

Discrete Mathematics



Syllabus

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What is Discrete Mathematics?

- the study of **discrete** (as opposed to continuous) objects.
- Which (do you think) are discrete objects?

characters? **digits?**

integers? **real numbers?**

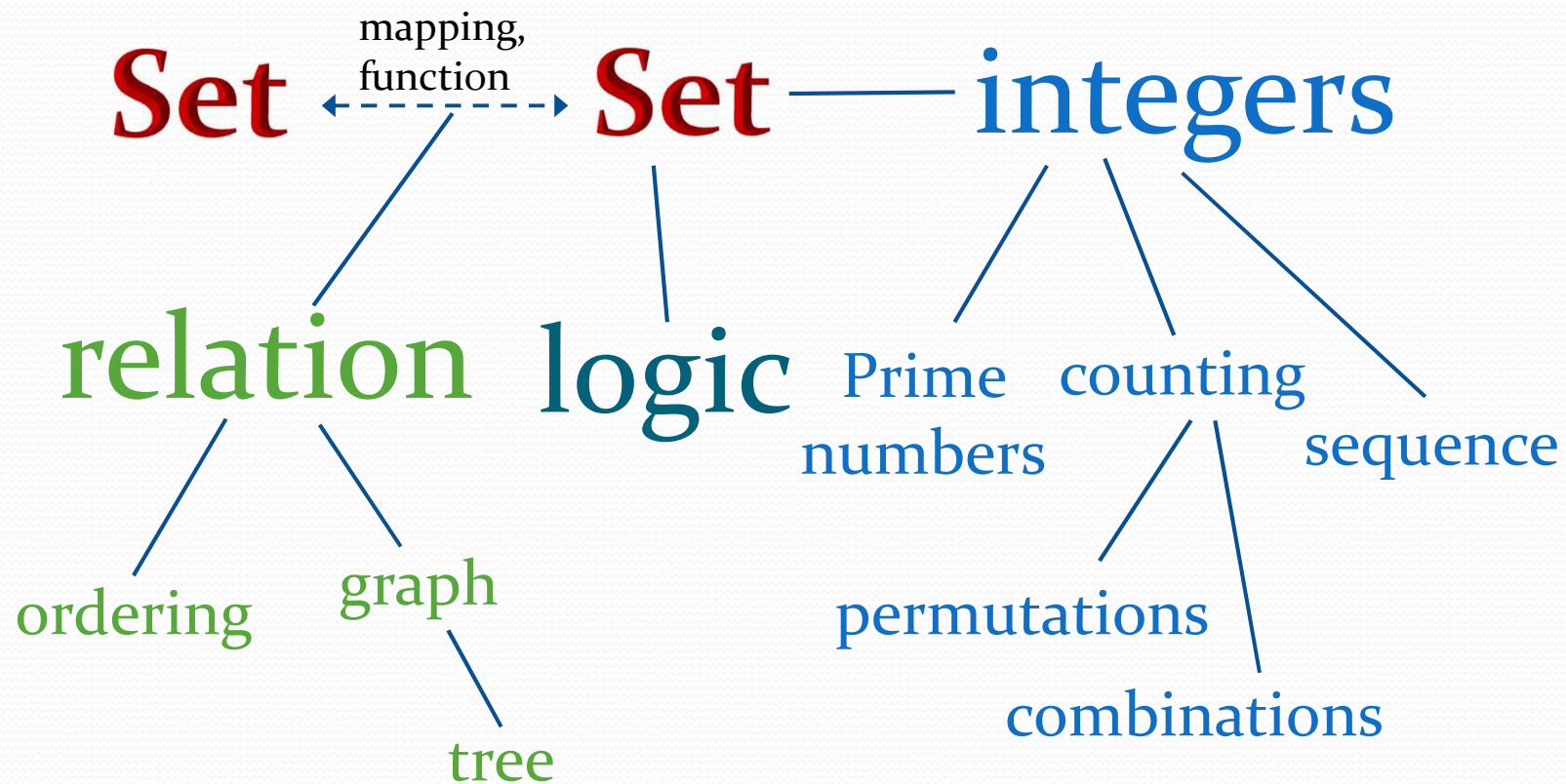
set like {a, b, c}? **time?**

length? **Answer of 'do you like me'?**

What's the difference?

- Discrete objects are countable (could be infinitely many)
 - we *can enumerate* elements of an infinite set S (*list* all elements of S in a sequence)
- Continuous objects are not countable
 - All real numbers in $[0, 1]$
 - All 'time' in 1 second (time can be as small as possible)
 - The area within a circle

Examples of Discrete Objects



Examples of discrete objects

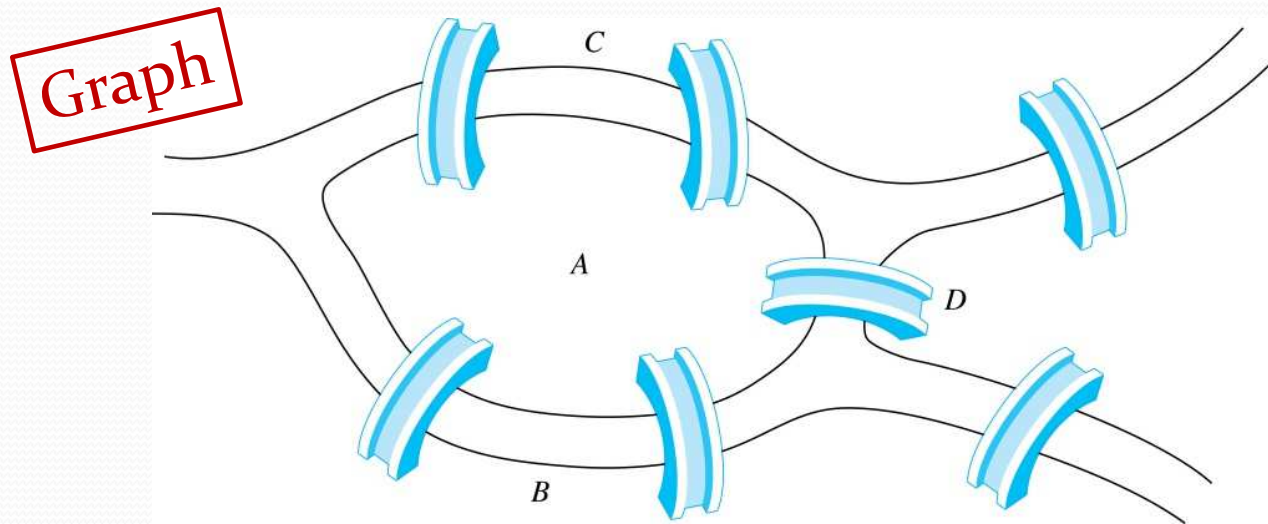
- integers
- steps taken by a computer program
- distinct paths to travel from point A to point B on a map along a road network
- ways to pick a winning set of numbers in a lottery
- ways to choose a password following specific rules
- number of valid Internet addresses
- possible paths between two cities using a transportation system

How much does it matter?

- provides the mathematical background needed for
 - all subsequent courses in **computer science** and
 - all subsequent courses in the many branches of discrete mathematics.

Some Problems Related to Discrete Mathematics (1/4)

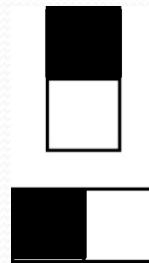
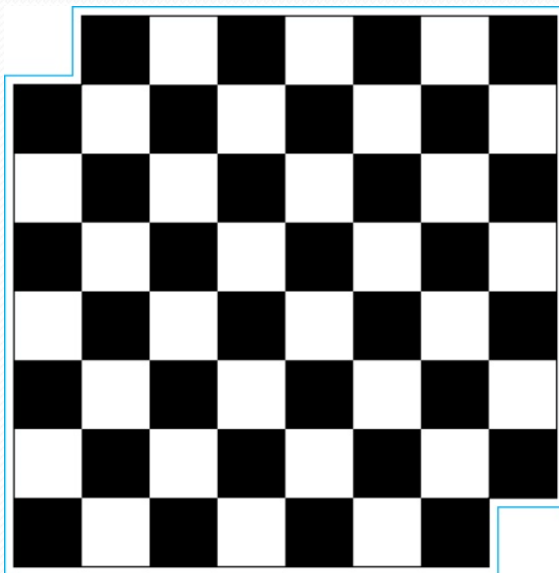
- Can we find a path that **crosses each bridge exactly once** and **returns to the starting point**?



The 7 Bridges of Königsberg

Some Problems Related to Discrete Mathematics (2/4)

- Can we tile the following checkerboard using dominos?

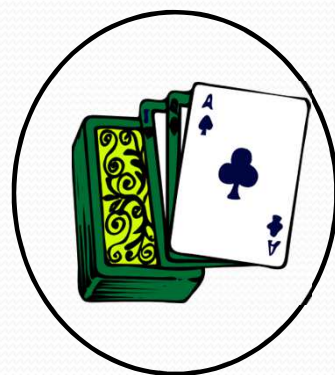


Dominos

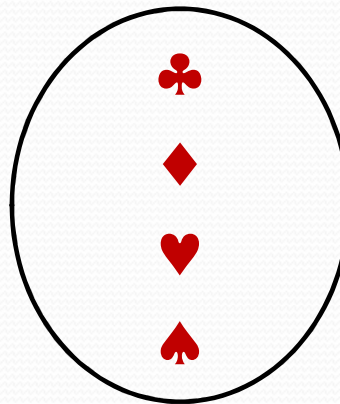
Proof

Some Problems Related to Discrete Mathematics (3/4)

- From a standard deck of 52 cards, how many cards must be selected to guarantee that **at least three cards of the same suit** are chosen?



52
cards



4 suits

Counting

Some Problems Related to Discrete Mathematics (4/4)

- An island has two kinds of inhabitants, *knights*, who always tell the truth, and *knaves*, who always lie.
- You go to the island and meet A and B.
 - A says “B is a knight.”
 - B says “The two of us are of opposite types.”

Question: What are the types of A and B?



Logic

Goals of a Course in Discrete Mathematics

(1/3)

- **Mathematical Reasoning:**
 - ability to read, understand, and construct mathematical arguments and **proofs**.
- **Combinatorial Analysis:**
 - techniques for **counting** objects of different kinds.
- **Discrete Structures:**
 - abstract **mathematical structures that represent objects (sets) and the relationships between them (relations, graphs, trees)**.

Goals of a Course in Discrete Mathematics

(2/3)

- **Algorithmic Thinking:** (not really covered in this course)
 - an algorithm is a **sequence of steps** that can be followed to solve any instance of a particular problem.
- Algorithmic thinking involves
 - **specifying** algorithms
 - **analyzing** the memory and time required by an execution of the algorithm
 - **verifying** that the algorithm will produce the correct answer.

Goals of a Course in Discrete Mathematics

(3/3)

- **Applications and Modeling:** (quite demanding)
 - appreciate and understand the wide range of **applications** of the topics in discrete mathematics
 - develop the ability to **develop new models** in various domains.
- Concepts from discrete mathematics have been used
 - to address problems in computing
 - to **solve** problems in many areas such as chemistry, biology, linguistics, geography, business, etc.

Discrete Mathematics is a Gateway Course

- Topics in discrete mathematics will be important in many courses that you will take in the future:
 - **Computer Science:** Computer Architecture, Data Structures, Algorithms, Programming Languages, Compilers, Computer Security, Databases, Artificial Intelligence, Networking, Graphics, Game Design, Theory of Computation,

Discrete Mathematics is a Gateway Course

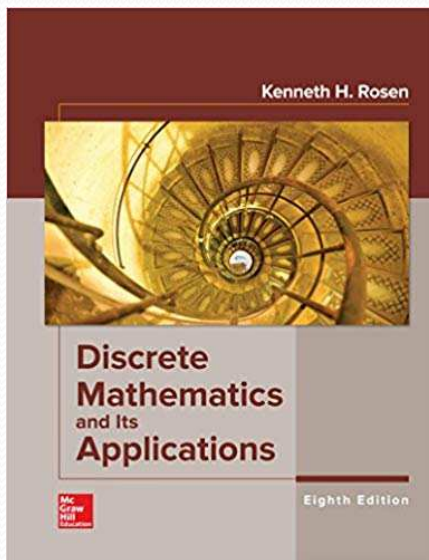
- **Mathematics:** Logic, Set Theory, Probability, Number Theory, Abstract Algebra, Combinatorics, Graph Theory, Game Theory, Network Optimization, ...
 - The concepts learned will also be helpful in continuous areas of mathematics.
- **Other Disciplines:** You may find concepts learned here useful in courses in philosophy, economics, linguistics, and other departments.

Course Enrollment

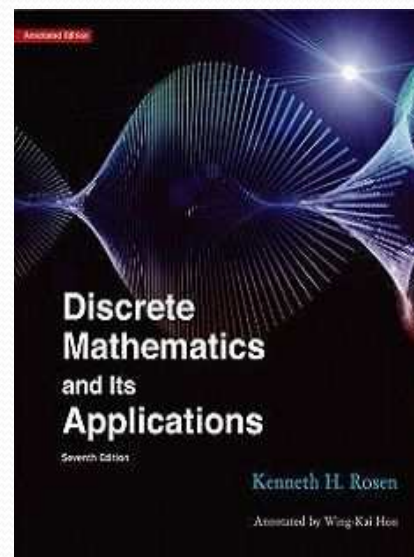
- There may be still some seats left.
- ~~Max. 10% extra seats are available even though the class is full~~
- ~~These extra seats are reserved to students that are retaking this course.~~
- ~~Qualified students should have my signature on the course adding form for approval.~~

Text Book

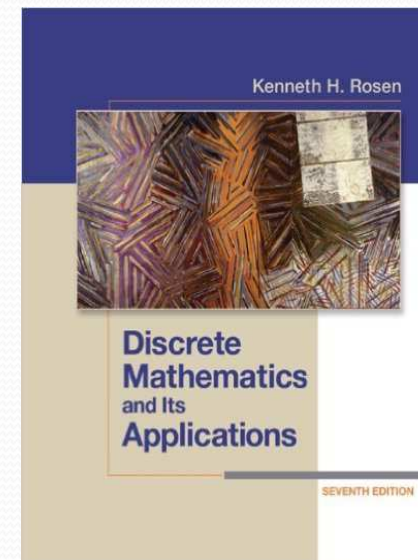
- Kenneth H. Rosen, Discrete Mathematics and Its Applications, 8th Ed., 2019. (7th ed. is fine)



8th ed.



annotated edition (7th ed.)



7th ed.

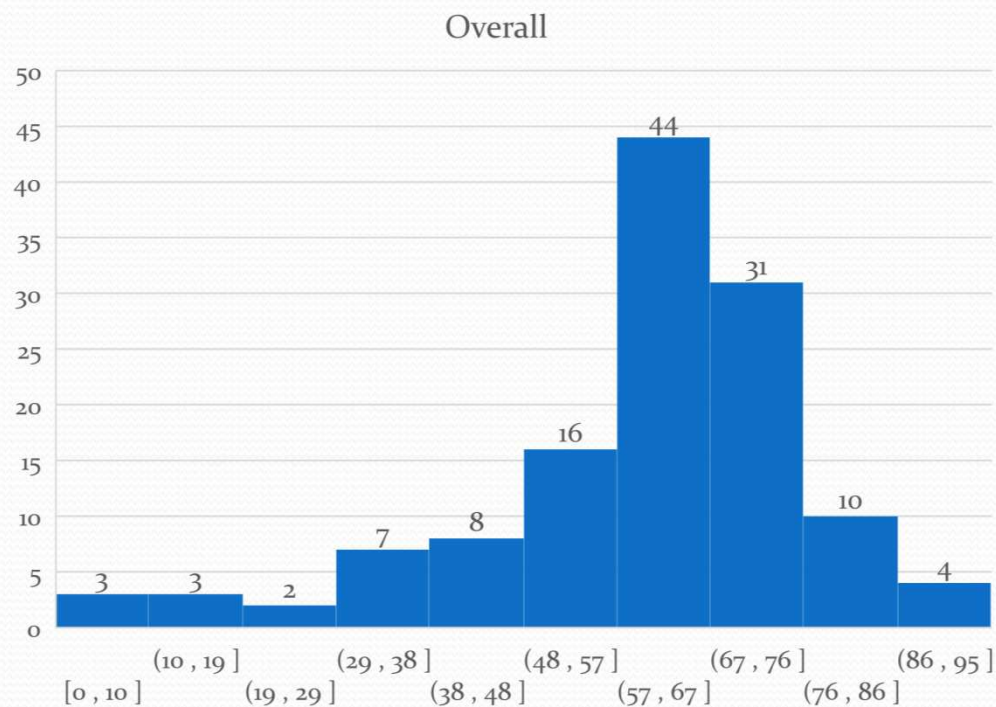
Schedule (tentative)

week	contents	week	contents
1	The Foundations: Logic and Proofs	9	Counting
2	Basic Structures: Sets, Functions, Sequences, Sums, and Matrices (1/2).	10	Advanced Counting Techniques(1/2)
3	Basic Structures: Sets, Functions, Sequences, Sums, and Matrices (2/2).	11	Advanced Counting Techniques(2/2)
4	Number Theory (1/2).	12	Relations (1/2)
5	Number Theory (2/2)	13	Relations (2/2)
6	Induction and Recursion (1/2)	14	Graphs (1/2)
7	Induction and Recursion (2/2)	15	Graphs (2/2)
8	Review and Mid-term Exam.	16	Final Exam.

Scoring Policy

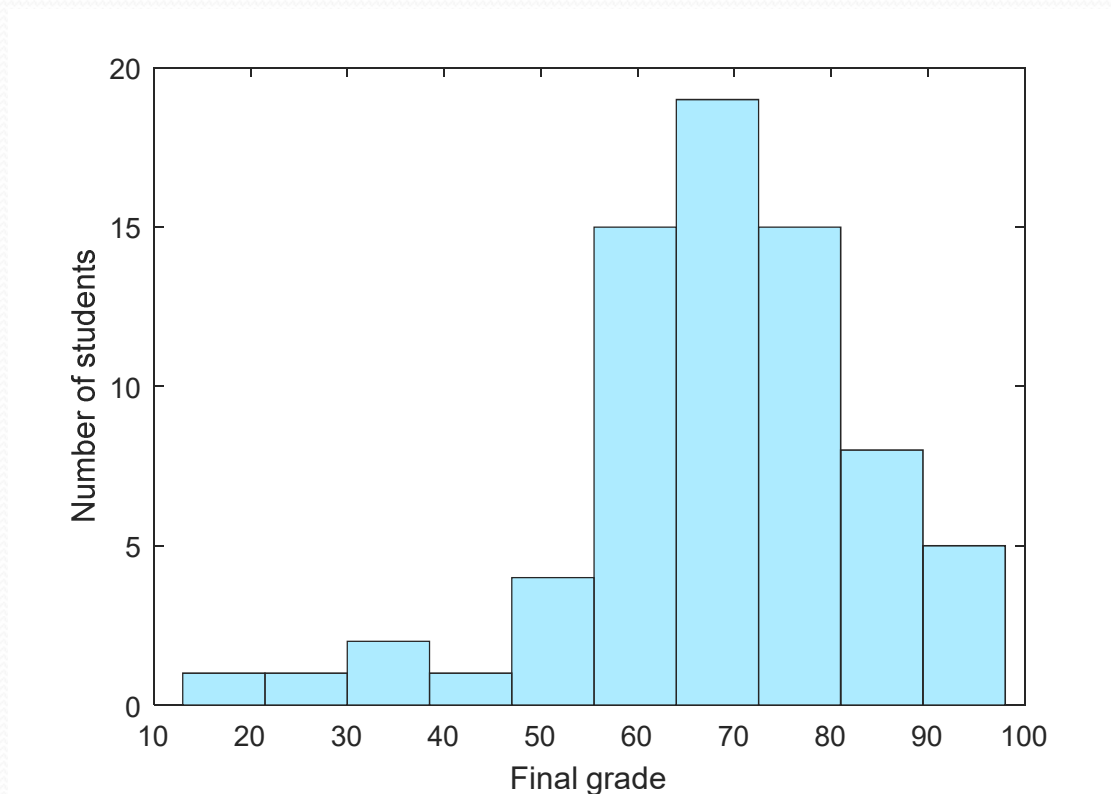
- (0%) self-study exercises
- (40%) quizzes.
 - 7 quizzes in 2017 and 2018. 8 quizzes in 2019 and 2020.
 - 8 quizzes expected this year (2022).
- (30%) Mid-term exam.
- (30%) Final exam.

Statistics (Spring, 2017)



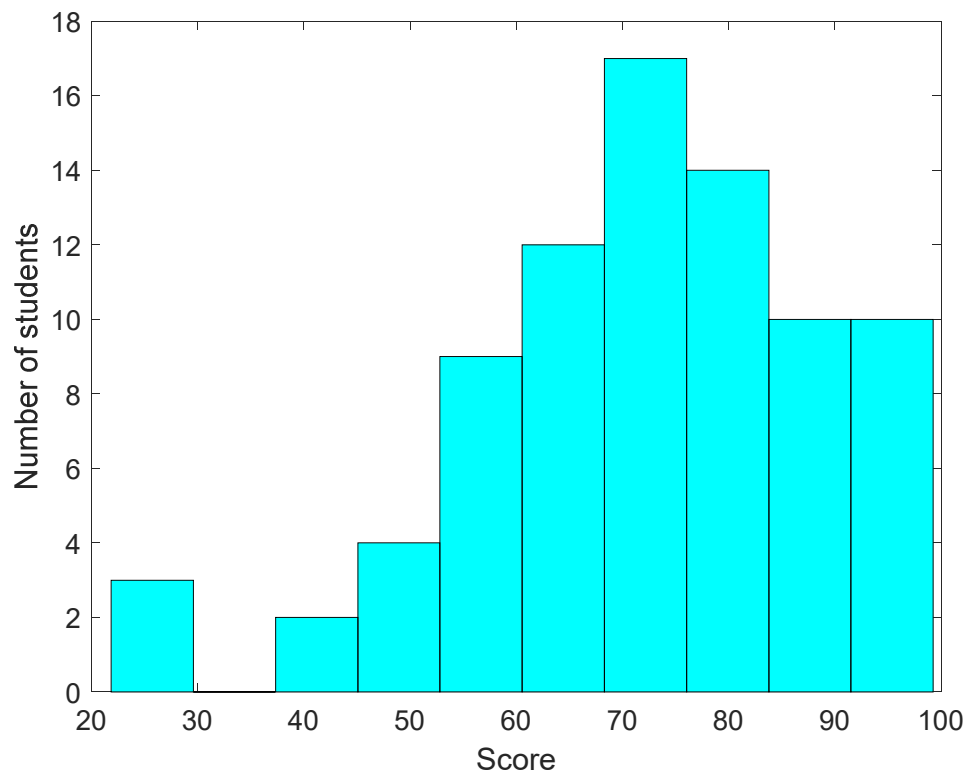
128 students
in total

Statistics (Spring, 2018)



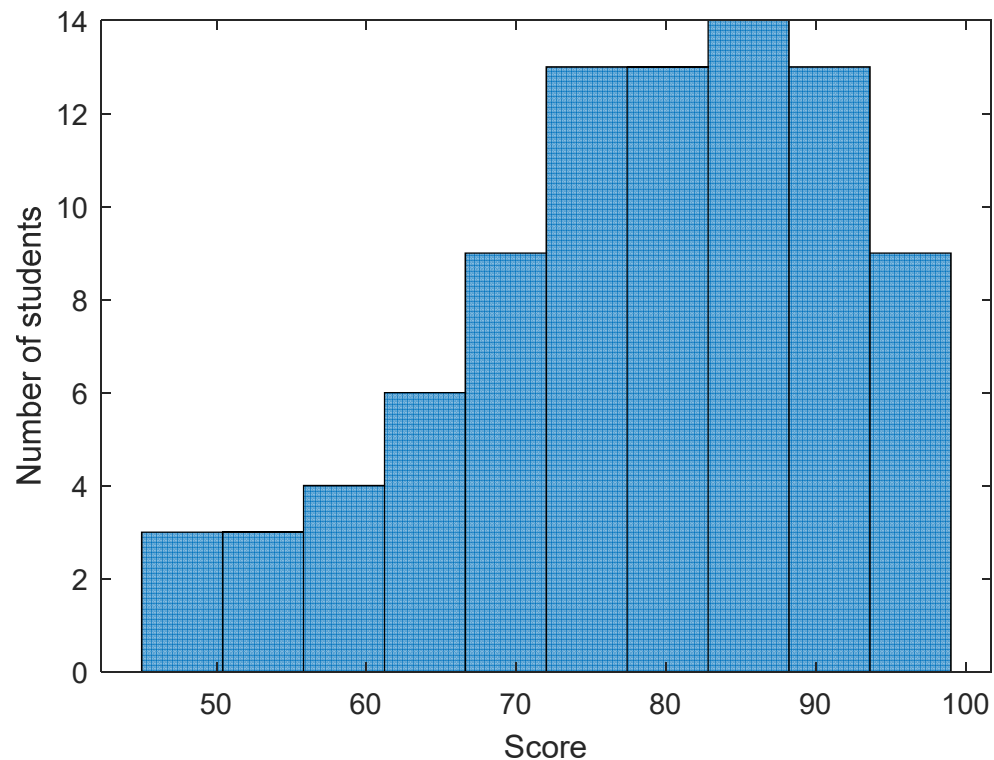
71 students
in total

Statistics (Spring, 2019)



81 students
in total

Statistics (Spring, 2020)



87 students in total
(excluding those
who did not take the
final exam.)

Average = 78.1

Teaching Assistants (TAs)

Name	E-mail	Phone	TA Time/Place
Yu (Mr.)	xcutionrirk@gmail.com	ext. 56674	13:30-15:10 Thr. @ES703B
Simon (Mr.)	shanan2224466@gmail.com	ext. 56674	13:00-15:00 Mon. @ES703B

ES: Microelectronics and Information Systems Research Center (電子與資訊研究中心)

Course Materials

- Slides are placed in new e3 system: <https://e3new.nctu.edu.tw/>
- All announcements are available in new e3 system:
<https://e3new.nctu.edu.tw/>
- Companion website provided by the author of the textbook:
<http://www.mhhe.com/rosen>
- Instructor's e-mail address: lhyen@nctu.edu.tw