Stability and Fairness of Native AP Selection Games in IEEE 802.11 Access Networks

Li-Hsing Yen, Jia-Jun Li, and Che-Ming Lin
Department of Computer Science and Information Engineering
National University of Kaohsiung, Kaohsiung, Taiwan 811, R.O.C.

***Abstract*—This paper models access point (AP) selections by IEEE 802.11 wireless stations (WSs) as a native game, where each WS's goal is to maximize its achievable throughput. We have proven the stability of this game (Nash equilibrium), and shown that selfish behavior of individual WS in fact improves overall bandwidth fairness among WSs. Thorough simulations were conducted to demonstrate the quality of the analytical results.**

I. Introduction

IEEE 802.11 wireless local area networks have been widely deployed as wireless infrastructures providing data access services in home, corporate, and public environments. In such environments, a wireless station (WS) with an IEEE 802.11 interface sends and receives frames via an access point (AP) to network infrastructure, and all APs in service constitute an access network. However, traffic load in an access network may not be fairly shared by all serving APs due to the uncoordinated nature of AP selections among WSs. More specifically, WSs typically select and associate with an AP with the highest received signal strength. This problem motivates many load-balancing schemes for IEEE 802.11 networks [1] with a design goal to make WS-AP associations load-aware, preventing WSs from making associations with congested APs. The ultimate goal is to either increase overall system throughput or maintain bandwidth fairness among WSs.

In this paper, we analyze the problem of AP selections under the framework of game theory. Game theory provides a mathematical modeling for the study of competition strategies in a game where players have conflicting benefits or goals. For the last decade, game theory has been used to analyze duty/resource sharing problems in wireless networks. In these games, selfish players usually bring in undesired results (uneven load distribution or unfair resource share), and researchers have to introduce incentive or punitive mechanisms to force cooperations among players. For example, a commonly-adopted mechanism is to design a synthetic utility function for players that penalizes selfish behaviors. The goal is to let games naturally fall into stable states called Nash equilibria where system's interest could potentially benefit.

Our framework differs from previous ones in that we consider *native* AP selections. That is, WSs select and re-select APs merely for their own interest (specifically, achievable throughput that a WS may receive from a selection). No other external incentive/punitive mechanisms are introduced to ensure stability or fairness. We shall prove that a Nash equilibrium exists even in this context, which eliminates the possibility of unstable association transitions (change of AP selections). Furthermore, we show that selfish yet rational behaviors under the proposed framework naturally improve bandwidth fairness, which was not expected previously.

II. Background and Related Work

WSs in an IEEE 802.11 access network compete for bandwidth offered by APs. Clearly, a WS's utility depends on not only its own association choice, but also other WS's. This is why game theory becomes a useful tool to apply here. Intuitively, WSs should select an AP that is the least crowded to maximize its achievable throughput. Games with player's objective defined to minimize the number of other users that share the same selection are known to be *crowding games* [2]. In the literature, crowding games has been used to model network selections by mobile users [3], [4]. However, this framework does not well apply to IEEE 802.11 networks as achievable throughput of WSs in an AP is not necessarily a homogeneously-decreasing function of WS population there. The irregularity comes from two design features of IEEE 802.11. One is its non-deterministic MAC (Medium Access Control) scheme, which does not guarantee any bandwidth share to participants. The other is the provision of multiple link rates in IEEE 802.11 a/b/g networks, which may give rise to an undesirable phenomenon called *performance anomaly* [5]. Performance anomaly refers to the effect that when links operating at different rates coexist within an AP, throughputs of high-rate links will all degrade to the level of the lowest-rate link. Performance anomaly not only impairs achievable throughputs of WSs, but also makes AP's actual capacity a variable. Consider the example of Fig. 1, where two IEEE 802.11b APs are serving four WSs. WS3 there could choose either AP1 or AP2 to associate with. Achievable throughputs of these two choices, based on the analysis of [5], are shown in Table I. We can see that selecting AP1 yields a better result, though AP1 is more crowded than AP2. AP1 is also a better choice from the perspective of system's benefit, as selecting it has a higher total achievable throughput than selecting the counterpart. Perception of performance anomaly can yield better performance result. But this cannot be characterized in crowding games.

The AP selection problem under consideration is modeled as a *noncooperative dynamic* game. In a noncooperative game, players do not cooperate with each other to seek system's

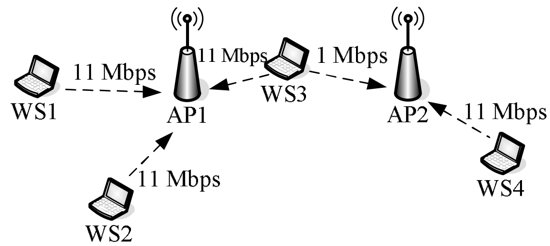

Fig. 1. A scenario illustrating performance anomaly

TABLE I
ACHIEVABLE THRIUGHPUT IN THE SCENARIO OF FIG. 1

WS3's choice	Achievable throughput (Mbps)				Total
	WS1	WS2	WS3	WS4	(Mbps)
AP1	2.19	2.19	2.19	8.01	14.58
AP2	3.62	3.62	0.81	0.81	8.86

benefit. A noncooperative game is dynamic if players take turns to make their decisions, knowing what decisions have already been done. In our model, an associated WS will reassociate with another AP if that reassociation improves its achievable throughput. The achievable throughput in recognition of the effect of performance anomaly can be computed with analytical results from prior work in [5], [6]. Here we assume WSs pursuing its own throughput improvement rather than the balance of workloads among APs (e.g., [7]). Although a lightly-loaded AP in principle offers a high achievable throughput and selecting an AP with the least load helps load balancing among APs, we argue that, from WS's perspective, AP selections based on achievable throughput are direct and more "natural" than AP selections based on load balancing. Several other approaches also proposed AP selections based on achievable throughput (potential bandwidth) [8], [9], [10]. Another issue of load-based AP selections comes from the fact that the notion of AP's load is not well defined in IEEE 802.11 networks. It could be the number of WSs associated with an AP, frame drop rate of AP's transmission queue during real-time sessions [11], or the total time that an AP takes to provide each WS one unit of traffic [12], [7].

Although there have been many approaches proposed for AP selections, only few of them treat the problem under the framework of game theory. Mittal *et al.* [13] introduced an AP selection game which differs from our setting in that WSs may need to travel some distance to reach an AP. The cost of an AP selection is measured by the AP's load and the traveling distance required by that selection. With this cost model, Mittal *et al.* proposed a simple greedy algorithm that brings the game to a Nash equilibrium under the condition of even WS distribution and absence of dynamic WS arrivals and departures. However, the ability to measure physical distance between WSs and APs, as required by this model, is not yet a primitive feature in today's wireless networks. Shakkottai *et al.* [14] studied the problem of a WS associating with multiple APs and splitting its traffic among these APs (*link-layer multihoming*). They used the model of *population game* [15], which implies that the impact of individual WS's selection on other WS's utilities is infinitesimal. Although link-layer multihoming is possible for WSs using a single wireless interface card [16], this technique is not yet mature and widely adopted. The population game model also dose not generally apply to IEEE 802.11 networks. Jiang *et al.* [17] considered base station (BS) selections by mobile users, where each user selfishly chooses a BS that gives her the highest achievable throughput. This work assumes that the throughput each user can receive is controlled by the BS, and that the number of users is enormous so as to apply the population game model. Both assumptions do not fit in IEEE 802.11 networks.

Besides throughput, *fairness* is also a typical criterion for AP selection problems. In the context of bandwidth sharing, *max-min* [18] is a commonly-adopted metric for fairness particularly when bandwidth requestors have different bandwidth demands. With an objective to maximize the minimum share of a requestor whose demand is not fully satisfied, basic principles of max-min fairness are to allocate bandwidth to requestors in increasing demands, to ensure no requestor receives bandwidth more than its demand, and to equally split the remaining bandwidth to requestors with unsatisfied demands. If we use a tuple to denote the set of allocated bandwidth of each requestor sorted in a nondecreasing order, then a bandwidth allocation is max-min fair when the corresponding tuple has the highest lexicographical value among all. A similar notion, *min-max* fairness, can be defined if the share of workloads among APs is concerned. Bejerano *et al.* [12] have studied AP selections that achieve min-max fairness of AP workloads. They proved that, unless link-layer multihoming is allowed, a min-max load balanced association does not imply a max-min fair bandwidth allocation and vice versa.

Max-min fairness well applies to cases where resource requestors have limited demands. In our problem setting, however, every WS has an unlimited bandwidth demand; it could actually consume all bandwidth available to it. For this kind of bandwidth sharing, *balance index* [19] can be used to quantify the fairness of bandwidth share among all competitors. For a bandwidth allocation consisting of n portions numbered 1 to n, let B_i, $1 \leq i \leq n$, denote the amount of bandwidth allocated to the ith portion. The balance index β is defined as

$$\beta = \frac{(\sum B_i)^2}{n \times \sum B_i^2}. \quad (1)$$

The value of β becomes 1 when all requestors get an equal share, and it approaches $1/n$ in case of extremely imbalanced allocations.

III. NATIVE AP SELECTION GAME

We consider an IEEE 802.11 network consisting of m APs and n WSs. We assume that each WS can access at least one AP. Let $A = \{a_1, a_2, \ldots, a_m\}$ and $W = \{w_1, w_2, \ldots, w_n\}$ be the sets of all APs and WSs, respectively. We denote the set of APs that w_i can access (i.e., the strategy set of w_i) by A_i, where $1 \leq i \leq n$. For a possible AP-WS association, the

WS's *utility* is defined to be achievable throughput of the WS resulted from that association.

We define a *configuration* (a strategy profile) to be an n-tuple $C = (c_1, c_2, \ldots, c_n)$, where $c_i \in A_i$ represents w_i's association choice. For a specific w_i, we may sometimes express C as $C = (c_i, C_{-i})$, where $C_{-i} = (c_1, c_2, \ldots, c_{i-1}, c_{i+1}, \ldots, c_n)$ denotes all other WS's associations other than w_i's. Function $u_i(C)$ gives w_i's utility with respect to configuration C. This function is realized by prior work in [5], [6]. The AP selection game is defined as $\Gamma = [W; A; \{u_i\}_{i=1}^{n}]$.

A. Stability

Definition 1: Nash Equilibrium: Given a game $\Gamma = [W; A; \{u_i\}_{i=1}^{n}]$, a configuration $C^* = (c_1^*, c_2^*, \ldots, c_n^*)$ is a Nash equilibrium if $\forall i \in \{1..n\} : \forall c_i \in A_i :: u_i(c_i^*, C_{-i}^*) \geq u_i(c_i, C_{-i}^*)$.

In other words, Nash equilibrium is a configuration where no WS can further increase its own utility by unilaterally changing its choice. Nash equilibrium is not necessarily a Pareto optimal strategy. A configuration $C = (c_1, c_2, \ldots, c_n)$ is Pareto optimal if and only if there exists no other configuration $C^{'} = (c_1^{'}, c_2^{'}, \ldots, c_n^{'})$ such that $\forall i \in \{1..n\} : u_i(C^{'}) \geq u_i(C)$ and $\exists j \in \{1..n\} : u_j(C^{'}) > u_j(C)$.

Recall that in our model, an associated WS can re-associate with another AP if that reassociation improves its achievable throughput. The reassociation action may trigger another WS's reassociation and so on. If Nash equilibria do not exist in this game, reassociation activities will last and the system cannot enter a stable state. We shall now show the existence of Nash equilibrium in the native AP selection game.

Let $\Sigma = \{C_1, C_2, \ldots, C_k\}$, where $k = |A_1| \times |A_2| \times \cdots \times |A_n|$, denote the configuration space, i.e., the set of all possible configurations. In our game model, a transition from one configuration to another naturally happens when some WS discovers that it may benefit from such transition and thereby conducts an association change. For simplicity, we assume that only one association change is conducted at a time; simultaneous transitions are serialized in some arbitrary order. Denote that transition relation on Σ by '$\rightsquigarrow$'. Formally, for any two configurations C_i and C_j, $C_i \rightsquigarrow C_j$ if $u_r(C_i) < u_r(C_j)$, where w_r is the only WS that has different association choices between C_i and C_j.

If there exists no Nash equilibrium, then for any configuration $C_i \in \Sigma$, there must exist some other configuration $C_j \in \Sigma$ such that $C_i \rightsquigarrow C_j$. Since the strategy space is finite, nonexistence of Nash equilibrium implies that there must be a series of configurations $C_1^{'}, C_2^{'}, \ldots, C_p^{'}$, where $p \leq k$, such that $C_1^{'} \rightsquigarrow C_2^{'}$, $C_2^{'} \rightsquigarrow C_3^{'}$, $\ldots, C_p^{'} \rightsquigarrow C_1^{'}$. We shall prove the existence of Nash equilibrium by showing that such series does not exist.

According to [5], all WSs that associate with the same AP receive equal amount of throughput that is governed by the lowest-rate link. Let $t(a)$ be the throughput of any WS residing in AP a. In case that no WS associates with a, we let $t(a)$ be a's real or nominal capacity. For each configuration $C_i \in \Sigma$, let $T_i = (\alpha_i^1, \alpha_i^2, \ldots, \alpha_i^m)$ be an m-tuple of APs such that $t(\alpha_i^1) \leq t(\alpha_i^2) \leq \cdots \leq t(\alpha_i^m)$. Let $\Theta = \{T_1, T_2, \ldots, T_k\}$. We also define a binary relation $\prec$ on Θ as follows. For $T_i, T_j \in \Theta$, we have $T_i \prec T_j$ if $\exists k \in \{1..m\} : t(\alpha_i^k) < t(\alpha_j^k)$ and, if $k > 1$, $\forall l : 1 \leq l < k :: t(\alpha_i^l) = t(\alpha_j^l)$. It is not hard to see that '$\prec$' is a precedence relation [20], i.e., it is antisymmetric and transitive.

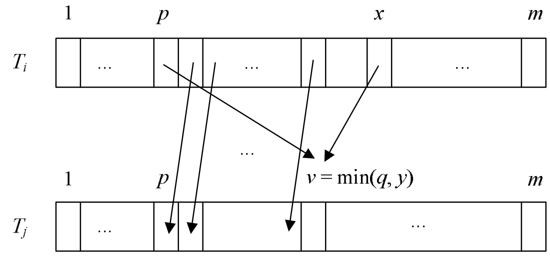

Fig. 2. Rank mapping from T_i to T_j when $v = \min\{q, y\} > p$

Theorem 1: $\forall C_i, C_j \in \Sigma : C_i \rightsquigarrow C_j \Rightarrow T_i \prec T_j$.

Proof: Without loss of generality, assume that $C_i \rightsquigarrow C_j$ because some WS w_r changes its AP from a_k to a_l. Let a_k be the pth and qth element in T_i and T_j, respectively. In other words, $a_k = \alpha_i^p = \alpha_j^q$. Similarly, let $a_l = \alpha_i^x = \alpha_j^y$. $C_i \rightsquigarrow C_j$ implies that $u_r(C_i) < u_r(C_j)$, which in turn implies that

$$t(\alpha_i^p) < t(\alpha_j^y). \tag{2}$$

$t(a_k)$ will be increased due to w_r's association migration, which means

$$t(\alpha_i^p) < t(\alpha_j^q). \tag{3}$$

By (2), (3), and the fact that a_k and a_l are the only two APs whose throughput is changed by $C_i \rightsquigarrow C_j$, the first $p-1$ elements in T_i hold their ranks in T_j. Thus we have

$$\forall s : 1 \leq s \leq p-1 :: t(\alpha_i^s) = t(\alpha_j^s). \tag{4}$$

Now consider the relation between $v = \min\{q, y\}$ and p. By (2), (3), and (4), it is impossible that $v < p$. If $v = p$, then we have the proof by (4) and either (2) or (3). If $v > p$, then α_i^p must change its rank from the pth element in T_i to at least the vth element in T_j, and all APs in between change their ranks accordingly. See Fig. 2. We therefore have

$$\forall s : p \leq s \leq v-1 :: t(\alpha_i^s) \leq t(\alpha_i^{s+1}) = t(\alpha_j^s) \tag{5}$$

and

$$t(\alpha_i^v) = t(\alpha_j^{v-1}) \leq t(\alpha_j^v). \tag{6}$$

If we can eliminate the possibility of condition $\forall s : p \leq s \leq v :: t(\alpha_i^s) = t(\alpha_j^s)$, then the theorem is proven by (4), (5), and (6). The condition at hand holds only if $t(\alpha_i^s) = t(\alpha_i^{s+1})$ for all s, $p \leq s \leq v-1$ and $t(\alpha_i^v) = t(\alpha_j^v)$, which in turn implies that $t(\alpha_i^p) = t(\alpha_j^v)$. The derived result contradicts with (2) and (3), and we thus prove the theorem. ■

Since $\prec$ is a precedence relation, Theorem 1 implies that a configuration transition loop cannot exist, and suffices to be a proof for the existence of Nash equilibrium in our game model.

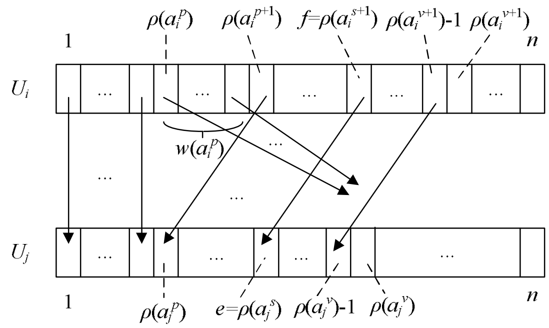

Fig. 3. Rank mapping from U_i to U_j when $v = \min\{q, y\} > p$

B. Fairness

We shall now address the fairness issue of the game. The definition of max-min fairness refers to only one configuration. To quantify the degree of fairness for other feasible configurations, we propose using the lexicographical value of its bandwidth-share tuple as the measurement of a configuration's fairness. More precisely, for each configuration $C_i \in \Sigma$, let $U_i = (\mu_i^1, \mu_i^2, \ldots, \mu_i^n)$ be a tuple of all terminal's utilities (with respect to C_i) sorted in a nondecreasing order. We said that C_j is *fairer* than C_i if $U_i \prec U_j$.

We can derive U_i from T_i by seeing that all WSs associating with the same AP receive identical throughput. Let $w(\alpha_i^k)$ be the number of WSs associating with α_i^k in T_i, where $1 \le k \le m$. Define

$$\rho(\alpha_i^k) = \begin{cases} 1 & k = 1, \\ 1 + \sum_{l=1}^{k-1} w(\alpha_i^l) & 2 \le k \le m. \end{cases} \tag{7}$$

Given T_i, we let each AP $\alpha_i^k \in T_i$, $1 \le k \le m$, map to $w(\alpha_i^k)$ consecutive elements in U_i. These elements, with ordinal numbers ranging from $\rho(\alpha_i^k)$ to $\rho(\alpha_i^{k+1}) - 1$, all have value $t(\alpha_i^k)$.

With the derivation of U_i from T_i, we shall further prove that if $T_i \prec T_j$, then U_j also has a higher lexicographical value than U_i. By Theorem 1, this means that if $C_i \rightsquigarrow C_j$, then C_j is fairer than C_i. The proof is outlined as follows. Since the first $p-1$ APs in T_i hold their ranks in T_j, all the first $\rho(\alpha_i^p) - 1$ elements in U_i also hold their ranks in U_j. If $v = \min\{q, y\} = p$, then the $\rho(\alpha_i^p)$th element in U_i is smaller than the corresponding element in U_j by either (2) or (3), and the proof is done. If $v > p$, then $\alpha_j^p, \alpha_j^{p+1}, \ldots, \alpha_j^{v-1}$ are $\alpha_i^{p+1}, \alpha_i^{p+2}, \ldots, \alpha_i^v$, respectively, since α_i^p changes its rank to at least the vth element in T_j. Therefore we have $\rho(\alpha_j^l) = \rho(\alpha_i^{l+1}) - w(\alpha_i^p)$ for all l, $p \le l < v$. See Fig. 3. From the proof of Theorem 1 we also know that $\exists l : p \le l < v :: t(\alpha_i^l) < t(\alpha_i^{l+1})$. Let s be the smallest such l. Let $d = \rho(\alpha_i^p)$, $f = \rho(\alpha_i^{s+1})$, and $e = \rho(\alpha_j^s) = f - w(\alpha_i^p)$. If $s \ne p$, then $\mu_j^l = \mu_i^l$ for all l, $d \le l < e$. We have the proof by seeing that $\mu_j^e = \mu_i^f > \mu_i^e$.

Although the proof is based on the notion of max-min fairness, in the next section we shall show through simulations that configuration transitions in the native AP selection game also improve bandwidth fairness in terms of balance index.

TABLE II
DISTANCE TO LINK RATE CONVERSION

Range of distance d (m)	Link rate (Mbps)
$0 \le d < 50$	11
$50 \le d < 80$	5.5
$80 \le d < 120$	2
$120 \le d < 150$	1
$d \ge 150$	0

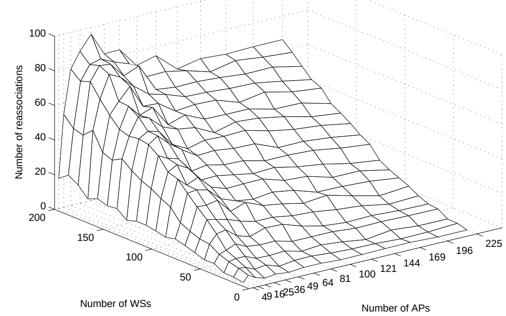

Fig. 4. Number of reassociations before Nash equilibrium

IV. NUMERICAL RESULTS

We conducted additional simulations to demonstrate our theoretical findings. The simulation setting is as follows. APs formed a square grid in a 600×600 (m^2) area with the dimension of the sides of the grid squares set to 2 to 15. Neighboring APs (also a border AP and the border of the area) were separated with equal distance. WSs were randomly uniformly distributed over the same region with the number of WSs varied 10 to 200 in increments of 10. The link rate between a WS and an AP is based on IEEE 802.11b and determined by their in-between distance (Table II). We also preclude unconnected WSs by randomly relocating such WSs. For each setting, 1000 trials were made for an average result.

We let each WS select an AP based on received signal strength initially. Here all APs were assumed identical transmitting power, and a simple path-loss model was adopted where the received signal strength decreases with the square of the traveling distance of the signal. After its initial association, a WS selected an AP to re-associate with for a higher achievable throughput. Fig. 4 shows the total number of reassociations before Nash equilibrium for each setting. As expected, all reassociation activities stopped after a limited number of times. The reassociation time generally increased with the number of WSs. The number of reassociations each WS made has a mean of 0.275 with standard deviation 0.118.

We also measured balance index for each trial twice. The first was after the initial association and the other after the Nash equilibrium. The change of the balance index was considered the improvement of fairness for the trial. Fig. 5 displays the average result. We can see that the degree of improvement generally increased with the number of WSs, particularly when the number of APs was few. The increase

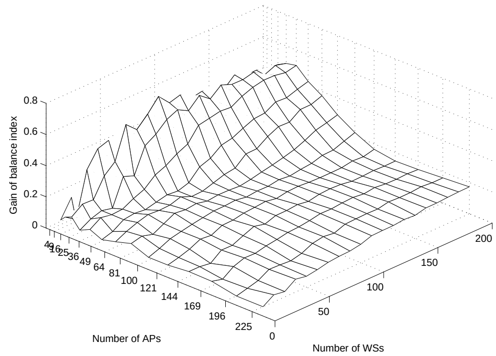

Fig. 5. Change of balance index after reassociations

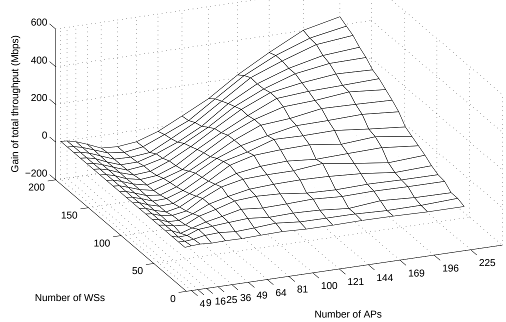

Fig. 6. Change of aggregated throughput after reassociations

of balance index was as high as 0.600, with mean and standard deviation 0.200 and 0.148, respectively.

The change of aggregated throughput (counting all WSs) due to reassociations was also investigated. Refer to Fig. 6. We can see that the activity of associations did not necessarily increase aggregated throughput. We found that for each number of APs, there was an optimal number of WSs for which the improvement of aggregated throughput was maximized. Deviation from this value diminishes the extent of improvement and might even degrade the aggregated throughput. This can be explained as, when the number of WSs is below the optimal value, reassociations help evenly redistribute WSs to other lightly-loaded APs and thus improves aggregated throughput. When the number of WSs is larger than the optimal value, a WS is likely to increase its throughput through associations at the price of decreasing other WS's throughput. Overall throughput may therefore suffer from such associations.

We observed that the optimal number of WSs was in proportional to the number of APs, which is reasonable. In case of 4 APs, the optimal number might be below 10; so reassociations always degraded aggregated throughput. The optimal number for 25, 49, 81, 121, and 169 APs were 30, 50, 90, 130, and 180, respectively.

V. Conclusions

This paper considers native AP selection games where WSs select APs merely to maximize their achievable throughput. We have proven that a Nash equilibrium exists in such games, which guarantees the convergence of configuration transitions. Furthermore, we show that association transitions triggered by selfish WSs in fact improve fairness of bandwidth share. Simulation results confirmed our analytical work.

References

[1] L.-H. Yen, T.-T. Yeh, and K.-H. Chi, "Load balancing in IEEE 802.11 networks," *IEEE Internet Computing*, vol. 13, pp. 30–38, Jan. 2009.

[2] I. Milchtaich, "Congestion games with player-specfic payoff functions," *Games and Economic Behavior*, vol. 13, no. 1, pp. 111–124, 1996.

[3] M. Cesana, N. Gatti, and I. Malanchini, "Game theoretic analysis of wireless access network selection: models, inefficiency bounds, and algorithms," in *Proc. 3rd Int'l Conf. on Performance Evaluation Methodologies and Tools*, Athens, Greece, Oct. 2008.

[4] M. Cesana, I. Malanchini, and A. Capone, "Modelling network selection and resource allocation in wireless access networks with non-cooperative games," in *The 5th IEEE Int'l Conf. on Mobile Ad Hoc and Sensor Systems*, Atlanta, USA, Oct. 2008, pp. 404–409.

[5] M. Heusse, F. Rousseu, G. Berger-Sabbatel, and A. Duda, "Performance anomaly of 802.11b," in *Proc. IEEE INFOCOM*, Mar. 2003, pp. 836–843.

[6] E. Garcia, D. Viamonte, R. Vidal, and J. Paradells, "Achievable bandwidth estimation for stations in multi-rate IEEE 802.11 WLAN cells," in *Proc. IEEE WoWMoM*, Helsinki, Finland, Jun. 2007, pp. 1–8.

[7] H. Gong and J. Kim, "Dynamic load balancing through association control of mobile users in WiFi networks," *IEEE Trans. on Consumer Electronics*, vol. 54, no. 2, pp. 342–348, May 2008.

[8] Y. Fukuda and Y. Oie, "Decentralized access point selection architecture for wireless LANs: Deployability and robustness," in *Proc. IEEE VTC2004-Fall*, Sep. 2004, pp. 1103–1107.

[9] S. Vasudevan, K. Papagiannaki, C. Diot, J. Kurose, and D. Towsley, "Facilitating access point selection in IEEE 802.11 wireless networks," in *Proc. ACM SIGCOMM/USENIX Internet Measurement Conference*, Berkeley, CA, USA, Oct. 2005.

[10] L.-H. Yen and T.-T. Yeh, "SNMP-based approach to load distribution in IEEE 802.11 networks," in *Proc. IEEE VTC2006-Spring*, Melbourne, Australia, May 2006.

[11] O. Brickley, S. Rea, and D. Pesch, "Load balancing for QoS enhancement in IEEE802.11e WLANs using cell breathing techniques," in *IFIP Mobile and Wireless Communication Networks Conf.*, Marrakech, Morocco, Sep. 2005.

[12] Y. Bejerano, S.-J. Han, and L. Li, "Fairness and load balancing in wireless LANs using association control," in *Proc. ACM MobiCom'04*, Oct. 2004, pp. 315–329.

[13] K. Mittal, E. M. Belding, and S. Suri, "A game-theoretic analysis of wireless access point selection by mobile users," *Computer Communications*, vol. 31, no. 10, pp. 2049–2062, 2008.

[14] S. Shakkottai, E. Altman, and A. Kumar, "Multihoming of users to access points in wlans: A population game perspective," *IEEE Journal on Selected Areas in Communications*, vol. 25, no. 6, pp. 1207–1215, Aug. 2007.

[15] W. H. Sandholm, *Population Games and Evolutionary Dynamics*. University of Wisconsin, 2006.

[16] R. Chandra, P. Bahl, and P. Bahl, "MultiNet: Connecting to multiple IEEE 802.11 networks using a single wireless card," in *Proc. IEEE INFOCOM 2004*, Hong Kong, Mar. 2004.

[17] L. Jiang, S. Parekh, and J. Walrand, "Base station association game in multi-cell wireless networks," in *Proc. of IEEE WCNC*, 2008.

[18] D. P. Bertsekas and R. Gallager, *Data networks*. New Jersey, USA: Prentice-Hall, 1992.

[19] D.-M. Chiu and R. Jain, "Analysis of the increase and decrease algorithms for congestion avoidance in computer networks," *Computer Networks and ISDN Systems*, vol. 17, pp. 1–14, 1989.

[20] C. L. Liu, *Elements of Discrete Mathematics*, 2nd ed. McGraw-Hill Inc., 1985, p. 145.