Discrete Mathematics Homework #1

Sec. 1.1

2. Which of these are propositions? What are the truth values of those that are propositions?

- **a)** Do not pass go.
- **b)** What time is it?
- **c)** There are no black flies in Maine.

14. Let p, q, and r be the propositions

p : You get an A on the final exam.
q : You do every exercise in this book.
r : You get an A in this class.

Write these propositions using p, q, and r and logical connectives (including negations).

- **a)** You get an A in this class, but you do not do every exercise in this book.
- **b)** You get an A on the final, you do every exercise in this book, and you get an A in this class.
- **c)** To get an A in this class, it is necessary for you to get an A on the final.
- **d)** You get an A on the final, but you don't do every exercise in this book; nevertheless, you get an A in this class.
- **e)** Getting an A on the final and doing every exercise in this book is sufficient for getting an A in this class.
- **f)** You will get an A in this class if and only if you either do every exercise in this book or you get an A on the final.

18. Determine whether each of these conditional statements is true or false.

- **a)** If $1 + 1 = 3$, then unicorns exist.
- **b)** If $1 + 1 = 3$, then dogs can fly.
- **c)** If $1 + 1 = 2$, then dogs can fly.
- **d)** If $2 + 2 = 4$, then $1 + 2 = 3$.

27. State the converse, contrapositive, and inverse of each of these conditional statements.

- **a)** If it snows today, I will ski tomorrow.
- **b)** I come to class whenever there is going to be a quiz.
- **c)** A positive integer is a prime only if it has no divisors other than 1 and itself.

30. How many rows appear in a truth table for each of these compound propositions?

- **a)** $(q \to \neg p) \vee (\neg p \to \neg q)$
- **b)** $(p \vee \neg t) \wedge (p \vee \neg s)$
- **c)** $(p \to r) \vee (\neg s \to \neg t) \vee (\neg u \to v)$
- **d)** $(p \wedge r \wedge s) \vee (q \wedge t) \vee (r \wedge \neg t)$

Sec. 1.2

In Exercises 1–6, translate the given statement into propositional logic using the propositions provided.

4. To use the wireless network in the airport you must pay the daily fee unless you are a subscriber to the service. Express your answer in terms of w: "You can use the wireless network in the airport," d: "You pay the daily fee," and s: "You are a subscriber to the service."

12. Are these system specifications consistent? "If the file system is not locked, then new messages will be queued. If the file system is not locked, then the system is functioning normally, and conversely. If new messages are not queued, then they will be sent to the message buffer. If the file system is not locked, then new messages will be sent to the message buffer. New messages will not be sent to the message buffer."

Exercises 19–23 relate to inhabitants of the island of knights and knaves created by Smullyan, where knights always tell the truth and knaves always lie. You encounter two people, A and B. Determine, if possible, what A and B are if they address you in the ways described. If you cannot determine what these two people are, can you draw any conclusions?

20. A says "The two of us are both knights" and B says "A is a knave."

Sec. 1.3

8. Use De Morgan's laws to find the negation of each of the following statements.

- **a)** Kwame will take a job in industry or go to graduate school.
- **b)** Yoshiko knows Java and calculus.
- **c)** James is young and strong.
- **d)** Rita will move to Oregon or Washington.

13. Use truth tables to verify the absorption laws.

- **a)** $p \vee (p \wedge q) \equiv p$
- **b)** $p \wedge (p \vee q) \equiv p$

Each of Exercises 16–28 asks you to show that two compound propositions are logically equivalent. To do this, either show that both sides are true, or that both sides are false, for exactly the same combinations of truth values of the propositional variables in these expressions (whichever is easier).

17. Show that $\neg(p \leftrightarrow q)$ and $p \leftrightarrow \neg q$ are logically equivalent.

61. Determine whether each of these compound propositions is satisfiable.

- **a)** $(p \vee \neg q) \wedge (\neg p \vee q) \wedge (\neg p \vee \neg q)$
- **b)** $(p \to q) \wedge (p \to \neg q) \wedge (\neg p \to q) \wedge (\neg p \to \neg q)$
- **c)** $(p \leftrightarrow q) \wedge (\neg p \leftrightarrow q)$