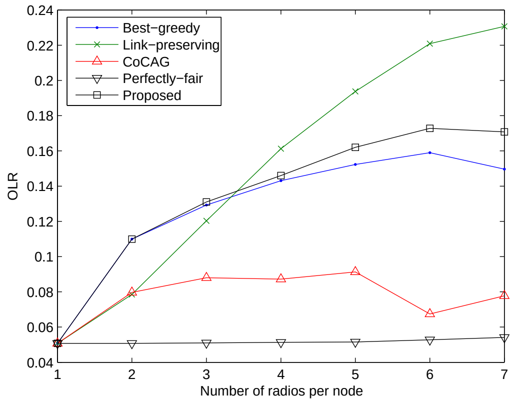

Fig. 22 OLR versus the number of radios with transmission range set to 500 m.

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