

# On Complexity in Wireless Network Localization

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## Abstract

Considerable literature on wireless network localization has assumed that each wireless station has a spherical radio range. This assumption, however, is generally untenable because wireless signals are subject to physical propagation of electromagnetic waves. In consequence, radio signals become irregular, making position assessment liable to errors. As a remedy, this study presents a convex hull-based localization scheme allowing for radio coverage irregularity. We exploit a mobile anchor node to assist in locating target stations in a non-idealized space containing obstacles. Our scheme operates without reliance upon any ranging measurements between radio transceivers. Qualitative and quantitative comparisons indicate that our approach outperforms counterpart schemes in terms of localization accuracy, yet at the expense of insignificant overhead.

**Keywords:** wireless network, localization, mobile anchor

## I. INTRODUCTION

Localization is essential to wireless network applications such as location-based services or objects tracking that require locating certain targets accurately. GPS (Global Positioning System), despite widespread use, is unavailable indoors and unaffordable for a vast set of legacy devices or lower-end wireless nodes such as wireless sensors. Accordingly, much research effort has been focused on developing other means than GPS with customized functionalities. They are characterized by operational aptness and assumptions for the communication environment in question.

Localization schemes can broadly be categorized as range-based or range-free. The former exploit signal metrics like angle of arrival, time of arrival, time difference of arrival, node-to-node distances, and received signal strength indication (RSSI.) With specific hardware, range-based schemes typically achieve higher accuracy. By contrast, range-free schemes represent another less expensive mainstream with coarse positioning accuracy that operates without resort to ranging but connection information of the network [2], [3], [5]. Among others, grid-scan algorithms were introduced in [5] for fast locating possible areas of interest, so as to improve the accuracy of estimated locations.

Further, the mobile anchor brings another practical dimension to strengthen conventional range-based and range-free

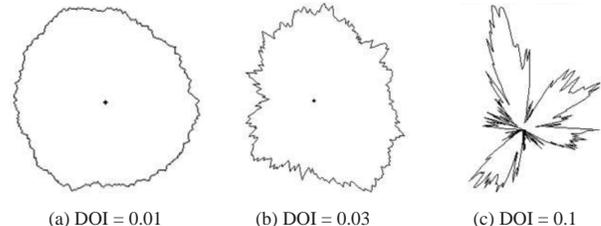


Fig. 1. Radio coverage under different degrees of irregularity (DOI) [11].

schemes. Such mobile anchor-assisted treatment works in a way that a few mobile anchor nodes aware of their positions are required to beacon for localization purpose while moving about, as opposed to range-based schemes demanding a larger number of positioning-capable devices. A smaller number of anchor nodes, compared with traditional range-free schemes, are involved collaboratively in the localization process. Overall this implies a cost-effective solution. For expository surveys on state-of-the-art schemes, we refer the reader to [1].

Considerable studies on wireless network localization assumed a spherical radio range. This assumption, however, does not generally hold in pragmatic situations, because wireless signals are subject to air temperature, humidity, propagation path loss, reflection, diffraction, refraction, and scattering of electromagnetic waves. Consequently, radio coverage becomes irregular, making position assessment liable to nontrivial error [8]. As a remedy, this paper proposes a range-free approach that uses a mobile anchor node to assist locating target stations in a variety of adverse circumstances as illustrated in Fig. 1. Our approach distinguishes itself from counterpart schemes (Table I) in dealing with radio irregularity. Inspired from a convex hull algorithm, our approach computes the most likely circumcenter from a convex polygon representative of radio range. Our design is indeed complementary to previous localization schemes as well.

The remainder of this paper is organized as follows. The next section elaborates on our proposed scheme. Performance evaluation is provided in Section 3. Lastly Section 4 concludes this work.

## II. THE PROPOSED APPROACH

Our approach is primarily based on computational geometry and triangle geometry, as shall be clarified shortly.

TABLE I  
COMPARISON OF MOBILE ANCHOR-BASED SCHEMES

	Sichitiu <i>et al.</i> [6]	Ssu <i>et al.</i> [7]	Liao <i>et al.</i> [4]	Yu <i>et al.</i> [9][10]	Our scheme
Accuracy	low	medium	medium	high	higher
Pre-calibration*	yes	no	no	no	no
Anchor trajectory	flexible	rectilinear	flexible	rectilinear	flexible
Assumed radio range	spherical	irregular	spherical	irregular	irregular
Beacon packets	many	3	3	4	many

\*Pre-calibration is to find a relation of signal strengths to the distances from a reference site, shaping a probability distribution function.

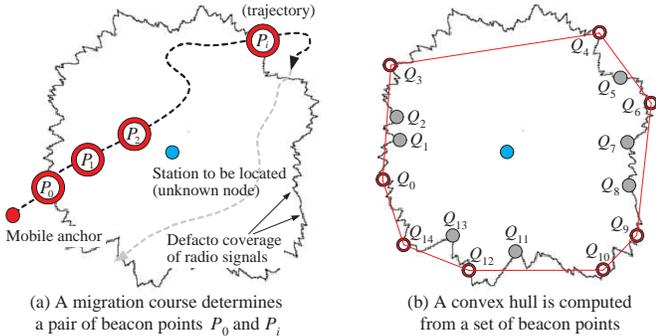


Fig. 2. Selecting a set of beacon points. Collection of beacon points need not be extensive; a few tens would suffice. Beacon points in subfigure (b) result from (a) but are relabeled for uniform presentation.

### A. System Model

Under discussion is a network of stationary stations to be located. A mobile anchor node is tasked to migrate within the network, broadcasting beacon packets periodically in course of movement. Each beacon is of the form  $\langle id, (x, y) \rangle$ , where  $id$  identifies the originating mobile anchor and  $(x, y)$  represents the anchor's current coordinate (say, from GPS readings.) Any concerned station in range receiving beacon packets may keep them for future use. As shown in Fig. 2(a), the station receives a sequence of beacon packets for one course at different times. In the sequence, the initial and the last beacons seen by the station are termed *beacon points* that delimit a course across the coverage of a station. A course concludes when the station cannot receive further beacons after a nominal interval has elapsed; the next received beacon will thus start another new course. As the mobile anchor continues moving about, various migration courses are taking shape, producing a plurality of beacon points for subsequent reference.

### B. Location Estimation

Given a set of  $n$  beacon points, we leverage the well-known Graham scan or Preparata-Hong algorithm to compute the convex hull for these points with time complexity  $O(n \log n)$ . As exemplified in Fig. 2(b), we single out all vertices of the convex polygon along its boundary that signifies the coverage of the centric station. Beacon points internal to the convex are considered less dominant.

With the computed convex hull in place, we proceed to find the longest chord joining two vertices on the hull. With reference to Fig. 2(b), Fig. 3 depicts that  $\overline{Q_3 Q_{10}}$  is the chord

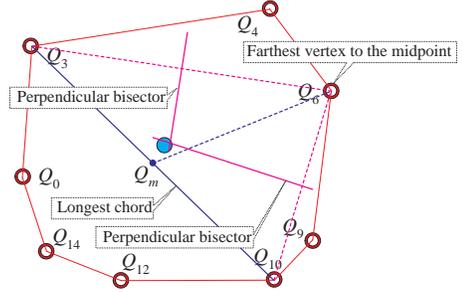


Fig. 3. The longest chord joining two vertices on the convex hull gives a midpoint, from which the farthest vertex is then resolved. The three vertices form a triangle characterizing the circumcenter. The circumcenter is taken for the coordinate of the target station.

in question. We then determine another non-colinear vertex ( $Q_6$  in this example) lying farthest away from the midpoint  $Q_m$  of the chord. These three vertices determine a triangle. Accordingly we assume the triangle's circumcenter to be the location of the target station, i.e., the intersection of any two perpendicular bisectors of the three sides. More specifically, the location is estimated by first selecting three proper vertices. Let  $Q_i = (x_i, y_i)$ ,  $Q_j = (x_j, y_j)$ , and  $Q_k = (x_k, y_k)$  be such selected vertices with known coordinates forming a triangle. Therefore we have two perpendicular bisectors of sides  $Q_i Q_k$  and  $Q_j Q_k$ , respectively:

$$\begin{cases} (x_i - x_k)x + (y_i - y_k)y = \frac{x_i^2 - x_k^2 + y_i^2 - y_k^2}{2} \\ (x_j - x_k)x + (y_j - y_k)y = \frac{x_j^2 - x_k^2 + y_j^2 - y_k^2}{2} \end{cases} \quad (1)$$

Solving the above linear system of equations gives  $(x, y)$ , the circumcenter suggesting the target location.

Note that the longest chord is chosen as the straight line segment closest to the diameter of an envisioned circle indicative of an ideal coverage (represented as a dashed circle in Fig. 5.) The chord makes a best possible reference base whereby localization with joint use of other vertices can be achieved despite adverse effects of shadow fading. The selection of such a chord is essential, as shall be discussed in Section II-C.

Our algorithm is summarized in Fig. 4, where  $\mathcal{B}$  is the collected set of beacon points. Each beacon point  $q$  is of the *coordinate* data type, with  $q.x$  and  $q.y$  denoting the  $x$  and  $y$  positions, respectively, on a plane. In line 4,  $\text{Convex\_Hull}(\mathcal{B})$  is the process of picking vertices out of  $\mathcal{B}$  on the convex polygon into another set  $\mathcal{Q}$ . We mention in passing that  $\zeta$  represents the square of a distance between two coordinates;  $\sqrt{\zeta}$  expresses

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1 var  $\mathcal{B}$ : set of coordinate; // set of beacon points
2  $\varsigma$ : real; // square of distance, initially 0
3 begin
4  $\mathcal{Q} := \text{Convex\_Hull}(\mathcal{B});$  // find the convex hull for  $\mathcal{B}$ 
5 for all  $q \in \mathcal{Q}$  do // find two endpoints of the longest chord
6   for all  $q' \in \mathcal{Q} - \{q\}$  do
7     if  $\varsigma < (q.x - q'.x)^2 + (q.y - q'.y)^2$  then
8       begin
9          $\varsigma := (q.x - q'.x)^2 + (q.y - q'.y)^2;$ 
10         $Q_i, Q_j := q, q'$ 
11      end
12     $\varsigma := 0;$ 
13  for all  $q \in \mathcal{Q} - \{Q_i, Q_j\}$  do // find the third vertex  $Q_k$ 
14    if  $\varsigma < (q.x - \frac{Q_i.x+Q_j.x}{2})^2 + (q.y - \frac{Q_i.y+Q_j.y}{2})^2$  then
15      begin
16         $\varsigma := (q.x - \frac{Q_i.x+Q_j.x}{2})^2 + (q.y - \frac{Q_i.y+Q_j.y}{2})^2;$ 
17         $Q_k := q$ 
18      end
19    Substituting  $Q_i, Q_j,$  and  $Q_k$  into Eq. 1 yields the solution
20 end

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Fig. 4. The proposed localization algorithm.

the in-between length. Our algorithm, however, avoids square root calculation to reduce computational burdens.

### C. Discussion

Let us now investigate the effectiveness of our approach. Fig. 5 depicts the discrepancy between estimate and actual locations of a target station in three scenarios, namely when the length of the median  $\overline{Q_m Q_k}$  is equal to, longer than, or shorter than half the length of the longest chord  $\overline{Q_i Q_j}$ . Fig. 5(a) shows the case where the two line segments are equal in length. In this case, the circumcenter coincides with the station's position. Fig. 5(b) shows that vertices  $Q_i, Q_j,$  and  $Q_k$  form an acute triangle whose circumcenter is enclosed. In contrast, Fig. 5(c) depicts an obtuse triangle with an exterior circumcenter. Fig. 5(b)(c) imply that if reference points for localization were ill chosen (chords deviating from the diameter of the dashed circle), the resulting circumcircle will fall off evidently, causing inaccurate location assessment. Since our scheme operates based on a near-diameter chord, our resolved circumcenter in each scenario of Fig. 5 is not distant from the target location, matching pretty well.

## III. PERFORMANCE

This section provides simulation results to validate our scheme. We developed a multithreaded simulator in Java that mimicked a mobile anchor node moving in a random walk fashion over a  $500 \times 500$  m<sup>2</sup> space, where  $N = 200$  stationary stations were randomly deployed. These  $N$  stations represent unknown nodes to be located. While the communication range  $R$  of each node was assumed to be 20 meters, radio irregularity models as in [11] were adopted here. For accuracy measurement, letting the exact coordinate  $(x_s, y_s)$  of each station  $s$  be known *a priori*, our approach was then carried out to yield the estimate location  $(\hat{x}_s, \hat{y}_s)$  in radio irregularity

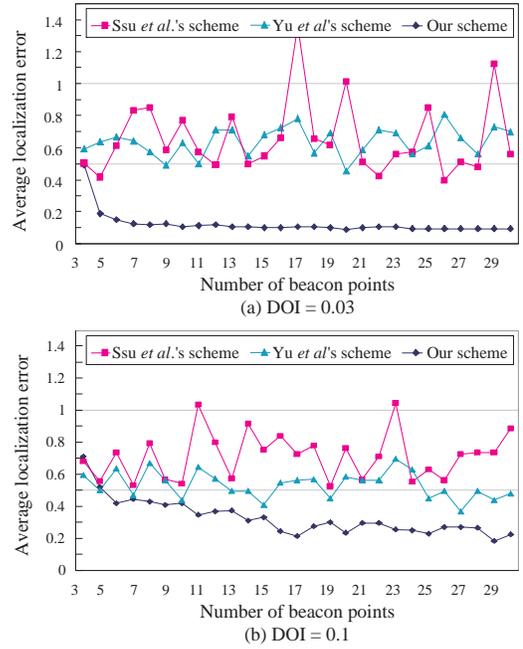


Fig. 6. Localization errors versus numbers of beacon points.

contexts. The in-between discrepancies account for an average localization error, defined by

$$\text{Localization error} = \frac{\sum_{s=1}^N \sqrt{(\hat{x}_s - x_s)^2 + (\hat{y}_s - y_s)^2}}{N \cdot R}. \quad (2)$$

Our approach is compared against Ssu *et al.*'s scheme [7] and Yu *et al.*'s scheme [9][10], arguably best-known localization methods with mobile anchors. Note that every  $(\hat{x}_s, \hat{y}_s)$  of each localization scheme assumes the mean value of 500 estimates resulting from 500 repeated simulations.

Fig. 6 shows how the three subject schemes perform under different degrees of irregularity. Subfigure (a) indicates that on average our approach outperforms Ssu *et al.*'s scheme by around 82% and Yu *et al.*'s scheme by 80%, respectively. Subfigure (b) indicates our outperformance to these two counterpart schemes by 75% and 61%, respectively. Our approach also achieves comparatively stable accuracy throughout. Observe that counterpart schemes bring about fluctuant errors along the horizontal axis, because either underwent different movement trajectories out of 500 simulations and the selected beacon points might thus vary drastically.

Since the availability of more beacon points favors localization accuracy, we are now concerned with how many points are cost effective enough. In case DOI is 0.03, Fig. 7 depicts that no more than 20 among hundreds of beacon points serve as convex-hull vertices (other points interior to the convex hull can be left out.) Further experiments in other DOI settings reflect a similar trend that 20 to 30 beacon points reasonably meet our demands.

In view of the foregoing trend, we next restrict attention to a sample of 26 beacon points. Accordingly, Fig. 8 compares the incurred average localization errors over a range of DOI. This

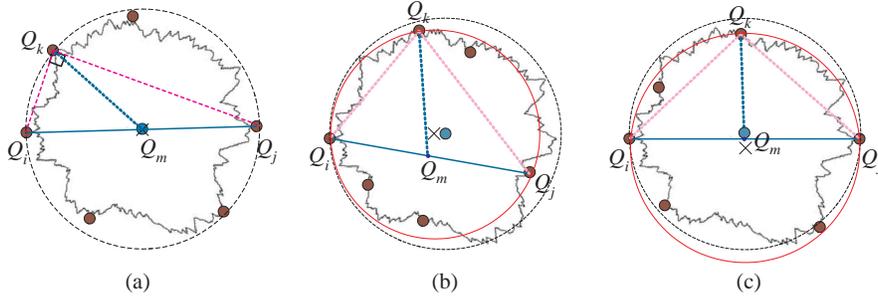


Fig. 5. Illustrating how estimate and actual locations of a target differ when  $\overline{Q_m Q_k}$  is (a) equal to, (b) longer than, or (c) shorter than half  $\overline{Q_i Q_j}$ . The cross ('x') marks the circumcenter, our estimate location. The colored dot at the center of each subfigure represents the actual location.

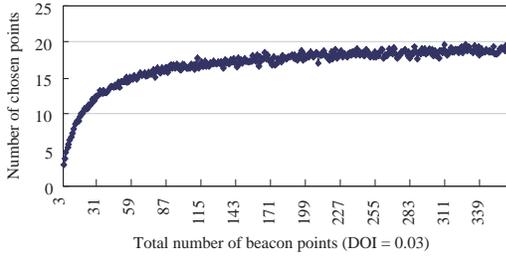


Fig. 7. The number of chosen points by our scheme versus the total number of available beacon points.

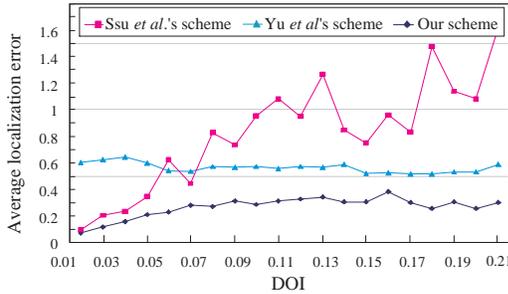


Fig. 8. Localization errors versus DOI.

figure reveals that errors caused by Ssu *et al.*'s scheme increase with DOI, whereas inaccuracy by Yu *et al.*'s scheme appears independent of varying DOI. A conclusive finding is that our approach retains its advantage over counterpart schemes by an appreciable amount. Our average performance gain amounts to 78% and 63%, respectively, relative to Ssu *et al.*'s scheme and Yu *et al.*'s scheme. Besides, our error of estimation is kept from exceeding a certain level in all circumstances, implying reliable performance.

#### IV. CONCLUSION

Location information of wireless network entities is essential for many applications like route discovery, disaster relief, location-based services, and so forth. While localization has become more and more important, there are a multitude of wireless nodes whose correct locations are hard to acquire. In

circumstance that radio signal propagation is influenced under physical conditions, difficulty in accurate signal measurements and sampling complicates the localization process. Therefore, radio irregularity tolerant localization warrants closer study.

This paper presented a mobile anchor-assisted localization scheme with remarkable accuracy that withstands various radio signal irregularity. In our architecture, neither costly hardware nor excessive communication is needed for located stations, making an affordable solution to large quantities of wireless stations or end devices without rich computing resources. Overall performance evaluation concludes that our approach is effective in practice. Our design tenet lends itself to diverse aspects of applications including network assisted handover, the Wi-Fi Positioning System, and object tracking in wireless sensor networks.

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