Discrete Mathematics

Syllabus

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What is Discrete Mathematics?

- the study of discrete (as opposed to continuous) objects.
- Which (do you think) are discrete objects?

characters? **digits?**

integers? **real numbers?**

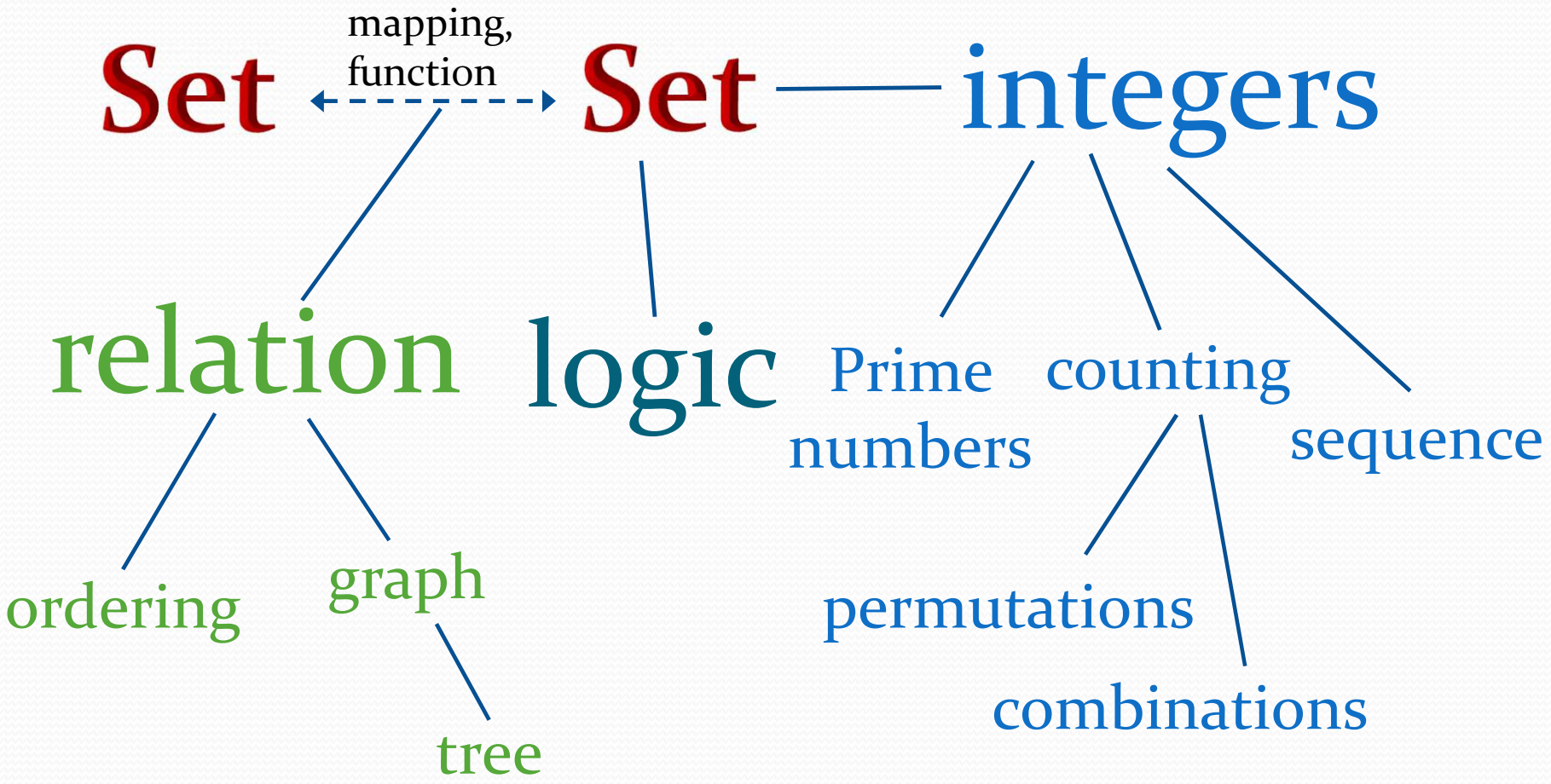
set like {a, b, c}? **time?**

length? **Answer of 'do you like me'?**

What's the difference?

- Discrete objects are countable (could be infinitely many)
  - we can *enumerate* elements of an infinite set S (*list* all elements of S in a sequence)
- Continuous objects are not countable
  - All real numbers in [0, 1]
  - All 'time' in 1 second (time can be as small as possible)
  - The area within a circle

Examples of Discrete Objects



Examples of discrete objects

- integers
- steps taken by a computer program
- distinct paths to travel from point A to point B on a map along a road network
- ways to pick a winning set of numbers in a lottery
- ways to choose a password following specific rules
- number of valid Internet addresses
- possible paths between two cities using a transportation system

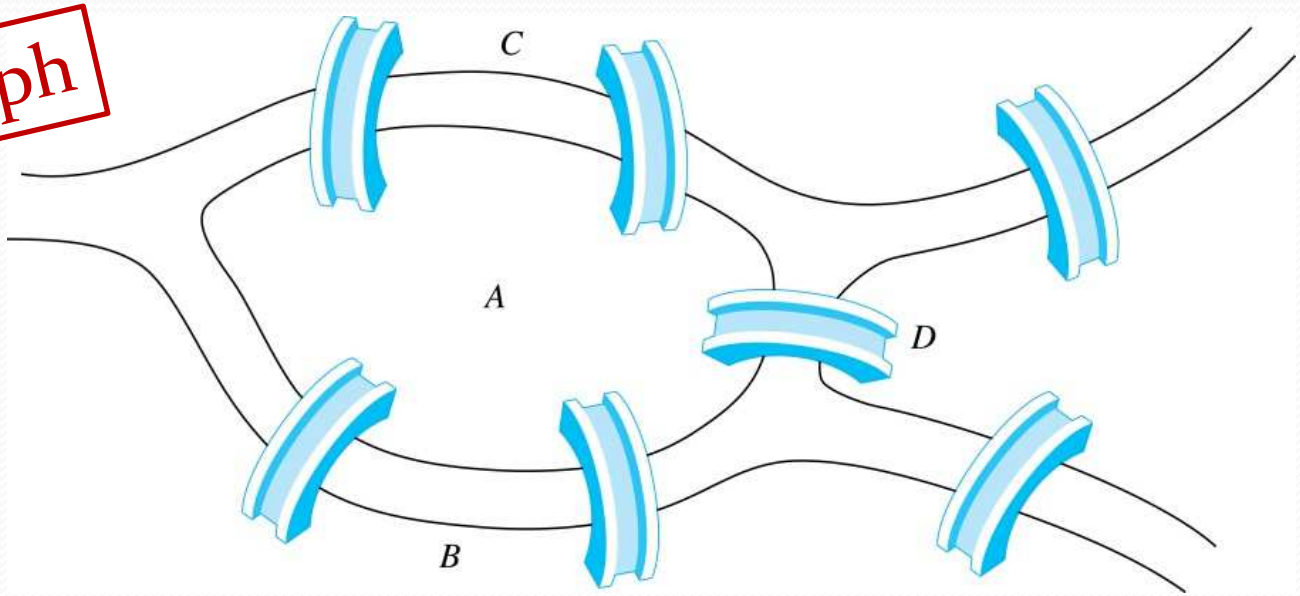
How much does it matter?

- provides the mathematical background needed for
  - all subsequent courses in computer science and
  - all subsequent courses in the many branches of discrete mathematics.

Some Problems Related to Discrete Mathematics (1/4)

- Can we find a path that crosses each bridge exactly once and returns to the starting point?

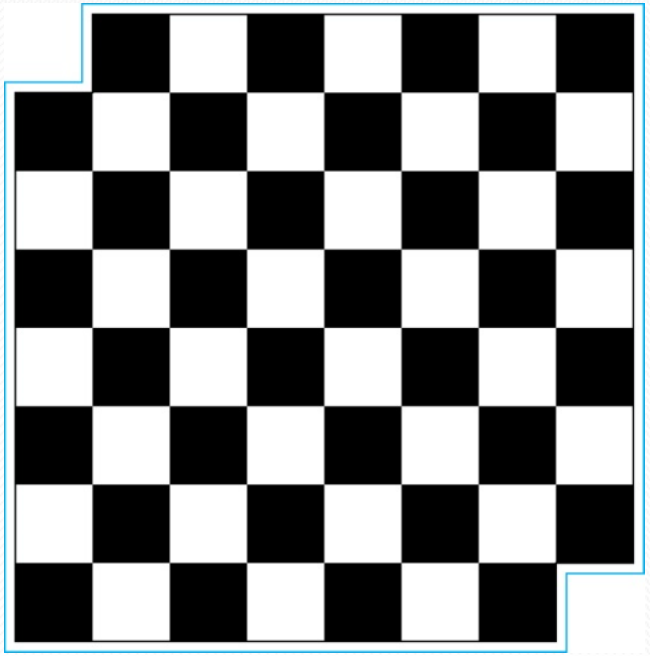
Graph

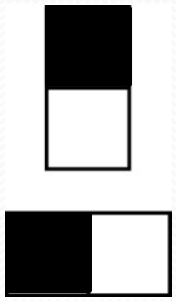


The 7 Bridges of Königsberg

Some Problems Related to Discrete Mathematics (2/4)

- Can we tile the following checkerboard using dominos?



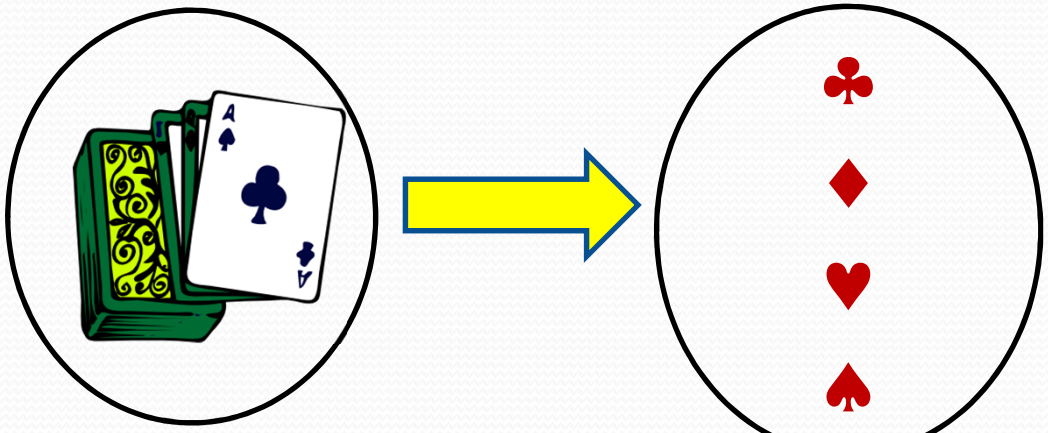


Dominoes

Proof

Some Problems Related to Discrete Mathematics (3/4)

- From a standard deck of 52 cards, how many cards must be selected to guarantee that at least three cards of the same suit are chosen?



52
cards

4 suits

Counting

Some Problems Related to Discrete Mathematics (4/4)

- An island has two kinds of inhabitants, *knights*, who always tell the truth, and *knaves*, who always lie.
- You go to the island and meet A and B.
  - A says "B is a knight."
  - B says "The two of us are of opposite types."

Question: What are the types of A and B?

Logic

Goals of a Course in Discrete Mathematics (1/3)

- **Mathematical Reasoning**:
  - ability to read, understand, and construct mathematical arguments and proofs.
- **Combinatorial Analysis**:
  - techniques for counting objects of different kinds.
- **Discrete Structures**:
  - abstract mathematical structures that represent objects (sets) and the relationships between them (relations, graphs, trees).

Goals of a Course in Discrete Mathematics (2/3)

- **Algorithmic Thinking**:
  - an algorithm is a sequence of steps that can be followed to solve any instance of a particular problem.
- Algorithmic thinking involves
  - specifying algorithms
  - analyzing the memory and time required by an execution of the algorithm
  - verifying that the algorithm will produce the correct answer.

Goals of a Course in Discrete Mathematics (3/3)

- **Applications and Modeling**:
  - appreciate and understand the wide range of applications of the topics in discrete mathematics
  - develop the ability to develop new models in various domains.
- Concepts from discrete mathematics have been used
  - to address problems in computing
  - to solve problems in many areas such as chemistry, biology, linguistics, geography, business, etc.

Discrete Mathematics is a Gateway Course

- Topics in discrete mathematics will be important in many courses that you will take in the future:
  - **Computer Science**: Computer Architecture, Data Structures, Algorithms, Programming Languages, Compilers, Computer Security, Databases, Artificial Intelligence, Networking, Graphics, Game Design, Theory of Computation, ……

Discrete Mathematics is a Gateway Course

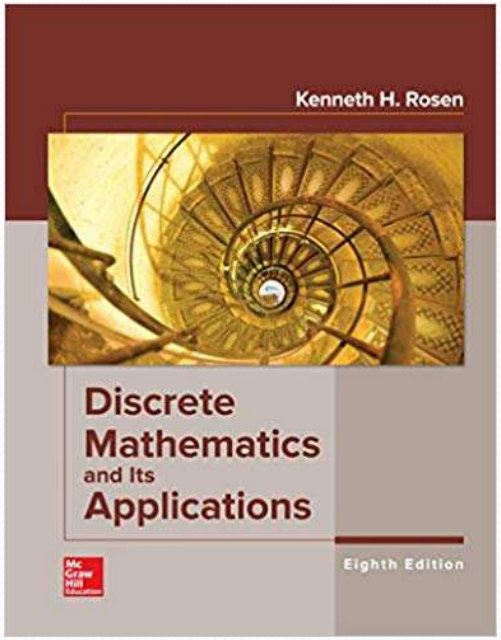
- **Mathematics**: Logic, Set Theory, Probability, Number Theory, Abstract Algebra, Combinatorics, Graph Theory, Game Theory, Network Optimization, …
  - The concepts learned will also be helpful in continuous areas of mathematics.
- **Other Disciplines**: You may find concepts learned here useful in courses in philosophy, economics, linguistics, and other departments.

Course Enrollment

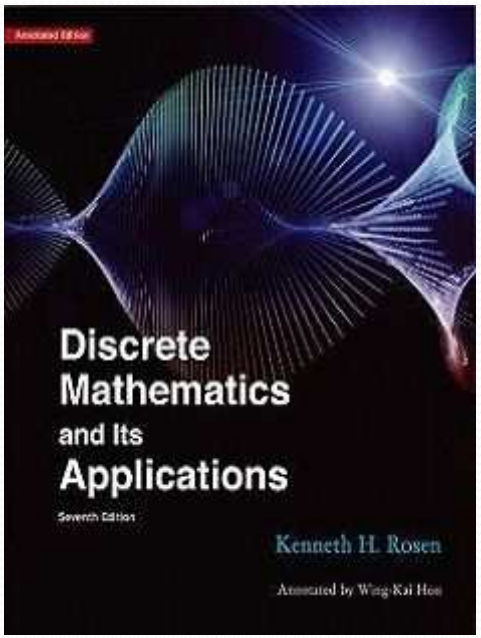
- There may be still some seats left.
- Max. 10% extra seats are available even though the class is full
- These extra seats are reserved to students that are retaking this course.
- Qualified students should have my signature on the course adding form for approval.

Text Book

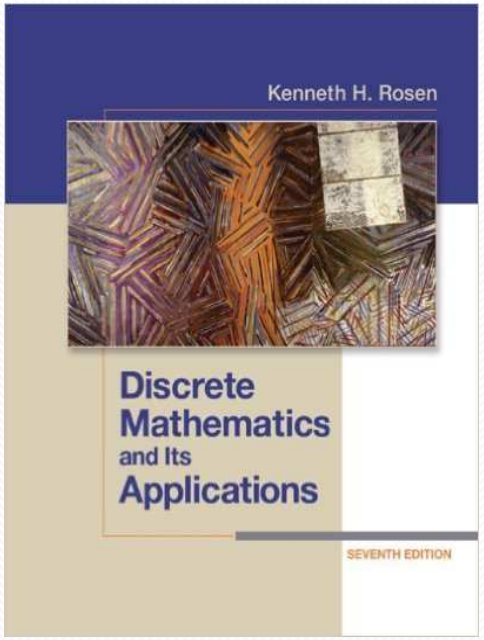
- Kenneth H. Rosen, Discrete Mathematics and Its Applications, 8th Ed., 2019. (7^{th} ed. is fine)



8^{th} ed.



annotated edition (7^{th} ed.)



7^{th} ed.

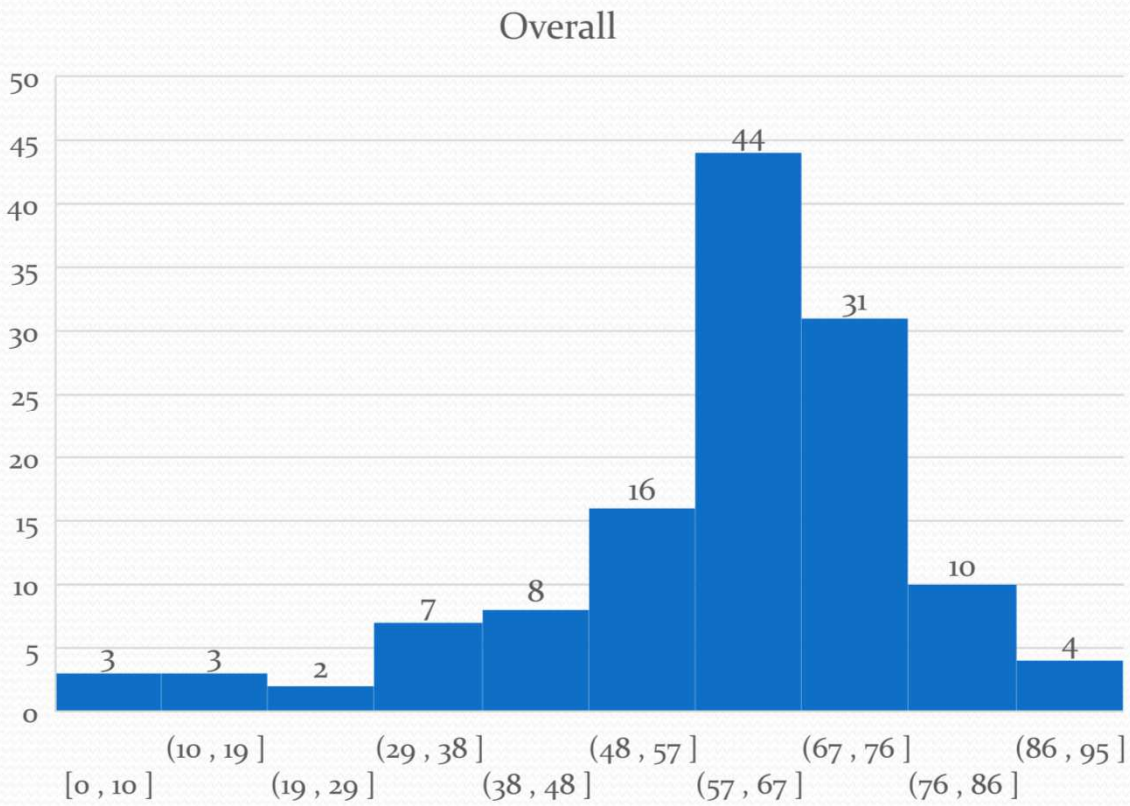
Schedule (tentative)

week	contents	week	contents
1	The Foundations: Logic and Proofs (1/2)	10	Counting (1/2)
2	The Foundations: Logic and Proofs (2/2)	11	Counting (2/2)
3	Basic Structures: Sets, Functions, Sequences, Sums, and Matrices (1/2).	12	Advanced Counting Techniques(1/2)
4	Basic Structures: Sets, Functions, Sequences, Sums, and Matrices (2/2).	13	Advanced Counting Techniques(2/2)
5	Number Theory (1/2).	14	Relations (1/2)
6	Number Theory (2/2)	15	Relations (2/2)
7	Induction and Recursion (1/2)	16	Graphs (1/2)
8	Induction and Recursion (2/2)	17	Graphs (2/2)
9	Review and Mid-term Exam.	18	Final Exam.

Scoring Policy

- (40%) Homework and quizzes.
  - Homework: 6 in 2017, 4 in 2018, 3 in 2019.
  - Mostly self-study homework this year (2020).
  - 7 quizzes in 2017, 2018. 8 quizzes in 2019.
    - 8 or 9 quizzes this year (2020).
- (30%) Mid-term exam.
- (30%) Final exam.

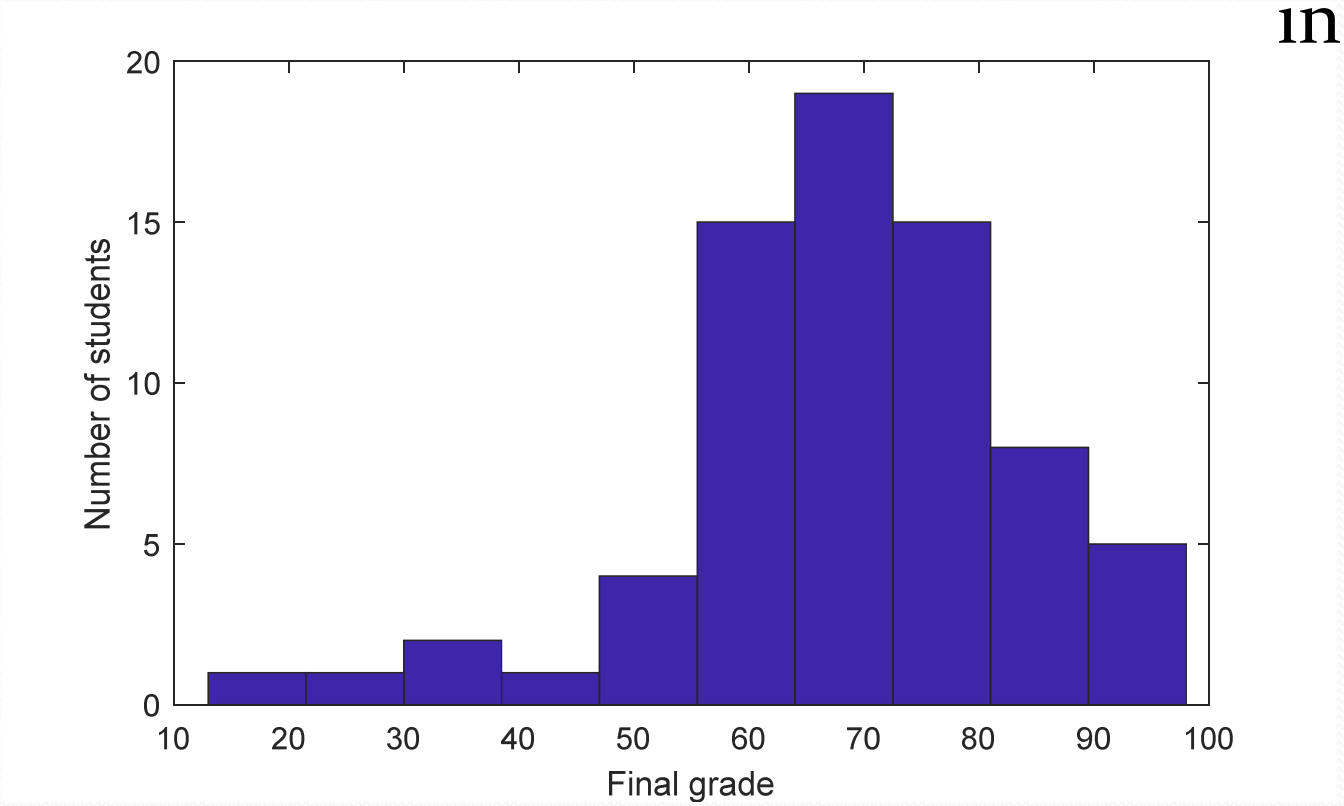
Statistics (Spring, 2017)



128 students in total

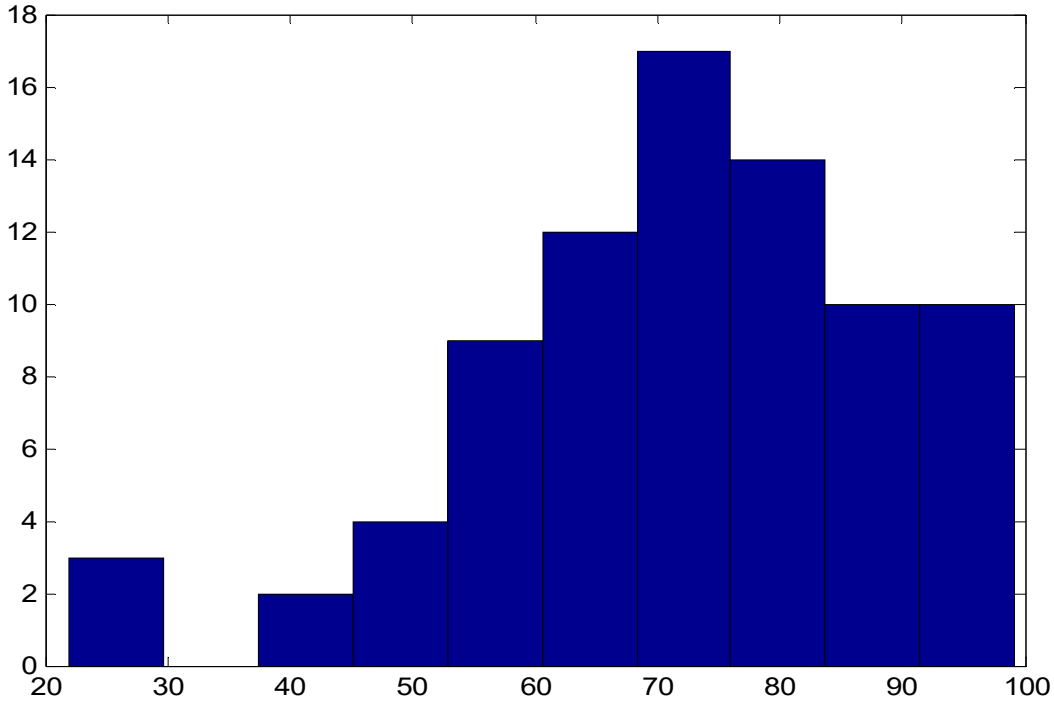
Statistics (Spring, 2018)

71 students in total



Statistics (Spring, 2019)

81 students in total



Teaching Assistants (TAs)

Name	E-mail	Phone	TA Time/Place
Albert	albert860728@gmail.com	ext. 56674	19:00-21:00 Tue. @ES703B
CHEN Feng Yang	st9105323@gmail.com	ext. 56674	13:00-15:00 Thu. @ES703B

ES: Microelectronics and Information Systems Research Center (電子與資訊研究中心)

Course Materials

- Slides are placed in new e3 system: https://e3new.nctu.edu.tw/
- All announcements are available in new e3 system: https://e3new.nctu.edu.tw/
- Companion website provided by the author of the textbook: http://www.mhhe.com/rosen
- Instructor's e-mail: lhyen@nctu.edu.tw